A METHOD FOR SELECTING HAZARD-CONSISTENT GROUND MOTIONS AND ESTIMATING SEISMIC DEMAND HAZARD CURVES

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Abstract

This paper presents a method to select unscaled ground motions for estimating seismic demand hazard curves (SDHCs) in performance-based earthquake engineering (PBEE). Currently, SDHCs are estimated from a probabilistic seismic demand analysis (PSDA), where several ensembles of ground motions are selected and scaled to a user-specified scalar conditioning intensity measure (IM). In contrast, the method presented herein provides a way to select a single ensemble of unscaled ground motions for estimating the SDHC. In the context of unscaled motions, the proposed method requires three inputs: (i) a database of unscaled ground motions, (ii) a small vector of IMs for selecting ground motions, and (iii) the number of ground motions to be selected, \( n \); in the context of scaled motions, two additional inputs are needed: (i) a maximum acceptable scale factor, \( SF_{\text{max}} \), and (ii) a target fraction of scaled ground motions, \( \gamma \). Using a recently developed approach for evaluating ground motion selection and modification procedures, the proposed method is evaluated and is demonstrated to provide accurate estimates of the SDHC when ground motions are unscaled.

Keywords: selection of unscaled ground motions; Importance Sampling; hazard consistency; seismic demand hazard curve
1. Introduction

In performance-based earthquake engineering, response history analyses (RHAs) of structural models are typically performed for three different contexts: (i) intensity-based assessment, (ii) scenario-based assessment, and (iii) risk-based assessment [1, 2]. This paper focuses on a risk-based assessment, or a probabilistic seismic demand analysis (PSDA), which is the most comprehensive context among the three. The primary output of a PSDA is a plot of the annual rate of exceedance, \( \lambda \), as a function of the seismic demand, or engineering demand parameter (EDP); such a plot is referred to as a seismic demand hazard curve (SDHC), which is unique for a given structure at a given site [3]. In essence, an SDHC is computed from a PSDA by selecting several ensembles of ground motions and scaling each ensemble to a user-specified scalar conditioning intensity measure (IM); a detailed explanation of this approach may be found elsewhere (e.g., Section 2 of [4], among others).

The objective of performing RHAs in this study is to estimate the SDHCs of a given structure at a particular site. Once constructed, the SDHC may be used to determine the seismic demand for a given annual rate of exceedance, or conversely, the exceedance rate for a given structural capacity. Furthermore, the SDHC may be integrated with fragility and consequence functions to estimate hazard curves for damage and loss, respectively (see e.g., Chapter 9 of [5]).

There are three issues to the existing approach for computing SDHCs from a PSDA. First, the choice of the scalar conditioning IM may not be obvious for some structures, and yet it determines the influence from other IMs on the EDP [6]. Second, ground motions selected from a PSDA are almost always amplitude scaled to various levels of the scalar conditioning IM, potentially causing bias in the resulting demands. Third, several ensembles of scaled ground motions are typically needed to determine the SDHC for a wide range of exceedance rates.

The choice of the scalar conditioning IM and the effects of amplitude scaling are relatively unimportant when ground motions are carefully selected to be consistent with the seismic hazard [3, 6]. More formally, the SDHCs are unbiased—irrespective of the extent of record scaling—when the corresponding ground motions are hazard-consistent with respect to IMs that are “sufficient” for the EDP in question [4, 7]. An IM, which may be scalar or vector valued, is defined to be sufficient with respect to an EDP when the EDP is influenced only by this IM and no other features of the ground motion [8, 9]. Because it is difficult to identify IMs that are sufficient for EDPs of a complex, realistic structure, a method that permits selection of unscaled ground motions is desirable.

The method presented herein permits selection of a single ensemble of unscaled ground motions to estimate the SDHC. First, instead of a single scalar conditioning IM, a small vector of IMs is chosen (on the basis of the structure and EDPs considered) for selecting ground motions. Next, the theoretical probability distribution of this vector-valued IM, derived from probabilistic seismic hazard analysis (PSHA), is employed as the target for ground motion selection. By selecting a single ensemble of unscaled ground motions to be consistent with this target (utilizing the concept of Importance Sampling from statistics), hazard consistency is directly enforced. Finally, a case study is chosen to illustrate the proposed method.

2. Ground motion selection method

2.1 Overview

There are five inputs to the proposed method: (i) a database of prospective ground motions; (ii) a small vector of IMs for selecting ground motions, \( \text{IM} \); (iii) a maximum acceptable scale factor, \( SF_{\text{max}} \); (iv) a target fraction of scaled ground motions, \( \gamma \); and (v) the number of ground motions to be selected, \( n \). The database of prospective ground motions should contain unscaled motions that originate from tectonic environments, soil conditions, earthquake magnitudes, and source-to-site distances that are similar to those expected for the site under
consideration [10]. (In this paper, the proposed method is applied to synthetic ground motions in order to preliminarily evaluate its ability to estimate SDHCs.) The vector of IMs should be chosen based on our understanding of the dynamics of the structure; to reflect higher-mode, inelastic, and duration-sensitive response, the recommended vector is \( \text{IM} = \{ A(T_1), A(T_2), A(2T_1), D_{5-75} \} \), where \( T_1 \) and \( T_2 \) refer, respectively, to the first and kth mode of the structure, and \( D_{5-75} \) is the 5-75% significant duration. The parameters \( SF_{\text{max}} \) and \( \gamma \) should be chosen based on the sufficiency of IM: if IM is judged to be sufficient, then large scale factors may be employed; otherwise, the degree of scaling should be minimized. Finally, the number of ground motions \( n \) should be chosen based on a tolerable level of epistemic uncertainty in the SDHC estimates.

Fig. 1 presents a schematic of the proposed ground motion selection method. First, the analyst specifies the five inputs to the method and then performs PSHA of the given site to determine the target for ground motion selection. The target multivariate probability density function (PDF) for IM from PSHA is (see Section 2.1 of [11])

\[
   f_{\text{IM}}(x) = \sum_{i=1}^{N_{\text{rup}}} \frac{\nu(rup_i)}{v_0} \cdot f_{\text{IM}|rup}(x | rup_i)
\]

where \( N_{\text{rup}} \) is the total number of rupture scenarios considered in the PSHA, \( \nu(rup_i) \) is the annual rate of the \( i \)th rupture scenario occurring, \( v_0 \) is the annual rate of any rupture occurring, and \( f_{\text{IM}|rup}(x | rup_i) \) is the conditional multivariate PDF of IM given the \( i \)th rupture; the latter is assumed to be a multivariate lognormal PDF, whose mean vector and covariance matrices are determined with ground motion prediction models (GMPMs) and correlation models. Based on the user inputs, an Importance Function (IF) is constructed for IM (Section 2.2) and is then employed to select an ensemble of ground motions (Section 2.3). Next, the Importance Sampling weights, \( w_i \), computed from the selected ground motions, are used to confirm hazard consistency of the selected motions; if hazard consistency is judged to be unsatisfactory, then new ground motions may be reselected from the proposed method (Section 2.4). Finally, nonlinear RHAs of the structural model subjected to the final choice of ground motions are performed, and the results are combined with the Importance Sampling weights \( w_i \) to compute the SDHC.

Fig. 1 – Schematic of proposed method to select ground motions (adapted from [11]).

2.2 Database-driven Importance Function

The purpose of the IF in Fig. 1 is to select ground motions that are intense while simultaneously consistent with the target defined by Eq. (1). The proposed procedure achieves this goal in two steps: (i) randomly generate a single realization of IM from the IF and (ii) select the ground motion from the specified database whose corresponding value of IM agrees most closely to the one generated from the IF. Since ground motions are...
selected from a user-specified database of prospective motions, this database plays a major role in fulfilling the purpose of the IF. Because the database of prospective motions may be effectively enlarged by allowing ground motions to be scaled, we recommend an IF that is controlled by three inputs: (i) the specified database of unscaled ground motions, (ii) $SF_{\text{max}}$, and (iii) $\gamma$.

If ground motions are restricted to be unscaled, the proposed IF is a multivariate lognormal distribution whose parameters are determined from the database of unscaled ground motions; this IF is denoted by $g_u(x)$. To determine the mean vector and covariance matrix of this multivariate distribution — $\mu_{IF}$ and $\Sigma_{IF}$ — we first consider the corresponding marginal distributions. The marginal distribution of the $j$th IM of $\text{IM}$ is lognormal; its two parameters — $\mu_{IF,j}$ and $\sigma_{IF,j}$ — are computed from the mean and standard deviation of the observed values of $\ln(IM_j)$ in the database. An example of this lognormal distribution is shown in Fig. 2a, where the fitted complementary cumulative distribution function (CCDF) is compared with the empirical CCDF from the database of ground motions. Next, the correlation between the $j$th and $k$th IM, denoted by $\rho_{IF,jk}$, is computed from the correlation between the observed values of $\ln(IM_j)$ and $\ln(IM_k)$ in the database; the covariance between the $j$th and $k$th IM is then given by $\rho_{IF,jk}\sigma_{IF,j}\sigma_{IF,k}$. Repeating such calculations for all elements of $\text{IM}$ yields the desired mean vector and covariance matrix of $g_u(x)$.

The approach described in the preceding paragraph may also be applied to determine an IF when ground motions are all scaled by the same scale factor. When the given database of ground motions is deemed to lack an adequate number of strong ground motions, judged by comparing $g_u(x)$ against $f(x)$, it is natural to consider scaling the motions upwards. For example, suppose that it is desired to scale all the motions in the database by a factor of $SF_{\text{max}}$ in order to create a new database of prospective motions. The IMs computed from scaled ground motions may be different, depending on the type of IM (e.g., spectral acceleration increases linearly with scale factor, whereas many common measures of duration are unaffected by scaling) [12]. Applying the approach in the preceding paragraph to the new database of scaled ground motions leads to the IF denoted by $g_s(x)$, which is illustrated in Fig. 2b. Observe that $g_s(x)$ differs from $g_u(x)$ only in its mean vector; the covariance matrices are the same in both cases.

In general, the recommended IF is a two-component mixture of multivariate lognormals:
\[ g(x) = [1 - \gamma] \cdot g_u(x) + \gamma \cdot g_s(x) \]  \hspace{1cm} (2)

where \(0 \leq \gamma \leq 1\) may be interpreted as the fraction of scaled ground motions in the ensemble of \(n\) selected motions. An example of this IF is shown by the solid curve in Fig. 3a; for comparison, its two individual components — \(g_u(x)\) and \(g_s(x)\) — and the target PDF \(f(x)\) (Eq. (1)) are also shown. This two-component mixture distribution permits the selection of ground motions that are scaled by factors between unity and \(SF_{\text{max}}\), where the fraction of scaled motions is roughly controlled by \(\gamma\) (Fig. 3b). For instance, when \(\gamma = 0\), the general two-component IF reduces to \(g_u(x)\), whereas when \(\gamma = 1\), it reduces to \(g_s(x)\). Note that when \(SF_{\text{max}} = 1\), the general two-component IF also reduces to \(g_u(x)\), irrespective of \(\gamma\).

Fig. 3 – Illustration of the two-component Importance Function: (a) comparison of \(g(x)\) with \(\gamma = 0.5\) against its two individual components and the target PDF, and (b) the effect of \(\gamma\) on \(g(x)\) (adapted from [11]).

### 2.3 Selection and scaling of ground motions from randomly generated intensity measures

The IF is used to randomly generate \(n\) realizations of \(\text{IM}\), each denoted by \(\text{IM}_{IF}\), which in turn are used to select a corresponding ensemble of \(n\) ground motions. Consequently, the ability to randomly generate a vector of IMs is an important consideration in specifying the IF. When the IF is specified as either \(g_u(x)\) or \(g_s(x)\), a vector of IMs can be readily generated from the multivariate lognormal distribution (e.g., by using the \text{mvnrnd()}\ command in Matlab and then taking the exponential of the realization). When the IF is the two-component IF given by Eq. (2), a vector of IMs can be obtained in two steps: (i) identify one of the two components first by randomly sampling from the Bernoulli distribution with probability \(\gamma\) and (ii) then randomly generate a vector of IMs from the multivariate lognormal component identified.

After a collection of \(n\) vectors is randomly generated from the IF, a corresponding ensemble of ground motions is selected. For each of the \(n\) randomly generated vector-valued IMs, the database of prospective ground motions is searched for the optimal match while ensuring that no ground motion is duplicated. The optimal ground motion is defined as the one whose computed vector of (potentially scaled) IMs, denoted by \(\text{IM}_{P}\), agrees most closely with the current \(\text{IM}_{IF}\). The agreement between \(\text{IM}_{IF}\) and \(\text{IM}_{P}\) for an arbitrary prospective ground motion is quantified by the sum-of-squared-normalized-residuals (SSNR):
\[ SSNR = \sum_{j=1}^{N_{IM}} \left[ \frac{\ln(IM_{IF,j}) - \ln(IM_{p,j})}{\sigma_{IF,j}} \right]^2 \]  

where \( N_{IM} \) refers to the number of IMs in IM and \( \sigma_{IF,j} \) refers to the standard deviation of \( \ln(IM_{j}) \) from the IF. Thus, for a given realization \( IM_{IF,j} \), the selected ground motion (and corresponding scale factor) is the one whose value of SSNR is the smallest among all prospective motions. To avoid selecting duplicate motions, the selected motion is removed from the database before proceeding to the next realization of IM from the IF.

When scaled ground motions are of interest, the optimal scale factor for each prospective ground motion should be determined first before computing SSNR. To determine the optimal scale factor for a given \( IM_{IF,j} \), we first note the relationship between the scaled and unscaled values of the \( j \)th IM:

\[ IM_{p,j} = IM_{U,j} \times SF^{\alpha_j} \]  

where \( IM_{p,j} \) and \( IM_{U,j} \) refer, respectively, to the scaled and unscaled values of the \( j \)th IM, \( SF \) refers to a scale factor, and \( \alpha_j \) denotes how the \( j \)th IM changes with record scaling (e.g., \( \alpha_j = 1 \) for spectral acceleration and \( \alpha_j = 0 \) for 5-75\% significant duration). Substituting Eq. (4) into Eq. (3) shows that SSNR is a quadratic function of the scale factor, for a given \( IM_{IF,j} \). By minimizing SSNR with respect to \( SF \), the optimal scale factor may be derived for each prospective ground motion:

\[ SF_{optimal} = \exp \left\{ \frac{\sum_{j=1}^{N_{IM}} \left( \frac{\alpha_j}{\sigma_j} \right) \cdot \ln \left( \frac{IM_{IF,j}}{IM_{U,j}} \right)}{\sum_{j=1}^{N_{IM}} \left( \frac{\alpha_j}{\sigma_j} \right)^2} \right\} \]  

where \( IM_{IF,j} \) refers to the \( j \)th IM within the current \( IM_{IF} \) under consideration. The prospective ground motions in the database whose \( SF_{optimal} \) is greater than \( SF_{max} \) (or less than 1/\( SF_{max} \)) may be excluded before proceeding with the rest of the selection process. After the ensemble of \( n \) ground motions is selected, the corresponding Importance Sampling weights \( w_i \) must be computed before proceeding further; for a single selected ground motion, its value of \( w_i \) is computed by evaluating the target PDF at \( IM_t \) (Eq. (1)), evaluating the IF at \( IM_t \) (e.g., Eq. (2)), and then taking the ratio of the two PDF values (see Eq. (6) in reference [11]).

2.4 Estimating hazard curves

The ground motions selected from the proposed method should be hazard-consistent with respect to as many important IMs as possible before proceeding with RHAs. For a given IM of interest, the ground motions are judged to be hazard-consistent if their resulting estimate of the hazard curve is practically equal to that obtained from PSHA; otherwise, they are defined to be hazard-inconsistent [6]. The estimates of hazard curves for a given selection of ground motions are determined from Eq. (5) in reference [11]. A 95\% bootstrap confidence interval of the hazard curve estimate can be obtained by applying the bootstrap procedure [13] to the selected ground motions, generating a different estimate of the hazard curve per bootstrap sample; such confidence intervals may be useful for judging the discrepancy between the estimated and PSHA-based hazard curves. Fig. 4a presents an
example where the selected ground motions are hazard-consistent with respect to $A(1s)$; in contrast, Fig. 4b presents an example where the selected motions are hazard-inconsistent with respect to PGA at exceedance rates less than $10^{-5}$.

![Graph](image)

**Fig. 4** – An example of selected ground motions that are: (a) hazard-consistent with respect to $A(1s)$ at exceedance rates greater than $10^{-5}$ and (b) hazard-inconsistent with respect to PGA at exceedance rates less than $10^{-5}$ (adapted from [11]).

When hazard consistency of the selected ground motions is judged to be unsatisfactory, there are two options to explore. The first option is to reselect ground motions by matching the selected ground motions to another randomly generated collection of $n$ vector-valued IMs from the IF. If the reselected motions remain hazard-inconsistent, even after several reselections, then the next option is to modify the recommended two-component IF by reducing $SF_{\text{max}}$ and/or reducing $\gamma$.

There are four steps to estimate SDHCs from an ensemble of hazard-consistent ground motions. First, RHA of the structural model is performed for all $n$ ground motions. Second, the results from RHAs are partitioned into collapse and non-collapse cases. Third, the EDPs corresponding to the collapsed cases are replaced by appropriate values (e.g., drifts may be replaced by infinity and floor accelerations by PGA [6]). Applying Eq. (5b) in reference [11] to the latter values of EDP leads to the desired estimate of the SDHC.

### 3. An illustrative example

A four-story reinforced concrete frame is chosen to demonstrate the applicability of the proposed method to realistic buildings. This well-vetted frame has been studied by past researchers in various contexts (e.g., [14, 15, 16]) and consequently, details regarding its geometry and material properties may be found in such references. In essence, the frame satisfies the strong-column, weak-beam design philosophy and is modeled in OpenSEES [17], where the inelasticity is modeled by plastic hinges at the ends of beam-column elements; its four modal periods of vibration are as follows: $T_1=0.94$ sec, $T_2=0.30$ sec, $T_3=0.17$ sec, and $T_4=0.12$ sec. The frame is classified as collapsed when its displacement increases without bounds. Two EDPs are considered: (i) maximum interstory drift ratio (MIDR), defined as the largest peak interstory drift ratio among the four stories, and (ii) maximum floor acceleration (MFA), defined as the largest peak floor acceleration among the four stories and the ground.

The selected site and earthquake rupture forecast are identical to those shown in Fig. 2 of [7]. A single strike-slip fault, with an activity rate of $\nu_0 = \nu = 0.02$ earthquakes per year, is located 10 km away from the site.
Each earthquake is assumed to occur at a fixed distance of 10 km but with different magnitudes that are characterized by Youngs and Coppersmith PDF shown in Fig. 2b of [7]; the database of prospective ground motions is specified as $10^4$ synthetic motions that are simulated from the stochastic model by Yamamoto and Baker [14, 18], with input magnitudes from the uniform distribution shown in Fig. 2b of [7]. This example is chosen because benchmark SDHCs may be readily computed from synthetic ground motions to evaluate the proposed ground motion selection procedure [7].

Assuming that no IM is perfectly sufficient for the response of this complex, realistic frame, the proposed selection method is applied to only unscaled ground motions. Because the first and fourth mode periods of the four-story frame are $T_1=0.94$ sec and $T_4=0.12$ sec, respectively, the vector of IMs is specified as $\mathbf{IM} = \{ A(0.1s), A(1s), A(2s), D_{5-75}\}$; thus, $N_{IM} = 4$. In order to minimize the effects of epistemic uncertainty on the accuracy of the SDHC estimate in this example, $n$ is specified as 1000. For the case study site, the target probability distribution of IM from PSHA (Eq. (1)) reduces to

\[
Pr(M|m) = \sum IM_{IM} \cdot f_{IM|M}(x|m)
\]  

where the summation is over the number of magnitude bins chosen to discretize Youngs and Coppersmith magnitude PDF, $Pr(M=m)$ refers to the probability of an earthquake with magnitude $m$ occurring, and $f_{IM|M}(x|m)$ denotes the multivariate lognormal distribution of $\mathbf{IM}|M$ whose parameters are given by the benchmark-consistent prediction models [7]. Because $SF_{max}=1$ in this example, the general two-component IF in Eq. (2) reduces to $g_n(x)$; its mean vector and covariance matrix are derived from the $10^4$ unscaled ground motions. With the IF constructed, a subset of $n$ motions is selected from the specified database by first randomly generating $n$ realizations of the IM from the multivariate IF and then identifying $n$ ground motions whose corresponding unscaled values of IM agree most closely with these realizations.

Before proceeding with RHAs, the selected motions are checked for hazard consistency with respect to $\mathbf{IM} = \{ A(0.1s), A(1s), A(2s), D_{5-75}\}$. First, the importance sampling weights are determined by applying Eq. (6) in reference [11] to all IMs computed from the selected motions, where $f(x)$ is given by Eq. (6) and $g(x)$ is given by $g_n(x)$ as mentioned in the preceding paragraph. Then, a hazard curve is estimated for each of the four IMs, using Eq. (5a) in reference [11], and compared against the corresponding benchmark hazard curve. In this example, ground motions were reselected a few times in order to achieve hazard consistency for a wide range of exceedance rates. This consistency for a wide range of exceedance rates is confirmed in Fig. 5.
Fig. 5 – Hazard consistency of 1000 unscaled ground motions selected from $g(x) = g_u(x)$; confidence intervals from 100 bootstrap samples (adapted from [11]).

After hazard consistency is confirmed, SDHCs are estimated for both EDPs of the four-story frame. First, RHAs of the 4-story frame are performed, where the frame is subjected to the $n$ selected ground motions. Second, the collapse cases are identified and the EDPs of the collapse cases are replaced by appropriate values. Third, the SDHC estimate for each EDP is obtained by applying Eq. (5b) in [11] to the computed EDP values. These SDHC estimates are shown by dashed black curves in Fig. 6. In order to convey the epistemic uncertainty of the SDHCs, 95% bootstrap CIs are also shown in this figure, which were obtained by applying the bootstrap procedure to the EDPs and Importance Sampling weights of the selected ground motions, generating multiple bootstrapped SDHCs.

Fig. 6 demonstrates that the SDHC estimates from the proposed method, using a large number of unscaled ground motions, are accurate because the benchmark SDHCs are approximately covered by the 95% confidence intervals. The agreement between the proposed estimate and the associated benchmark is excellent for both EDPs, at a wide range of exceedance rates. Such excellent agreement is likely due to the fact that the selected motions are hazard-consistent with respect to many different features of the ground motion, even though only four IMs were chosen to select ground motions. The selected motions in this example are consistent with the hazard for a wide range of IMs because (i) the motions are unscaled and (ii) the four IMs are chosen to be strongly correlated with many other features of the ground motion.
4. Conclusions

A novel ground motion selection method is presented in this paper. For a given structure at a given site, the method provides a single ensemble of ground motions and a corresponding collection of Importance Sampling weights for estimating SDHCs. The method requires five inputs: (i) a database of prospective ground motions; (ii) a small vector of IMs for selecting ground motions, \( \mathbf{IM} \); (iii) a maximum acceptable scale factor, \( SF_{\text{max}} \); (iv) a target fraction of scaled ground motions, \( \gamma \); and (v) a specified number of ground motions to be selected, \( n \). Based on the author’s experience, the following inputs are recommended: (i) \( \mathbf{IM} = \{A(T_4), A(T_1), A(2T_1), D_{5-75}\} \), (ii) \( SF_{\text{max}} \leq 4 \), and (iii) \( \gamma \leq 0.5 \). The choice of \( n \) depends on a trade-off between the number of RHAs one is willing to perform and the epistemic uncertainty in the resulting SDHC estimates that one is willing to tolerate.

This method provides the following advantages:

1. the ability to estimate SDHCs from a single ensemble of ground motions, which may be particularly useful for high-performance computing using OpenSEES;
2. the option to select ground motions that are scaled to varying degrees, including the important case of selecting only unscaled ground motions; and
3. the means to directly enforce hazard consistency with respect to a specified vector of IMs.

The method is limited in that the resulting SDHC estimates depend appreciably on the choice of the IF (i.e., in this paper, choices for the specified database of prospective ground motions, \( \mathbf{IM}, SF_{\text{max}} \), and \( \gamma \)). Since the IF may significantly affect the SDHC estimates, it is very important to ensure that the selected ground motions are indeed hazard-consistent with respect to a wide range of IMs before proceeding with RHAs and SDHC estimation.

6. References


