

Registration Code: S-R1463151189

# FRAMEWORK DEVELOPMENT FOR MULTI-AXIAL REAL-TIME HYBRID SIMULATION TESTING

G. Fermandois-Cornejo<sup>(1)</sup>, B.F. Spencer, Jr.<sup>(2)</sup>

(1) PhD Candidate, University of Illinois at Urbana-Champaign, United States, fermand2@illinois.edu
(2) Newmark Endowed Chair of Civil Engineering, University of Illinois at Urbana-Champaign, United States, bfs@illinois.edu

#### Abstract

Real-time hybrid simulation is an efficient and cost-effective cyber-physical dynamic testing technique for performance evaluation of structural systems subjected to earthquake loading with rate-dependent behavior. A loading assembly with multiple actuators is required to impose realistic boundary conditions on physical specimens. However, such a testing system is expected to exhibit significant actuator dynamic coupling and suffer from potential time delays that are associated to servo-hydraulic system dynamics and control-structure interaction (CSI). One approach to reduce experimental errors considers a multi-input, multi-output (MIMO) approach to design controllers for accurate reference tracking and noise rejection. In this paper, a framework for multi-axial real-time hybrid simulation (maRTHS) testing is presented. The methodology consists in designing a cyber-physical system with multiple actuators with real-time control in Cartesian coordinates. For this matter, kinematic transformations between actuator space and Cartesian space are derived to control all six-degrees-of freedom of the moving platform in 3D Cartesian space. Then, a frequency domain identification technique with non-linear optimization tool is used to derive a model of the MIMO transfer function. Further, a Cartesian-domain model-based feedforward-feedback controller is implemented for delay compensation and to increase the robustness of the reference tracking for given model uncertainty. The framework is implemented using the 1/5<sup>th</sup>-scale Load and Boundary Condition Box (LBCB) located at the University of Illinois at Urbana-Champaign. To validate the proposed framework, a small scale structural systems with a physical cantilever rubber column specimen is considered. For real-time execution, the numerical substructure, kinematic transformations and controllers are implemented over an embedded system with a microcontroller and digital signal processor. Finally, the stability of the real-time system is demonstrated for an illustrative example of a single-degree-of-freedom structure subjected to earthquake loading. The test results shows that the framework can accurately provide excellent reference tracking performance.

Keywords: real-time hybrid simulation; multiple actuator; dynamic coupling; kinematic transformations; model-based compensation.



## 1. Introduction

Societies seek to understand the risks of natural hazards and promote mitigation plans for the development of resilient communities. As part of this task, reliable performance assessment of structural systems is required to conduct vulnerability assessment and risk management of civil infrastructure. From the different methods available, experimental testing is considered an essential ingredient for the development of building codes, providing understanding of the behavior of structural systems and construction materials, as well as allowing calibration of numerical models for reliable, cost-effective analysis and design. Quite often, experimental testing is limited by physical constraints such as laboratory size and equipment payloads, and also by project budget and timeframe.

Hybrid simulation is an experimental testing method that has proven to be an attractive approach to conduct experimental testing of structural components. The original concept was proposed by Hakuno et al. [1], then Takanashi et al. [2] was the first to incorporate digital computers to control the experimental equipment for online testing. Substructure hybrid simulation consists in physical testing of only the critical components of the structural system that are of interest (i.e. where damage is expected), meanwhile the rest of the structure is modeled numerically. Both physical and numerical components are connected by actuators and sensors, forming a feedback loop to solve the equations of motion at every time step.

For dynamic testing, real-time hybrid simulation (RTHS) testing has demonstrated many advances and improvements of in the past 20 years. Testing is executed in real-time, meaning that all calculations, imposed boundary conditions on physical specimens, measured forces and displacements, and digital data acquisition, must be performed at very short time intervals, typically less than 10 msec. Furthermore, the boundary conditions must also be imposed at fast rates, meaning that dynamic actuators are required for this task. Therefore, the fundamental condition to execute RTHS tests is that fast hardware and software is required to allow a stable and accurate result. In addition, RTHS testing results have been compared with conventional shaking table tests for both steel specimens [3] and concrete specimens [4], with very good agreement between the two testing methods. Real-time hybrid simulation (RTHS) studies have been carried out for a number of structural systems with rate-dependent components, such as passive energy dissipation devices [5–7]; semi-active control devices [8–11]; and sliding bearing devices [12]. As such, RTHS testing is an accurate, cost-effective, flexible, and repeatable alternative to conventional shake table testing.

Nonetheless, RTHS testing has not yet reached maturity; many challenges remain to be solved in the areas of servo-hydraulic delay compensation, control design, digital communications, and fast numerical computations, among others. In particular, there are no specific studies of RTHS where it is required to control multiple dynamic actuators at a single connection to impose three-dimensional boundary conditions to physical specimens. This ability is required to advance the state-of-the-art of RTHS.

Research in the field of three-dimensional, multi-axial, hybrid simulation was explored in the past ten years, but for slow speed (quasi-static) loading rates. In 2006, a multi-dimensional hybrid simulation framework based on a servo-hydraulic simulator that could impose six-degree-of-freedom (6DOF) load and boundary conditions to a physical specimen for seismic performance evaluation purposes was proposed [13]. The loading system, called *Load and Boundary Condition Box* (LBCB), is currently located in the *Newmark Civil Engineering Laboratory* (NCEL) from University of Illinois at Urbana-Champaign (UIUC). The LBCB consist on six actuators mounted to a boxed frame, and each moving piston is connected in parallel configuration to a rigid platform (i.e. end effector) for controlled 6DOF rigid body motion, as shown on Fig. 1. Three large-scale LBCBs are available for experimental testing at the NCEL laboratory. Also, three 1/5<sup>th</sup>-scale versions are available at the same facility, primarily intended for training and academic purposes. A series of (slow) hybrid simulation tests have been conducted in this facility using the LBCB loading assemblies for seismic vulnerability assessment, from semi-rigid steel moment frames in a low rise building [14], to reinforced concrete piers in curved bridges [15].



Fig. 1 – Load and Boundary Condition Box (LBCB)

The literature provides some results on the use of coupled dynamic actuators in RTHS testing. For instance, two [16–19] and three [20, 21] coupled actuators have been reported to date. The degree of coupling is different in each case, from actuators connected to rigid brackets to impose multiple boundary conditions at one point [16], to actuators that are connected to one physical specimen in multiple points [e.g., 19, 21]. Regardless, four or more coupled actuators in RTHS testing has not been reported in the literature, and this identified research gap is key to allow for 6DOF dynamic loading using earthquake simulators such as the LBCB system.

In this paper, a novel framework to conduct real-time hybrid simulation testing of physical specimens with three-dimensional, multiple boundary conditions, is presented. Section 2 will briefly explain the proposed framework. Subsequently, Section 3 will introduce the necessary tasks for framework implementation, namely: kinematic transformation algorithm, external sensor calibrations, system identification and control design, and Section 4 will briefly illustrate the methodology with an example. Finally, Section 5 will provide some final remarks and an outline of future research.

### 2. Problem Formulation

#### 2.1 Substructuring technique with multiple interface degrees of freedom

The dynamic response of the structural system of interest can be obtained by solving the equations of motion:

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{C}\dot{\mathbf{u}} + \mathbf{R}^{int}(\mathbf{u}, \dot{\mathbf{u}}) = \mathbf{F}$$
(1)

where  $\mathbf{u}, \dot{\mathbf{u}}, \ddot{\mathbf{u}}$  are displacement, velocity and acceleration vectors, respectively, at the location of the degrees of freedom (DOF) of the structural system; **M** is the mass matrix, **C** is the damping matrix,  $\mathbf{R}^{int}$  is the internal (restoring) force vector, and **F** is the external force vector (e.g. earthquake loading). Instead of solving this problem for the entire domain, it is possible to subdivide the domain into smaller components, and by defining coupling between components, to solve reduce the order of large and complex structural systems. This technique is known as *substructuring*. Typically, in hybrid simulation, the domain of the problem is divided in two as shown in Fig. 2: a numerical subdomain ( $\Omega^N$ ) and an experimental subdomain ( $\Omega^E$ ), each having its own boundary conditions for prescribed displacements ( $\Gamma_g^N$  and  $\Gamma_g^E$ ) and prescribed loads ( $\Gamma_h^N$  and  $\Gamma_h^E$ ). In particular, by arranging the displacement vector of the associated numerical subdomain as  $\mathbf{u}^N = {\{\mathbf{u}_i^N \ \mathbf{u}_b^N\}}^T$ , where the subscripts "i" and "b" refer to internal and boundary degrees of freedom, the equations of motion for the numerical subdomain can be expressed as follows:

$$\mathbf{M}^{N}\ddot{\mathbf{u}}^{N} + \mathbf{C}^{N}\dot{\mathbf{u}}^{N} + \mathbf{R}^{N}(\mathbf{u}^{N},\dot{\mathbf{u}}^{N}) = \mathbf{F}^{N} - \mathbf{R}^{E}$$
(2)

where  $\mathbf{R}^N$  is the internal force vector from the numerical component, that could be chosen to be either a linear or nonlinear relationship, depending exclusively on the application. In addition,  $\mathbf{R}^E$  is the reaction force vector from the experimental component, where  $\mathbf{R}^E = \{\mathbf{0}_i \ \mathbf{r}_b^E\}^T$ , which means that only force coupling exists between



boundary DOF of numerical and experimental components through the coupling force vector  $\mathbf{r}_b^E$ . This coupling force includes the effects of mechanical, damping, inertial, and experimental external.

To obtain an admissible solution, compatibility and equilibrium must be satisfied for the degrees of freedom at the boundary ( $\Gamma_b$ ) between numerical and experimental subdomains. Therefore, and algorithm is considered to prescribe displacements and forces at the boundary during the time-stepping numerical integration of the dynamic system. In short, after solving  $\mathbf{u}^N$  from (2), the values of  $\mathbf{u}_b^N$  are extracted and prescribed over the boundary of the experimental component. Then, the coupling force  $\mathbf{r}_b^E$  is measured at the boundary of the experimental component and prescribed back into (2) to continue with the next time step of the numerical integration until the simulation is complete.



Fig. 2 - Substructuring of dynamic system with multiple boundary DOF

#### 2.2 Framework for Multi-axial Real-time Hybrid Simulation

A sketch of the framework for multi-axial real-time hybrid simulation (maRTHS) is presented in Fig. 3, where the multiple-degrees-of-freedom (MDOF) boundary conditions are imposed to the physical specimen by a modular multi-actuator loading assembly. The procedure for maRTHS starts with the time-stepping solution of the equations of motion from the numerical substructure, and the target Cartesian displacements at the boundary between numerical and experimental components are obtained. Then, the target Cartesian coordinates ( $\mathbf{x}_{target}$ ) are processed by an outer-loop controller, which is the component responsible for minimizing the error between measured and target Cartesian displacements, i.e.  $\mathbf{e} = \mathbf{x}_{meas} - \mathbf{x}_{target} \approx \mathbf{0}$ . The output of outer-loop controller is a command Cartesian coordinates ( $\mathbf{x}_{cmd}$ ) that needs to be transformed to command actuator signal through a kinematic transformation block. Then, each single actuator command stroke is processed by the inner-loop servo-controller, and the motion of the moving platform is obtained through an external motion measurement system, which provides the measured Cartesian coordinates ( $\mathbf{x}_{meas}$ ) that are essential for feedback control purposes. Finally, after reading individual actuator forces, the restoring Cartesian forces are calculated using kinematic relationships and the virtual work principle. The measured restoring Cartesian forces ( $\mathbf{r}_{meas}$ ) are applied to the numerical substructure at the boundary degrees of freedom, and then solve the numerical substructure for the next time step until the simulation is complete.



Fig. 3 - Framework for multi-axial, real-time hybrid simulation (maRTHS) testing

This framework includes multi-actuator and multi-sensor systems, therefore it requires significant calibration effort to allow for accurate motion tracking in global Cartesian coordinates of the loading platform. In addition, a model-based control design is used for compensation of servo-hydraulic dynamics, with explicit consideration of multi-actuator dynamic coupling effects for improved accuracy and robustness.

## 3. Implementation of small-scale maRTHS testing

#### 3.1 Overview of experimental setup

The development of the proposed framework was achieved by using the experimental resources from Newmark Civil Engineering Laboratory at University of Illinois, Urbana-Champaign. In particular, one small-scale LBCB loading assembly was chosen to complete the goals of this research. The small-scale LBCB is a 1/5<sup>th</sup>-scale version of the full-scale LBCB, also available in the same experimental facility, and was manufactured by Shore Western Manufacturing, Inc. The small-scale LBCB carries the same features and control capabilities of the full-scale version, and it has been used for academic and training purposes. For the proposed research, this device will become the workbench to develop and debug control algorithms and testing procedures for the maRTHS framework.

The small-scale LBCB consists in six servo-hydraulic actuators connected in a parallel configuration. Two long actuators, with a maximum stroke of 101.6 mm (4 in), are primarily oriented with the *x* global coordinate, meanwhile four shorter actuators, with a maximum stroke of 50.8 mm (2 in), are oriented with the *y* and *z* global coordinates. A dedicated hydraulic power supply with a capacity of 10 gpm (gallons per minute) at 3,000 psi is provided to operate the actuators. Meanwhile, an analog servo-controller is in charge of commanding the motion of each individual actuator. The specifications for the small-scale LBCB were previously introduced by [22], and are presented in Table 1 for further reference.



Degree of	Coordinate	Displacement	Force
Freedom		(mm or deg)	(kN or kN-m)
Translation	$u_x$	$\pm 50.8$	±8.9
	$u_y$	±25.4	±4.5
	$u_z$	±25.4	±12.3
Rotation	$ heta_x$	±16.0	±1.13
	$ heta_y$	±12.0	±2.03
	$ heta_z$	±12.0	±1.13

Table 1 – Capacity specifications for small-scale LBCB



The implementation of the proposed framework is summarized Fig. 4. To control the synchronized motion of the actuators in Cartesian coordinates, a digital signal processor is connected to the analog servo-controller. The hardware of choice is a dSpace DS1103PPC controller board with a PPC 750GX, 1 GHz processor, and 20 A/D channels and 8 D/A channels, both with 16-bit resolution, are available for external device communications. A host PC is connected directly to the DSP board via fiber optics. The host PC stores all the programming code and preferences required for maRTHS. The programming development environment is Matlab/Simulink. Then, using dSpace Real-Time Interface (RTI), the programs are translated to C language, loaded to the DSP board and compiled for real-time execution. Virtual instrument interfaces can also be developed to check parameters of the simulation on the fly, by using dSpace ControlDesk software. The DSP board has an interface with the analog servo controller in order to receive input (Meas) and output (Cmd) signals in order to control the multi-actuator system.



Fig. 4 - Framework implementation using small-scale LBCB equipment

### 3.2 Overview of Real-time system

The proposed maRTHS real-time system is shown in Fig. 5. This is composed of three main subsystems: (i) *numerical component*, where the numerical substructure model, external loading, and time-stepping integration algorithm are declared; (ii) *outer-loop controller* (Fig. 6), where the model-based compensation for servo-hydraulic dynamics are defined; and (iii) *physical component* (Fig. 7), where kinematic transformations, calibration corrections, and digital-analog signal conversion are provided to communicate with external devices in real-time.



RTHS loop

Fig. 5 - Block diagram of maRTHS experimental testing system



Fig. 6 - Block diagram of outer-loop controller subsystem



Fig. 7 - Block diagram of Cartesian-domain physical subsystem

### 3.3 Methodology

For implementation of the maRTHS real-time system, the following steps should be considered in preparation for the desired experimental test:

#### (i) Real-time kinematic transformations

The target displacements from the numerical substructure are applied to the physical specimen by using multiple servo-hydraulic actuators attached to the loading platform. The actuator's piston can only be commanded to move along its axis; therefore, if the multiple actuators of the loading assembly are not aligned with the global Cartesian system of coordinates, a kinematic transformation between Actuator and Cartesian space coordinates [23] will be required for successful maRTHS testing. Hence, the objective is to develop an explicit solution for



inverse and forward kinematic transformation problems using an external sensor approach attached to the loading platform, that will also account for any flexibility of the reaction frame [24]. Six linear potentiometers are connected to the loading platform, and are oriented as close as possible to the Cartesian axes to facilitate forward kinematic transformations in real-time.

#### (ii) Quasi-static calibration

Two calibration corrections are required to improve the accuracy of the loading assembly. First, command calibration is necessary in order to match both the command and real measured displacements from a reference value. Second, an external sensor calibration procedure is essential to match both the estimated displacements and the real measured displacements from a reference. In both cases, quasi-static loading is considered, without accounting for rate-of-loading effects yet. Also, the measured reference values are provided by Krypton K600 DMM, a contact-less dynamic measuring machine, that provides very accurate Cartesian position measuring of up to  $\pm 0.02$  mm in 3D space. After the calibration process, command errors ( $e_{cmd} = x_{cmd} - x_{meas}$ ) were reduced to less than 0.25 mm in translation, and 0.05 deg in rotation. In addition, the measurement error ( $e_{meas} = x_{meas} - x_{reference}$ ) are less than 0.2 mm in translation, and less than 0.03 deg in rotation. This results are considered as satisfactory for all purposes of framework development of maRTHS using the small scale facility, and are in agreement with the calibrations performed by [24].

#### (iii) System identification of multi-axial loading assembly

To provide good reference tracking and robustness properties to the real-time system, an accurate representation of the dynamics of the experimental setup is needed. The goal is to obtain a model of the multi-input, multi-output (MIMO) experimental system, that incorporates all the parameters from the servo-hydraulic actuators, the test specimen interaction, and any actuator dynamic coupling effects. To complete this task, a nonparametric frequency-domain system identification procedure is considered. The data collected to create the model is obtained from a multi-input random vibration excitation test of the physical component, with a particular frequency bandwidth between 0 to 25 Hz. The result of this stage is a  $6 \times 6$  transfer system matrix, which represents the direct command-to-measured response of the 6DOF rigid body motion in Cartesian space. Also, this result must show expected properties, such as DC (static) response (i.e. unity gain and zero phase for f = 0 Hz) for diagonal components, frequency-dependent phase (i.e. measured response is indeed delayed with respect to the command signal), and small but non-zero off-diagonal components of this transfer matrix showing expected dynamic coupling patterns, even the physical specimen attached to the small-scale LBCB is considered to be rather soft.

#### (iv) Model-based compensator design

To enable accurate and stable response of the maRTHS test, different alternatives of time-domain or frequencydomain delay compensation techniques have been presented in the literature. For the purposes of this research, a model-based controller [9, 25] has been employed for the outer-loop controller. The goals is to produce a multiinput, multi-output (MIMO) model-based controller to perform maRTHS testing. This outer-loop controller will be the capable of real-time stabilization of the experimental system if any time delay or dynamic coupling effect is present in the experimental system. In addition, the controller will allow increased accuracy on the overall system response, without the need of adding artificial damping. The control system is designed considering the global Cartesian displacements (target, command, and measured signals) of the physical specimen, quite different from previous RTHS solutions where single actuator feedback was considered. This configuration will ensure that the correct boundary conditions are imposed into the physical specimen, because Cartesian space control can ensure a more reliable tracking [26]. In particular, a feedforward-feedback control architecture is considered. The feedforward compensator is designed with information from an inverse model of the experimental system, and is primarily responsible of tracking the target displacements with zero phase delay error. Meanwhile, the feedback controller considers an LQG/LTR approach to provide additional robustness to the system when the feedforward is not able to perfectly track the target displacements due to model uncertainty. The performance of the designed controllers will be assessed in terms of analytical simulations and experimental tests for specimens with varying degree of relative stiffness.



#### 4. Illustrative Example

To demonstrate the effectiveness of this framework, successive tests and adjustments are considered to guarantee that the experimental results satisfy the main goals of this proposed research. The nominal controllers for maRTHS are tested by numerical simulation, and the system performance for difference excitation signals is studied. For example, a f = 1 Hz single-degree-of-freedom (SDOF) structure, with  $\zeta = 5\%$  intrinsic damping ratio, subjected to a 3% scaled El Centro ground motion in the  $u_x$  direction is considered. Both numerical and experimental columns are assumed to hold the same nominal properties, with a lateral stiffness of  $k_{num} = 19.9026$  N/mm. The numerical substructure is modeled using a state-space representation as follows:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{3}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u} \tag{4}$$

where  $\mathbf{x} = \{u_x \, \dot{u}_x\}^{\mathrm{T}}$  is the state vector,  $\mathbf{u} = \{\ddot{u}_{xg} \, r_{\text{phys}}\}^{\mathrm{T}}$  is the input vector, and  $\mathbf{y} = \{u_x \, \dot{u}_x \, \ddot{u}_x\}^{\mathrm{T}}$  is the measurement vector. The state space matrices are the following:



(a) Structural system (b) Experimental setup Fig. 8 – Example of a single-degree-of-freedom structure with a rubber column experimental specimen

Two cases are studied: no compensation, and model-based (feedforward+feedback) compensation. Good reference tracking is observed when the model-based compensation is considered, as shown in Fig. 9. Also, it is clear from the subspace synchronization plots shown in Fig. 10, that model-based compensation can effectively reduce experimental errors due to actuator dynamics. In terms of reference tracking, the root-mean-square (RMS) error is used as a performance index:

$$RMS_{error} = \sqrt{\frac{\left(x_{meas}[k] - x_{target}[k]\right)^2}{\left(x_{target}[k]\right)^2} \times 100\%}$$
(5)

where  $x_{\text{meas}}[k]$  and  $x_{\text{target}}[k]$  are the measured and target Cartesian displacement signal at the boundary, respectively. For no compensation case, the RMS error is estimated to be 9.1429%, meanwhile the case of



model-based compensation significantly reduces this error to 1.8839%. Even though model-based compensation successfully reduces the tracking errors during the experimental test, it is believed that more studies on the compensation efforts should be conducted in future research to improve the performance indices as much as possible for particular testing application scenarios.





## 5. Conclusions

In this paper, a framework for multi-axial real-time hybrid simulation has been presented. The framework consists in prescribing multiple degrees of freedom at the boundary between numerical and experimental substructures, by using a multi-actuator loading assembly with the addition of multi-transducer position measuring system. The complexities of the implementation are presented, and the methodology for kinematic transformations, equipment calibration, system identification, and control design are briefly discussed. The illustrative example shows that the method is able to reproduce boundary conditions at the physical specimen in an accurate, reliable and stable manner. Although, the boundary conditions of the example were rather simple, so future work on the aspect of multi-boundary conditions will be presented in a following study. Indeed, this small-scale implementation will provide a test-bed for future research applications in order to verify and evaluate



rate-dependent materials and components that can be used for the design of structural systems subjected to dynamic loading. Finally, this framework offers the opportunity to increase the class of structures that can be experimentally tested using the hybrid simulation technique, while enabling significant reductions on costs through substructuring methods. This opportunity opens a promising field that could incorporate new materials and structural systems into future versions of building design codes.

## 6. Acknowledgements

The first author gratefully acknowledges the financial support for his Doctorate Studies from CONICYT-Chile through Becas Chile Scholarship No. 72140204, and Universidad Tecnica Federico Santa Maria (Chile) through Faculty Development Scholarship No. 208-13.

## 7. References

- [1] Hakuno M, Shidawara M, Hara T (1969): Dynamic destructive test of a cantilever beam, controlled by an analogcomputer. *Proceedings of the Japan Society of Civil Engineers*, 1969:1–9. doi: 10.2208/jscej1969.1969.171\_1.
- [2] Takanashi K, Udagawa K, Seki M, Okada T, Tanaka H (1975): Nonlinear Earthquake Response Analysis of Structures by a Computer-Actuator On-Line System. *Bulletin of Earthquake Resistant Structure Research Center*, No. 8:Institute of Industrial Science, University of Tok..
- [3] Lamarche C-P, Tremblay R, Léger P, Leclerc M, Bursi OS (2010): Comparison between real-time dynamic substructuring and shake table testing techniques for nonlinear seismic applications. *Earthquake Engineering & Structural Dynamics*, 39:1299–1320. doi: 10.1002/eqe.994.
- [4] Saouma V, Haussmann G, Kang D, Ghannoum W (2014): Real-Time Hybrid Simulation of a Nonductile Reinforced Concrete Frame. *Journal of Structural Engineering*, 140:1–12. doi: 10.1061/(ASCE)ST.
- [5] Nakashima M, Kato H, Takaoka E (1992): Development of real-time pseudo dynamic testing. *Earthquake Engineering & Structural Dynamics*, 21:79–92. doi: 10.1002/eqe.4290210106.
- [6] Horiuchi T, Inoue M, Konno T, Namita Y (1999): Real-time hybrid experimental system with actuator delay compensation and its application to a piping system with energy absorber. *Earthquake Engineering & Structural Dynamics*, 28:1121–1141. doi: 10.1002/(SICI)1096-9845(199910)28:10<1121::AID-EQE858>3.0.CO;2-O.
- [7] Chae Y, Kazemibidokhti K, Ricles JM (2013): Adaptive time series compensator for delay compensation of servohydraulic actuator systems for real-time hybrid simulation. *Earthquake Engineering & Structural Dynamics*, 42:1697–1715. doi: 10.1002/eqe.2294.
- [8] Carrion JE, Spencer Jr. BF, Phillips BM (2009): Real-time hybrid simulation for structural control performance assessment. *Earthquake Engineering & Engineering Vibration*, 8:481–492. doi: 10.1007/s11803-009-9122-4.
- [9] Phillips BM, Spencer Jr. BF (2013): Model-Based Feedforward-Feedback Actuator Control for Real-Time Hybrid Simulation. *Journal of Structural Engineering*, 139:1205–1214. doi: 10.1061/(ASCE)ST.1943-541X.0000606.
- [10] Asai T, Chang C-M, Spencer Jr. BF (2014): Real-Time Hybrid Simulation of a Smart Base-Isolated Building. Journal of Engineering Mechanics, 0:4014128. doi: 10.1061/(ASCE)EM.1943-7889.0000844.
- [11] Chae Y, Ricles JM, Sause R (2014): Large-scale real-time hybrid simulation of a three-story steel frame building with magneto-rheological dampers. *Earthquake Engineering & Structural Dynamics*, 139:1215–1226. doi: 10.1002/eqe.2429.
- [12] Nakashima M (2001): Development, potential, and limitations of real-time online (pseudo-dynamic) testing. *Philosophical Transactions of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 359:1851– 1867. doi: 10.1098/rsta.2001.0876.
- [13] Elnashai AS, Spencer, Jr. BF, Kuchma DA, Yang G, Carrion JE, Gan Q, Kim SJ (2006): The Multi-Axial Full-scale Sub-structured Testing and Simulation (MUST-SIM) Facility at the University of Illinois at Urbana-Champaign. In: Adv. Earthq. Eng. Urban Risk Reduct. Kluwer Academic Publishers, pp 245–260.
- [14] Mahmoud HN, Elnashai AS, Spencer, Jr. BF, Kwon O-S, Bennier DJ (2013): Hybrid Simulation for Earthquake



Response of Semirigid Partial-Strength Steel Frames. *Journal of Structural Engineering*, 139:1134–1148. doi: 10.1061/(ASCE)ST.1943-541X.0000721.

- [15] Frankie T, Abdelnaby A, Silva P, Sanders D, Elnashai A, Spencer B, Kuchma D, Chang C-M (2013): Hybrid Simulation of Curved Four-Span Bridge: Comparison of Numerical and Hybrid Experimental/Analytical Results and Methods of Numerical Model Calibration. In: Struct. Congr. 2013. American Society of Civil Engineers, Reston, VA, pp 721–732.
- [16] Darby A, Williams M, Blakeborough A (2002): Stability and Delay Compensation for Real-Time Substructure Testing. *Journal of Engineering Mechanics*, 128:1276–1284. doi: 10.1061/(ASCE)0733-9399(2002)128:12(1276).
- [17] Bonnet PA, Lim CN, Williams MS, Blakeborough A, Neild SA, Stoten DP, Taylor CA (2007): Real-time hybrid experiments with Newmark integration, MCSmd outer-loop control and multi-tasking strategies. *Earthquake Engineering & Structural Dynamics*, 36:119–141. doi: 10.1002/eqe.628.
- [18] Jung R-Y, Shing PB, Stauffer E, Thoen B (2007): Performance of a real-time pseudodynamic test system considering nonlinear structural response. *Earthquake Engineering & Structural Dynamics*, 36:1785–1809. doi: 10.1002/eqe.722.
- [19] Gao X, Castaneda N, Dyke SJ (2014): Experimental Validation of a Generalized Procedure for MDOF Real-Time Hybrid Simulation. *Journal of Engineering Mechanics*, 140:4013006. doi: 10.1061/(ASCE)EM.1943-7889.0000696.
- [20] Phillips BM, Spencer, Jr. BF (2013): Model-Based Multiactuator Control for Real-Time Hybrid Simulation. *Journal of Engineering Mechanics*, 139:219–228. doi: 10.1061/(ASCE)EM.1943-7889.0000493.
- [21] Friedman A, Dyke S, Phillips BM, Ahn R, Dong B, Chae Y, Castaneda N, Jiang Z, Zhang J, Cha Y, et al. (2014): Large-Scale Real-Time Hybrid Simulation for Evaluation of Advanced Damping System Performance. *Journal of Structural Engineering*, 0:4014150. doi: 10.1061/(ASCE)ST.1943-541X.0001093.
- [22] Nakata N, Spencer, Jr. BF, Elnashai AS (2007): Multi-dimensional Mixed-mode Hybrid Simulation Control and Applications.
- [23] Merlet J-P (2006): *Parallel robots*, Second. Springer Science & Business Media.
- [24] Chang C-M, Frankie TM, Spencer Jr. BF, Kuchma DA (2015): Multiple Degrees of Freedom Positioning Correction for Hybrid Simulation. *Journal of Earthquake Engineering*, 19:277–296..
- [25] Carrion JE, Spencer, Jr. BF (2007): Model-based Strategies for Real-time Hybrid Testing. .
- [26] Paccot F, Andreff N, Martinet P (2009): A Review on the Dynamic Control of Parallel Kinematic Machines: Theory and Experiments. *The International Journal of Robotics Research*, 28:395–416. doi: 10.1177/0278364908096236.