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SIMPLIFIED METHOD FOR EVALUATING SEISMIC RESPONSE OF CIRCULAR REINFORCED CONCRETE SILOS

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Abstract

Silos are a type of unconventional structure widely used by industries and agro farms for storage of all types of grains. Because of the complexity of the seismic behavior of silo-grain systems, it is not trivial to evaluate the seismic responses quickly and reliably. This paper presents mathematical expressions to evaluate the following responses of interest in a circular reinforced concrete silo: the fundamental period of vibration, lateral displacements, shear forces and overturning moments. These expressions are based on a linear elastic model that can represent the structural behavior of silo-grain system, and the design spectrum of Peruvian standard E030 was considered as seismic action. The most relevant geometric and physical parameters that govern the behavior of silo-grain system were identified, and it was considered that the grain has a soil-like behavior. The linear elastic model of the silo is given by a spring-mass system, with a number of discretized elements representing its structure. With the model, differential equation of motion were formulated and were solved using spectral modal combination method, finding the responses of interest mentioned above. These results were compared with those obtained using a refined finite element model to validate the proposed model. A margin of error between 15% and 20% was achieved, which is acceptable in civil engineering. Finally, the simplified methods for calculating the response of interest were obtained from the validated response.

Keywords: Concrete Silo, Seismic Response, Seismic Analysis.

1. Introduction

Silos are structures used for storing farm products, cement and aggregates. These unconventional structures are distinguished by having a complex behavior, due to the interaction between the silo as structure and the stored material. The silo-grain system must respond appropriately to seismic ground motion, when they are located in seismic zones.

ACI code [1] and Eurocode [2] provides that, for purposes of seismic analysis, this type of structure should be evaluated considering 80% of the effective weight of the stored material, and that the material moves in conjunction with the silo walls, if a more refined analysis is not being performed. In turn, the Peruvian Earthquake Resistant Design Standard E030 [3] states 100% of the effective weight of the stored material should be considered to evaluate seismic responses. Recent research has attempted to describe their behavior using detailed studies with inelastic nonlinear models. However, these models represent an increase in the difficulty and time in obtaining reliable answers for these structures.

The aim of this research is to pose simplified methods to estimate the responses of interest of silos. This research presents a linear elastic structural model build from the representative parameters of the silo-grain system, capable of reproducing the behavior of this unconventional structure. From this model, it was possible to obtain the following seismic responses of interest: the lateral displacement, the base shear at any height and overturning moment at any height. In addition, dynamic and seismic analyses were performed to a family of silos using this model, in order to obtaining a large number of responses of interest. Then, it was possible to construct simplified methods from these responses. The advantage of having such expressions is the versatility and speed in determining reliable results for this type of structure.

2. Definition of Parameters

It is possible to obtain circular reinforced concrete silos through a set of geometric and physical parameters. The most relevant parameters defined are divided in two groups:

a) Geometric Parameters : Inner Diameter (D) , Total Height – Diameter Ratio (H/D) and Inner Diameter – Thickness Ratio (D/e)
b) Physical Parameters: Young Modulus, Poisson Coefficient, Density, Lateral Pressure Coefficient, Friction Coefficient.

Table 1 was generated from the definition of the geometrical parameters. The silo cases studied for the proposed model are the combination of the values presented in this table. Compressive strength equal to f'_c= 35 Mpa, and yield strength equal to f_y= 420 Mpa as properties of the reinforced concrete were considered. Hochstetten sand [4] was considered as stored material.

Table 1 – Silo Parameters.

Inner Diameter	D (m)	7	10	15	-	-
Total height/Inner Diameter Ratio	H/D	1.5	2.5	3.5	4.5	5.5
Inner Diameter/Thickness Ratio	D/e	30	40	50	60	-

(-) No value.

3. Proposed Model

3.1 Mathematical Approach

The model presented in this research is based on the model of Housner [5], as shown in Fig. 1. The properties of water were exchanged with the properties of the stored granular material, adding to the model properties lateral pressure coefficient (κ) and frictional coefficient between the stored material and the structure (μ). It was considered for the silo: the inner radius (R), height (h), convective mass

(M_0), the n[th] impulse weight (M_n), the height of the convective mass (h_0), the height of the nth impulsive mass (h_n) and the density of the stored material (ρ). Also the distances to the analyzed fiber (*x* and *y*), the velocity components (*u* and *v*), the rotation of the analyzed fiber (θ) for analysis purposes.

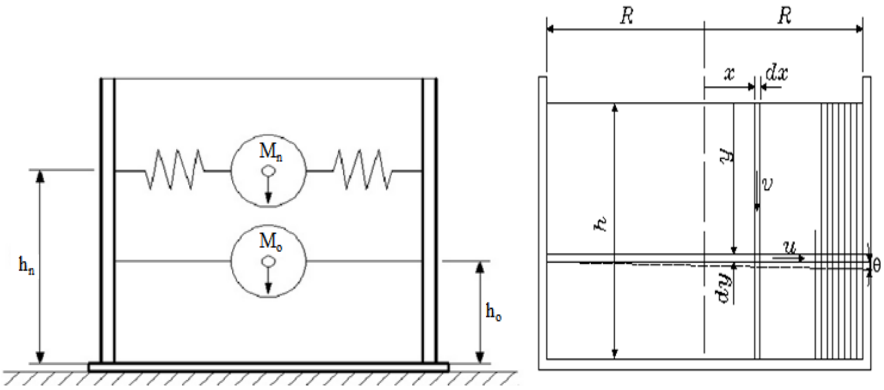


Figure 1– Housner Model

The pressure equation proposed by Housner [5] was modified by exchanging the stored material with the water, introducing the lateral pressure coefficient, as is shown in Eq. (1).

$$\frac{dp}{dy} = -\rho\kappa\dot{v} \quad , \qquad P = \int_0^h p dy \tag{1}$$

In addition, the dissipative force (F_d) and the friction coefficient (μ) were included as depicted in Eq. (2), which have appeared because of the friction between stored material and the silo.

$$F_d = 2\mu p dx dy \tag{2}$$

These modifications lead to the following changes in the differential equations of convective and impulsive mass proposed by Housner, as it is shown in Eqs. (3) and (4) respectively.

$$\frac{d^2\dot{u}}{dx^2} - \frac{3}{h^2}\dot{u} = 0 \qquad \rightarrow \qquad \frac{d^2\dot{u}}{dx^2} - \frac{3}{\kappa h^2}\dot{u} - \frac{\mu}{R}\frac{d\dot{u}}{dx} = 0 \tag{3}$$

$$\frac{2\pi R^6}{27}\frac{d^2\ddot{\theta}}{dy^2} - \frac{\pi R^4}{4}\ddot{\theta} = 0 \quad \rightarrow \quad \kappa\frac{2\pi R^6}{27}\frac{d^2\ddot{\theta}}{dy^2} - \mu\frac{4R^5}{27}\frac{d\ddot{\theta}}{dy} - \frac{\pi R^4}{4}\ddot{\theta} = 0 \tag{4}$$

Just as in Housner's model, there is a maximum height, for which Eqs. (1) to (4) are valid. This height is shown in Eq. (5). In the case of silos whose height is greater than this maximum height, stored material at the top has the behavior depicted in Eqs. (3) and (4) until it reaches a depth equal to the maximum height measured from the top. The stored material below this maximum height moves in conjunction with the silo walls. This maximum height can be obtained from Eq. (5).

$$\frac{2}{3}\kappa\frac{R^2}{h^2} = \left(1 + \frac{2}{3}j\kappa\frac{h^2}{R}\frac{\mathrm{senh}(j-i)R \cdot \mathrm{senh}(i+j)R}{\mathrm{senh}(2j)R}\right) \tag{5}$$

The harmonic solution of convective equations lead to the solutions shown in Eqs. (6) to (8) for the convective mass, the height of the convective mass, and its parameters, respectively.

$$M_0 = M\frac{h^2}{3R}\left(\frac{\mathrm{senh}(j-\ i)R}{\mathrm{senh}(2j)R}(i+j)\exp^{(i+j)R} + \frac{\mathrm{senh}(i+j)R}{\mathrm{senh}(2j)R}(i-j)\exp^{(i-j)R}\right) \tag{6}$$

$$h_0 = h\left[\frac{1}{M_0}\left(1 - \frac{2j\,\mathrm{senh}(j-i)R.\,\mathrm{senh}(i+j)R}{l(j^2-i^2)\mathrm{senh}(2j)R}\right) + \frac{3}{8}\right] \tag{7}$$

$$i = \frac{\mu}{2R}\,, \qquad j = \sqrt{\frac{\mu^2}{4R^2} + \frac{3}{\kappa h^2}} \tag{8}$$

The harmonic solutions of impulsive equations lead to the results shown in Eqs. (9) to (12) for natural frequency of vibration of the stored, impulsive mass, stiffness of impulsive mass, and height of impulsive mass, respectively.

$$\Omega^{-2} = -\frac{8R}{27g}\left[\frac{j^2 \exp^{\frac{-2ih}{R}}}{4i\,\mathrm{senh}^2\left(\frac{jh}{R}\right)} - \frac{j^2\cos^2\left(\frac{jh}{R}\right)}{4i\,\mathrm{senh}^2\left(\frac{jh}{R}\right)} - \frac{j^2\cosh\left(\frac{jh}{R}\right)\mathrm{senh}\left(\frac{jh}{R}\right)}{2\,\mathrm{senh}^2\left(\frac{jh}{R}\right)} - \frac{2i^2 - j^2}{4i}\right]$$

$$-\frac{R}{g}\left[\frac{j^2\exp^{\frac{-2ih}{R}}}{4i(i^2-j^2)\mathrm{senh}^2\left(\frac{jh}{R}\right)} + \frac{j\,\mathrm{senh}\left(\frac{2jh}{R}\right)}{4(i^2-j^2)\mathrm{senh}^2\left(\frac{jh}{R}\right)} - \frac{i\cosh\left(\frac{2jh}{R}\right)}{4(i^2-j^2)\mathrm{senh}^2\left(\frac{jh}{R}\right)} + \frac{1}{4i\,\mathrm{senh}^2\left(\frac{jh}{R}\right)}\right] \tag{9}$$

$$M_n = \pi\frac{R^4}{4}\rho.\,g^{-1}.\,\Omega^2 \tag{10}$$

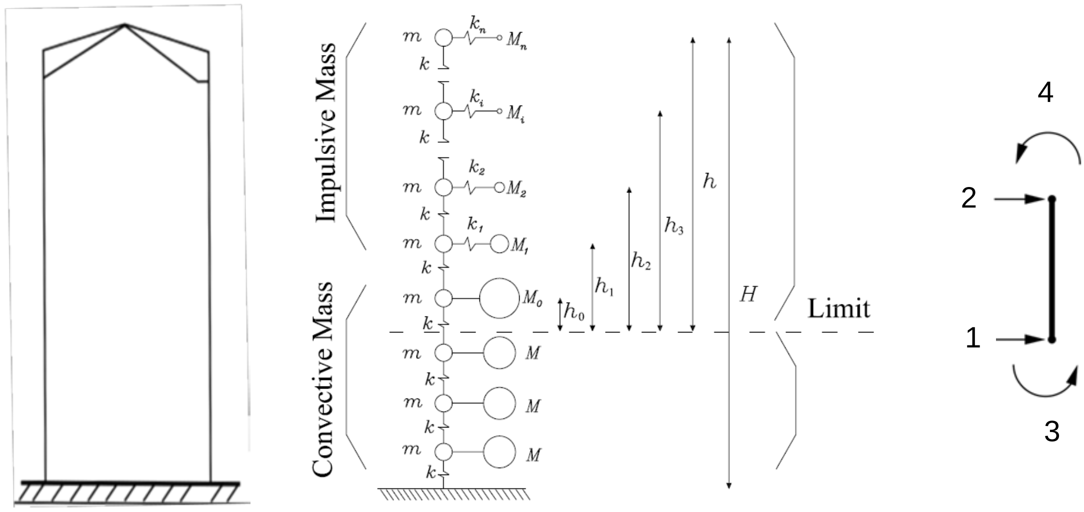
$$K_n = M_n.\,\Omega^2 \tag{11}$$

$$h_n = \frac{31}{57}\frac{\kappa.\,R.\,j.\exp^{-\frac{ih}{R}}}{\mathrm{senh}\left(\frac{jh}{R}\right)} + h - \frac{8}{27}\frac{\kappa.\,j.\,R}{\tanh\left(\frac{jh}{R}\right)} + \frac{8}{27}\kappa.\,i.\,R \tag{12}$$

The values of R in Eqs. (9) to (12) might be replaced by R/(2n-1), to calculate the n^{th} frequency of vibration of the stored material corresponding to the n^{th} mode. The value of n depends on the required precision; however, it is recommended the value of n = 1, since the influence of the impulsive mass when n >1 is very small.

3.2 Structural Model Proposed

From the expressions for evaluating the mass and stiffness of the stored material, it was possible to construct a simple structural model. The circular reinforced concrete silo was divided into a number of discrete beam-type elements, with which it was possible to build the mass matrix and the stiffness matrix of the structure. The mass and stiffness matrix of the stored material was added to the mass and stiffness matrix of the silo, respectively. (Fig. 2).



a) Silo [2] *b) Structural model proposed* *c) Beam type element*

Figure 2 – Structural Model

The following hypotheses were taken for the model shown in Fig. 2:

- Linear elastic behavior was assumed for concrete.
- It has been considered that the structure is embedded in the base.
- The stresses and strains due to gravitational actions were not considered.
- Contributions to the lateral stiffness due to the stored material or hoppers were not considered.
- Movement was considered in one direction.
- Lateral pressure coefficient, friction coefficient and density of stored material were considered the same throughout the height.
- Mohr-Coulomb and Rankine theories were considered valid for the stored material.

Using these guidelines, the mass and stiffness matrixes of the silo-grain system are shown in Eqs. (13) and (14) respectively (upper-case variables for stored material and lower-case variables for silo wall).

$$\boldsymbol{M_s} + \boldsymbol{M_g} = \boldsymbol{M_{s-g}} = \begin{pmatrix} m_{11}+M_0 & \cdots & m_{1n} & 0 & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ m_{n1} & \cdots & m_{nn}+M & \vdots & \vdots & \vdots \\ 0 & \cdots & 0 & M_1 & 0 & 0 \\ 0 & \cdots & 0 & 0 & M_2 & 0 \\ 0 & \cdots & 0 & 0 & 0 & M_3 \end{pmatrix} \tag{13}$$

$$\boldsymbol{K_s} + \boldsymbol{K_g} = \boldsymbol{K_{s-g}} = \begin{pmatrix} k_{11}+K_1 & k_{12} & k_{13} & \cdots & k_{1n} & -K_1 & 0 & 0 \\ k_{21} & k_{22}+K_2 & k_{23} & \cdots & \vdots & 0 & -K_2 & 0 \\ k_{31} & k_{32} & k_{33}+K_3 & \cdots & \vdots & 0 & 0 & -K_3 \\ \vdots & \cdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ k_{n1} & \cdots & \cdots & \cdots & K_{nn} & \vdots & \vdots & \vdots \\ -K_1 & 0 & 0 & \cdots & \cdots & K_1 & 0 & 0 \\ 0 & -K_2 & 0 & \cdots & \cdots & 0 & K_2 & 0 \\ 0 & 0 & -K_3 & \cdots & \cdots & 0 & 0 & K_3 \end{pmatrix} \tag{14}$$

4. Sensibility Analysis

The periods of vibration and mode shapes of the silo-grain system can be calculated using the matrixes depicted in Eqs. (13) and (14). To validate the dynamics of the proposed model, its responses were compared with the responses obtained by using an elastic finite element model. Both models were excited using a set of records scaled to a determined seismic hazard level.

4.1 Modal Analysis and Election of Seismic Hazard Level

Modal analysis was performed to find the seismic responses for linear systems. The n^{th} degrees of freedom system was reduced to "n" systems of one degree of freedom [6]. Newmark's constant average acceleration method (γ= 0.5 and β= 0.25) was used to find the numerical solution of modal analysis. Rayleigh's damping matrix was used considering a critical damping ratio equal to 5% (first and fifth mode). The systems were subjected to a set of acceleration records scaled [7] to be compatible with the spectrum design of the Peruvian code (Z= 0.35, S= 1.0) [3] with R=3 [7] (Fig. 3). This seismic hazard level was chosen just as a reference for the validation of the proposed model.

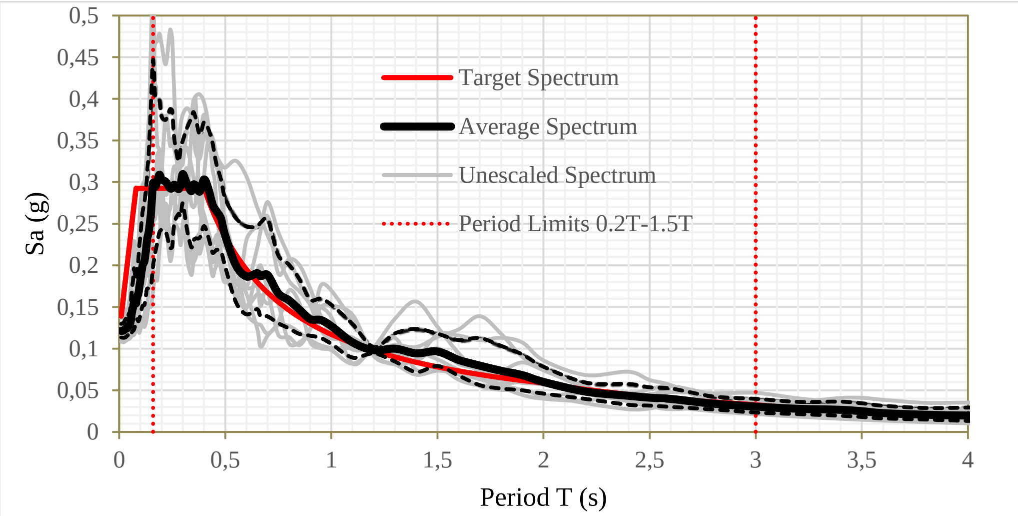


Figure 3. Pseudo-acceleration design spectrum: E030 [3] (red) and arithmetic mean of chosen records (black).

The ground motion records chosen are shown in Table 2. PEER ground motion database was used for selection and scaling the ground motion records [8]. Records were selected based on soil type (semi rigid to rigid) and avoiding scale factors bigger than 4. Records were scaled in order to perform a modal analysis for the proposed model and a time history analysis for the finite element model.

Table 2 – Ground motion records chosen and its scale factor.

SRN	Earthquake Name	Year	Station Name	M_w	R_{jb} (km)	Scale Factor
009	"Borrego"	1942	"El Centro Array #9"	6.50	56.88	1.85
015	"Kern County"	1952	"Taft Lincoln School"	7.36	38.42	0.68
040	"Borrego Mtn"	1968	"San Onofre - So Cal Edison"	6.63	129.11	3.06
065	"San Fernando"	1971	"Gormon - Oso Pump Plant"	6.61	43.95	1.63
166	"Imperial Valley-06"	1979	"Coachella Canal #4"	6.53	49.10	1.06
186	"Imperial Valley-06"	1979	"Niland Fire Station"	6.53	35.64	1.06
299	"Irpinia_ Italy-02"	1980	"Brienza"	6.02	41.73	3.19
328	"Coalinga-01"	1983	"Parkfield - Cholame 3W"	6.36	44.82	1.14

4.2 Analysis of Results and Model Validation

Fundamental periods of vibration of the generated silos were obtained by using finite element analysis, and were compared to those obtained from the proposed model. The maximum percentage of error obtained was -6.72% for the analyzed silos [9].

For seismic responses, Mean responses from modal analysis of silos with chosen records were compared with those obtained using finite element analysis to validate the proposed model. Lateral displacement values obtained are on average 5% higher. Base shear values obtained are on average 12% higher, and overturning moment values are on average 8% higher. The maximum percentage of error obtained for analyzed silos are shown in Table 3. Due to limited space, only 10 analyzed silos are shown.

Table 3. Percentage of error of seismic response of proposed model with respect to response obtained by using finite element analysis with linear elastic materials.

Analyzed Silos						% Error		
ID	D (m)	H/D	D/e	ρ (kg/cm^2)	T (sec)	D_{max}(%)	F_{max}(%)	M_{max}(%)
1	15	3.5	40	1500	0.7819	3.01	16.96	3.82
2	10	4.5	40	1500	0.8283	1.92	11.23	4.51
3	7	5.5	40	1500	0.8505	5.00	11.93	11.51
4	15	3.5	60	1500	0.9472	0.22	7.03	1.69
5	10	4.5	60	1500	1.0041	2.22	8.77	6.65
6	7	5.5	60	1500	1.0313	3.57	13.59	9.68
7	10	5.5	40	1500	1.2123	5.93	11.71	13.66
8	15	4.5	40	1500	1.24	3.53	13.13	7.95
9	10	5.5	60	1500	1.4684	1.40	4.66	6.16
10	15	4.5	60	1500	1.5015	8.13	11.33	11.88

A maximum percentage of error of 16.96% and a minimum percentage of error of 0.22% are shown in Table 3. These percentages of error are less than 20%, which is considered acceptable in Civil Engineering. Moreover, responses obtained by using proposed model are higher than those obtained by using finite element analysis in all the analyzed cases. This seismic response precision of the proposed model lead to the formulation of simplified methods for calculating seismic response of reinforced concrete silos.

5. Simplified method for calculating seismic response

After the validation of the proposed model, it was possible to propose a simplified method through a set of equations that they were found and that only depend on the parameters defining the silo, similar to what was done for chimneys [10]. A new set of cases was generated to verify this equations (Table 4). These cases were analyzed using the proposed model and the response spectrum method. Eqs. (15) to (18) allow to calculating the fundamental period of vibration (T), maximum lateral displacement (X_{max}), maximum base shear (F_{max}), and maximum overturning moment (M_{max}) for silos whose values of parameters are within the domain presented in Table 4. Physical and geometric properties, and the pseudo-acceleration spectrum are parameters included in these equations expresed in the International System of Units (SI).

$$T = \frac{2\pi}{\sqrt{\mathcal{a}\,\frac{K_{SM}}{M_{SM}}}} \tag{15}$$

$$X_{max} = \frac{1}{\mathcal{b}} . \frac{M_{SM}}{K_{SM}} . S_a \tag{16}$$

$$F_{max} = M_{SM} . \frac{\mathcal{c}}{C} . S_a \tag{17}$$

$$M_{max} = F_{max} . H . \mathcal{d} \tag{18}$$

The auxiliary variables $\mathcal{a}$, $\mathcal{b}$, $\mathcal{c}$ and $\mathcal{d}$ are defined in Eqs. (19) to (21).

$$\mathcal{a} = 4.658 - \frac{4.3390}{\left(\frac{H}{D}\right)} + \frac{0.0487}{\left(\frac{\rho}{1000}\right)^2} \quad , \qquad \mathcal{b} = \frac{-2.4141}{\left(\frac{H}{D}\right)} + 2.8366 \tag{19}$$

$$\mathcal{c} = \begin{cases} \dfrac{C}{0.0767 . Ln(T) + 1.3893} , & \text{if } C = 2.5 \\[2ex] \dfrac{C^2}{\left(\dfrac{-1.6211}{\left(\frac{H}{D}\right)^2} + 1.5730\right) C + \left(\dfrac{2.3400}{\left(\frac{H}{D}\right)^2} - 0.4321\right)} , & \text{If: } 0.5 < C < 2.5 \\[2ex] \mathcal{c} = 0.0748 + 1.0123C , & \text{If: } C < 0.5 \end{cases} \tag{20}$$

$$\mathcal{d} = \begin{cases} \dfrac{0.7649C - 0.1773}{C} , & \text{If: } C > 0.5, \\[2ex] \dfrac{0.4477C - 0.0228}{C} , & \text{If } C < 0.5, \end{cases} \tag{21}$$

The variables I_{SM} , E_{SM}, K_{SM} and M_{SM} are the inertia of the silo section, the elasticity modulus of concrete, the lateral stiffness of cantiliever beam of height H and the total mass of the silo respectively. They are defined in Eqs. (22) and (23). The variable S_a (pseudo-acceleration spectrum in meters per second) and C can be calculated from the E030 Peruvian code [3].

$$I_{SM} = \frac{\pi}{4}\left[\left(\frac{D}{2}+e\right)^4 - \left(\frac{D}{2}\right)^4\right] \tag{22}$$

$$K_{SM} = \frac{3.E.I_{SM}}{H^3}, \qquad M_{SM} = \pi H\left\{\rho_c\left[\left(\frac{D}{2}+e\right)^2 - \left(\frac{D}{2}\right)^2\right] + \rho\left(\frac{D}{2}\right)^2\right\}, \tag{23}$$

In addition, the envelop of lateral displacement, base shear and overturning moment along the height of analyzed silos were calculated. The envelope equations are depicted in Eqs. (24) to (26). These equations provide dimensionless values, therefore in order to obtain the required response it is necesary to multiply the result with some of the responses of interes listed in Eqs. (16) to (18) according to the case. The variable z/H is the normalized height of the silo.

Table 4 – Cases analyzed to verify simplified methods from Silos.

Inner Diameter	D (m)	7	8	10	12	15
Total height/Inner Diameter Ratio	H/D	1.5	2.5	3.5	4.5	5.5
Inner Diameter/Thickness Ratio	D/e	30	40	50	60	-
Density of stored material	ρ (kg/m^3)	600	1000	1500	1800	-
Lateral pressure coefficient	κ	0.36	0.5	0.64	-	-
Frictional Coefficient	μ	0.36	0.5	0.64	-	-
Pseudo-acceleration spectrum	S_a	All possible combinations from E030 Peruvian Code				

(-) No value.

$$x\left(\frac{z}{H}\right) = -0.6\left(\frac{z}{H}\right)^3 + \left(0.065\left(\frac{H}{D}\right)^2 + 0.64\left(\frac{H}{D}\right) - 0.128\right)\left(\frac{z}{H}\right)^2 + \left(0.065\left(\frac{H}{D}\right)^2 - 0.64\left(\frac{H}{D}\right) + 1.722\right)\left(\frac{z}{H}\right) \tag{24}$$

$$f\left(\frac{z}{H}\right) = \left(-0.35\left(\frac{H}{D}\right) + 0.36\right)\left(\frac{z}{H}\right)^3 + \left(0.39\left(\frac{H}{D}\right) + 1.37\right)\left(\frac{z}{H}\right)^2 - 0.15\left(\frac{z}{H}\right) + 1 \tag{25}$$

$$m\left(\frac{z}{H}\right) = \left(-0.08\left(\frac{H}{D}\right) + 1.1\right)\left(\frac{z}{H}\right)^3 + \left(-0.0843\left(\frac{H}{D}\right) - 0.4862\right)\left(\frac{z}{H}\right)^2 + \left(-0.16\left(\frac{H}{D}\right) - 1.6\right)\left(\frac{z}{H}\right) + 1 \tag{26}$$

Eqs. (27), (28) and (29) shows the responses of interest in terms of the normalized height.

$$\mathcal{X}\left(\frac{z}{H}\right) = x\left(\frac{z}{H}\right).X_{max} \tag{27}$$

$$\mathcal{F}\left(\frac{z}{H}\right) = \mathscr{f}\left(\frac{z}{H}\right).F_{max} \tag{28}$$

$$\mathcal{M}\left(\frac{z}{H}\right) = \mathscr{m}\left(\frac{z}{H}\right).M_{max} \tag{29}$$

The responses obtained by using Eqs. (15), (17), (18) and (20) were compared with the values obtained by using the SRSS rule for modal combination in response spectrum method. The maximum percentages of error were as follow: a) fundamental period of vibration: 1.53%, b) lateral displacement: 2.67%, c) base shear: 8.44%, and d) overturning moment: 7.82%. Due to limited space, Fig. 4 shows percentage of error of some silos cases evaluated for zone 4 and soil type S_0.

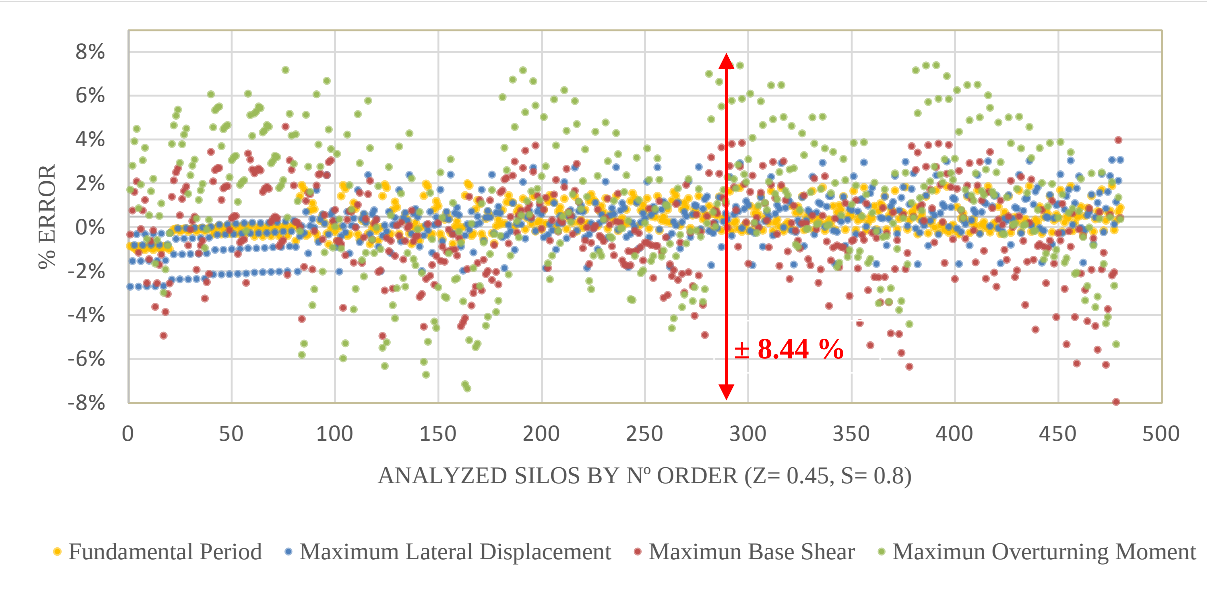


Figure 4. Percentage error of the different responses of the silos analyzed: simplified method versus spectral modal combination.

6. Conclusions and recommendations

6.1 Conclusions

a) The values obtained by using the proposed model are quite approximate to those obtained by finite element analysis. Results are on average 10% higher and in a band of 0% to 20%. Moreover, responses obtained by using proposed model are higher than those obtained by using finite element analysis in all the analyzed cases, making the model a bit conservative in addition to its accuracy.
b) The Slenderness or H/D ratio is the most incident parameter. The parameters D, D/e y ρ are also important in calculating seismic response. Although the parameters κ y μ happen to be less incident, these are useful for defining the convective and impulsive parameters.
c) The simplified methods for calculating seismic response gave very close values to those obtained by using spectral modal combination. The maximum error obtained for the proposed expressions was 8.44%. These equations have the advantage of offering quick results at the cost of only requiring the entry of geometrical and physical parameters of the studied system.

6.2 Recommendations

The authors suggest using nonlinear inelastic models for the stored material, if seismic responses of slender silos (H/D> 3.5) are needed. Seismic responses may be lower than actual in some cases if either a spectral modal combination or a time history analysis are performed, as verified by other authors [4, 11].

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