



FASTEST POSSIBLE NONLINEAR TIME-HISTORY SEISMIC ANALYSIS OF BRIDGE FRAME STRUCTURES

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Abstract

In California and elsewhere, bridge frame structures are designed to respond nonlinearly in high seismic areas, but displacement demand is typically calculated using linear-elastic methods. Nonlinear behavior is often recognized, however, in the displacement capacity determination using a pushover analysis. So there is a clear inconsistency between the way demand and capacity displacements are found, and it arises because of difficulty finding displacement demands using nonlinear analysis methods. A full nonlinear time-history analysis (NTHA) is required to accurately capture the seismic response of a bridge frame over time and to find the maximum displacement – the displacement demand. This more advanced approach of NTHA is solved by the stiffness method, and is not used in the design of everyday highway bridge structures because of the time it takes to run the analysis, instabilities, and overall added difficulty in model preparation, input and output compared to linear-elastic methods. Also because prior to the earthquake the input motion is not known, requiring many different ground motions and analyses to bound the results of a future earthquake. It is just not practical to use NTHA for the design of bridge structures with the currently available analysis tools that are based on the stiffness method.

A new method has been developed for NTHA of bridge frame structures that does not depend on the stiffness method or matrix mathematics. Rather it uses closed-form equations that are exact for each time increment. The incremental closed-form method (ICFM) is 1000s of times faster than the traditional stiffness method because (1) there are no simultaneous equations to solve, (2) there is no iteration required and (3) it is a stable solution scheme. This new method was initially presented by the author (prior to completion of the computer program) at the 15th World Conference on Earthquake Engineering (15WCEE) in Lisbon, Portugal and (with the computer program fully working) at the Ninth International Conference on Structural Dynamics (EURODYN 2014) in Porto, Portugal. An example multi-span bridge frame showed that as complexity was added to the model, the ICFM continued to outpace the stiffness approach, while producing the same results; with nonlinear behavior representing plastic hinges at all column ends, as well as banging and soil crushing behind seat-type abutments, the closed-form approach was more than 25,000 times faster than the stiffness method. Such speed increases will allow the ICFM to be used directly by design engineers for NTHA of everyday bridge frame structures, resulting in realistic displacement demands. Because of the tremendous time savings using the ICFM, there is no difficulty running multiple earthquake motions through the structure, one after the other, to determine displacement demand from a future, un-defined earthquake.

In the current paper, the author presents and discusses new features of the ICFM, and related computer program, that significantly furthers its capabilities.

Keywords: Time-History Analysis; Nonlinear Behavior; Bridge Frame; Plastic Hinge; Exact Closed-Form



1. Introduction

Nonlinear time-history analysis (NTHA) of a bridge frame structure is required to determine a realistic assessment of displacement demands from a severe earthquake. This is because bridge structures are designed to form plastic hinges at the column ends [1], protecting the footings and superstructure from inelastic action through capacity design principles [2]. Typically in California the supports at the two ends of the bridge are seat-type abutments [1], with a gap provided for temperature expansion that is closed once severe ground shaking occurs. The superstructure moves into the abutment backwall, which breaks off at its base and activates the full capacity of the soil behind the abutment. So the two critical nonlinear behaviors that must be included in a time-history analysis are (1) cyclic plastic hinging at column ends where the moment is maximum, and (2) engaging, crushing and subsequent gapping of soil behind abutments. Both of these behaviors have distinct hysteretic responses that must be included in the time-history analysis to have realistic results. P-Delta effects can also result in changes to the expected displacement demands and should be included.

Traditional NTHA of a bridge frame is conducted using the stiffness method, requiring multiple simultaneous equations to be solved at each time increment for the duration of the earthquake, and when change in stiffness is great many iterations are needed for convergence to be satisfied when nonlinear events occur. This leads to a very slow analysis for the thousands of time steps in a typical earthquake record that may not converge, regardless of the number of iterations, when significant nonlinearities and sudden strong shaking develop – the analysis stops at this point since all future responses of the bridge are dependent on the prior results. And, of course, the results of most interest with largest bridge demands come from the strongest ground shaking. Therefore when convergence problems do develop, not only is the remainder of the response not available, the analysis results up to that point can't be used since it is expected that larger results would have developed beyond that point in time. Because of the (1) slow analysis time, (2) convergence problems and (3) need to run multiple earthquake records, nonlinear time-history analysis is not used for everyday bridge design in California, choosing instead simplified linear-elastic methods. Displacement capacity of the bridge frame is found from monotonic nonlinear pushover analysis and compared to displacement demand from linear-elastic methods [1]. The concern is that the displacement demand from linear-elastic approximations could be in error by a factor of two or more, resulting in the incorrect conclusion that displacement capacity (which is determined realistically using nonlinear pushover analysis) is greater than displacement demand and that the bridge is safe from collapse.

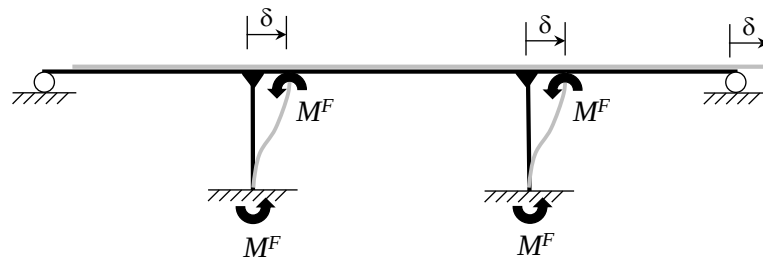
This paper presents a novel method for NTHA that can be used for everyday bridge design and, hence, replace the linear-elastic methods currently in use. The author has presented earlier versions of this method elsewhere [3, 4]. The closed-form method provides identical results to the stiffness method but does not require setting up and solving simultaneous equations nor does it require any iterations for nonlinear events to occur. Therefore it is a stable solution scheme that always converges. For each time step the exact incremental results are found in a closed-form and if a nonlinear event develops within the time step, the time when the event occurred is directly determined and the solution is backed up to that point, again with no iterations required. Since the solution at each time step is found from closed-form equations, the method can be considered analytical rather than numerical, which provides stability. Examples are provided in the paper that show how much faster and stable the new approach is compared to the traditional stiffness method, making this new and novel method a true replacement to the stiffness method and to the linear-elastic approaches currently used in design of bridge structures.

2. Theory of Incremental Closed-Form Method

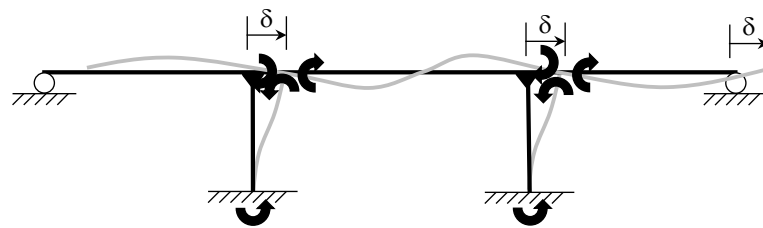
The Incremental Closed-Form Method (ICFM) was created by extending the closed-form approach [5] developed by the author. In the closed-form approach, final member-end-moments are found from equations for a single loading case, producing results that do not require the solution of simultaneous equations as in the stiffness method nor distributing moments back and forth as in moment distribution [6]. The closed-form

equations were derived from a combination of moment distribution and calculus. Once these end moments are known, statics allows the remainder of the member forces and reactions to be determined.

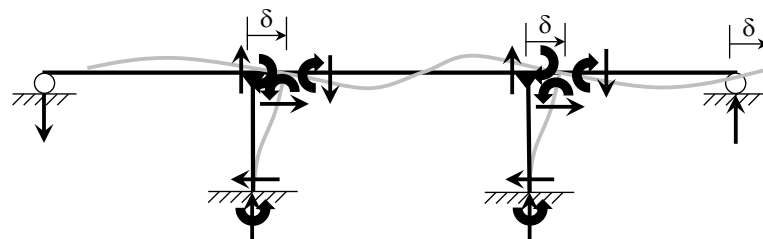
In the ICFM the closed-form approach is applied at each time step of an earthquake. First the change in frame displacement develops while the joints are fixed from rotation, resulting in fixed-end-moments at the column ends (Fig. 1a). Incremental member-end-moments are then found from the closed-form equations while holding the same displacement (Fig. 1b), followed by statics to solve for all other member forces and reactions (Fig. 1c).



(a) Incremental fixed-end-moments



(b) Closed-form equations provide final incremental member-end-moments



(c) Statics results in incremental shear forces, axial forces and reactions

Fig. 1 – Incremental member-end-moments

Nonlinear moment-rotation springs are given at the column ends, allowing any hysteretic behavior to be provided for the column plastic hinges at these critical locations (Fig. 2). When a nonlinear event develops within a time step, all of the results are returned to the prior time step, the time to the nonlinear event is found exactly and an additional increment is added at that time, with renumbering of subsequent increments (Fig. 3). This time shift requires no iterations.

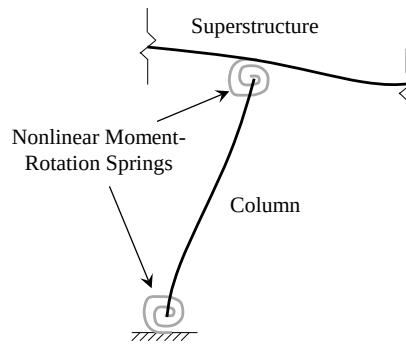


Fig. 2 – Column with end plastic hinges

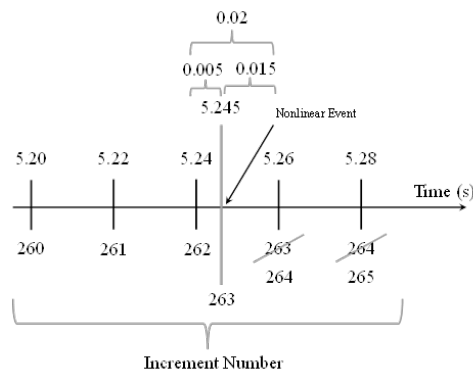


Fig. 3 – Nonlinear event time shift with no iterations required

A flowchart is given in Fig. 4 which demonstrates the order of ICFM analysis in a computer program written in FORTRAN by the author. The program uses the average acceleration method [7] which is unconditionally stable to advance the solution forward from one time step to the next. Based on the current frame stiffness and earthquake input over a given time step, incremental displacement, velocity, acceleration and member-end-moments are found, and summed to prior results in order to determine the total response up to that time. This is continued until all earthquake time steps have been completed.

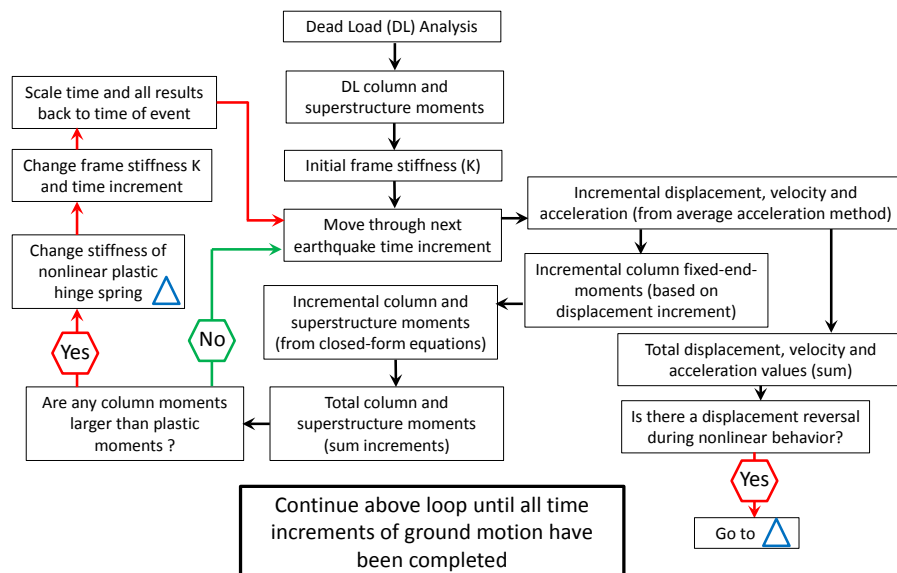


Fig. 4 – ICFM flowchart

3. Example Bridge Frame NTHA

A five-span, reinforced concrete bridge frame is given in Fig. 5 for the purpose of comparing the accuracy, speed and stability of the ICFM to the stiffness method. Provided in this figure is the elevation view of the entire frame and a cross-sectional view of the superstructure. Plastic moments can develop at both ends of each of the columns, with the plastic moment capacity provided in Fig. 6. Any type of moment-rotation hysteretic behavior can be programmed for the column ends representing cyclic plastic hinging (Fig. 7). At this point the elasto-plastic hinge (Fig. 7a) is fully working and verified while the Pivot Model [8] stiffness degrading plastic hinge (Fig. 7b) is currently being programmed and is expected to be fully working by conference time. And this should provide some interesting results with significant softening of the frame once plastic hinges have developed.

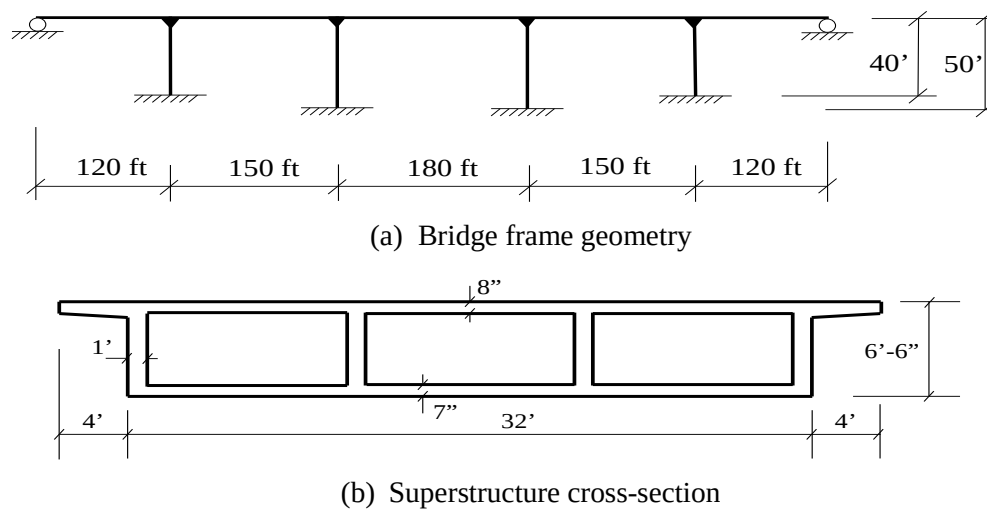


Fig. 5 – Bridge frame structure used in example NTHA

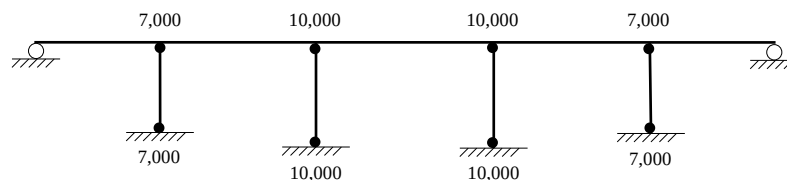
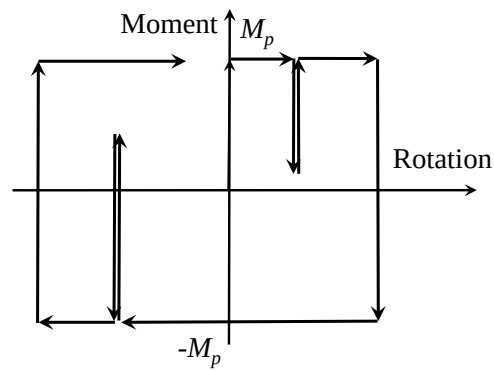
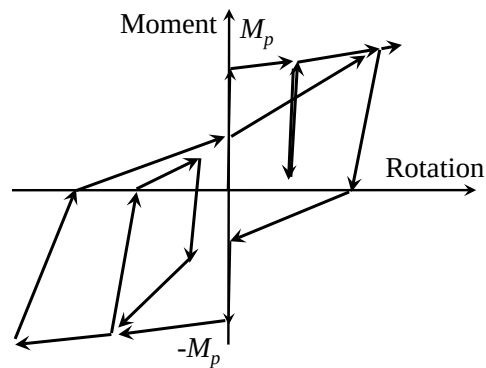


Fig. 6 – Column plastic moment values in kip-ft

Minimal required input for the ICFM makes the setup for nonlinear analysis simple (see Fig. 8 for the complete text input for the example bridge frame). Note that so long as they are consistent, any units can be used. The units themselves are not required in the input nor given in the output.



(a) Elasto-plastic



(b) Stiffness degrading (Pivot Hysteresis Model)

Fig. 7 – Different plastic hinge moment-rotation hysteresis models for ICFM

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Number of spans
5
Span, length, E, I
1 120 580464 439
2 150 580464 439
3 180 580464 439
4 150 580464 439
5 120 580464 439
Column, length, fixity, E, I
1 40 2 580464 22.46
2 50 2 580464 22.46
3 50 2 580464 22.46
4 40 2 580464 22.46
Span, wload, pload
1 10.93 0
2 10.93 0
3 10.93 0
4 10.93 0
5 10.93 0
Column, plastic moment capacity
1 7000 7000
2 10000 10000
3 10000 10000
4 7000 7000
Mass and damping
270.8 0.02
EQ file name
EQ.txt

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Fig. 8 – Text input file for ICFM (no units are required so long as consistent units are used)



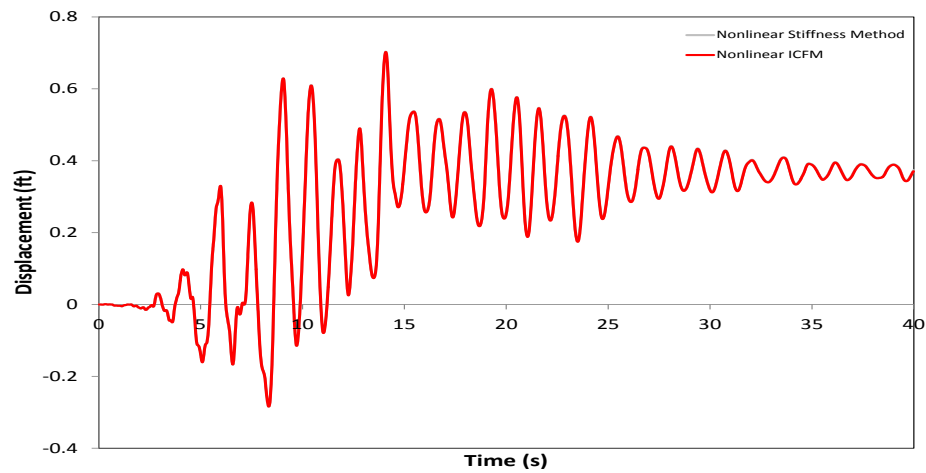
3.1 Analysis Results

Analysis times for three different cases are given in Table 1. There are two earthquake records used with no abutments included in resisting the longitudinal movement and one earthquake record where nonlinear behavior of the end abutments was included. As indicated in the table, in all cases the ICFM was thousands of times faster than the stiffness method, with increased relative efficiency as more details were added. With the abutments included in the response, the ICFM is more than 25,000 times faster than the stiffness method (given as Ratio of Times in Table 1), and in this case the stiffness method produced approximate results while the ICFM gave exact values for all time increments. For this one earthquake record the stiffness method took over 42 minutes of running time while the ICFM solved the problem in less than one tenth of a second.

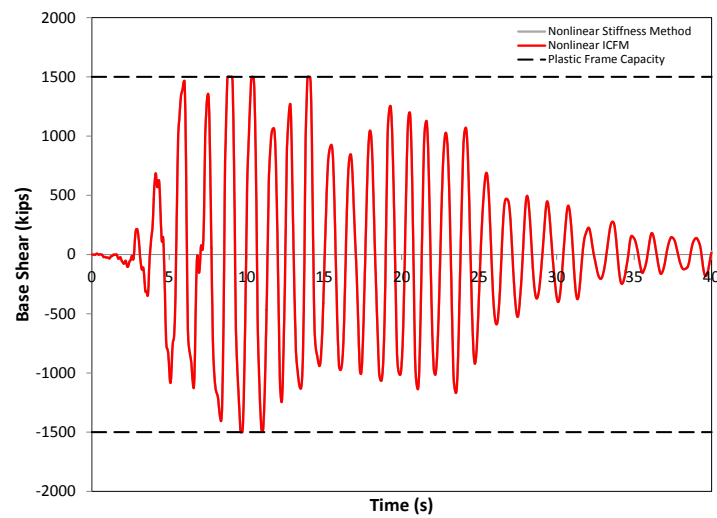
Table 1 – Solution times for ICFM and the Stiffness Method

Frame Condition	Earthquake File	Duration (s)	Time Increment (s)	Number of Increments	Stiffness Method Solution Time	Closed-Form Solution Time	Ratio of Times
Bridge Frame	1989 Loma Prieta EQ, Capitola Fire Station E/W Ch3	40	0.02	2,000	204.8 s (3.41 min)	0.06240 s	3,282
Bridge Frame with Abutments	1989 Loma Prieta EQ, Capitola Fire Station E/W Ch3	40	0.02	2,000	2,533 s (42 min)	0.0936 s	27,062
Bridge Frame	2010 Mexicali EQ, Calexico - El Centro Array 11 N/S Ch1	50	0.005	10,000	1,410 s (23.5 min)	0.2808 s	5,021

Nonlinear time-history analysis results are given for the ICFM and the stiffness method with the bridge subjected to the 1989 Loma Prieta earthquake record given in Table 1. Results provided in Figs. 9 through 12 do not include the longitudinal influence of the abutments, which is the same as providing large enough initial gaps so that the superstructure never impacts the backwall of either seat-type abutment. As is clear from these figures, the response of the bridge frame in terms of relative displacement (Fig. 9a), restoring force (or base shear) (Fig. 9b), and hysteretic force-deformation behavior (Fig. 10) the new analysis approach (ICFM) matches the stiffness method, capturing all nonlinear nuances for the duration of the earthquake – but at much faster speed as shown in Table 1. Both methods reach the plastic frame capacity of 1500 kips in the positive and negative directions several times in the earthquake (Fig. 9b and Fig. 10), but do not exceed this value indicating good convergence. This plastic frame capacity occurs when all plastic hinges have developed and can be determined by simply summing the column plastic shears, which are found from adding the plastic moment capacities at the two column ends (given in Fig. 6) and dividing by the column height. Fig. 11 demonstrates how much less force develops in the frame when nonlinear column behavior is considered compared to linear-elastic response with the same initial frame stiffness. By adding P-Delta effects, negative stiffness developed causing the stiffness method to not converge while the ICFM was able to complete the analysis successfully (Fig. 12). It is interesting to see that larger displacements occurred with P-Delta included, and in the direction of maximum displacement.



(a) Relative displacement



(b) Base shear

Fig. 9 – Time-history responses of bridge frame from ICFM and the Stiffness Method

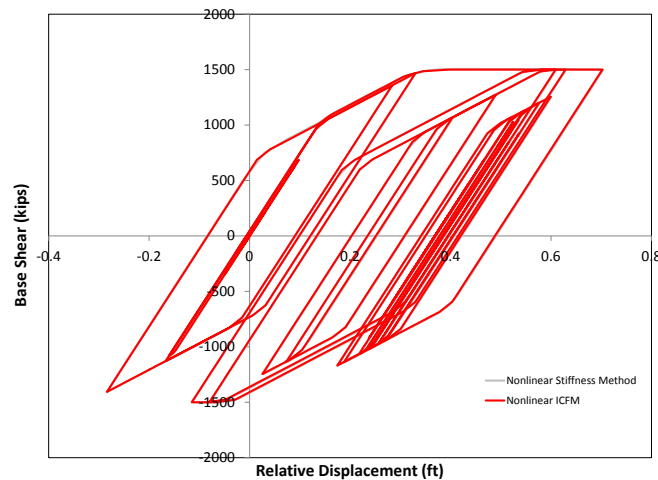


Fig. 10 – Hysteretic force-displacement time-history responses of bridge frame from the ICFM and the Stiffness Method

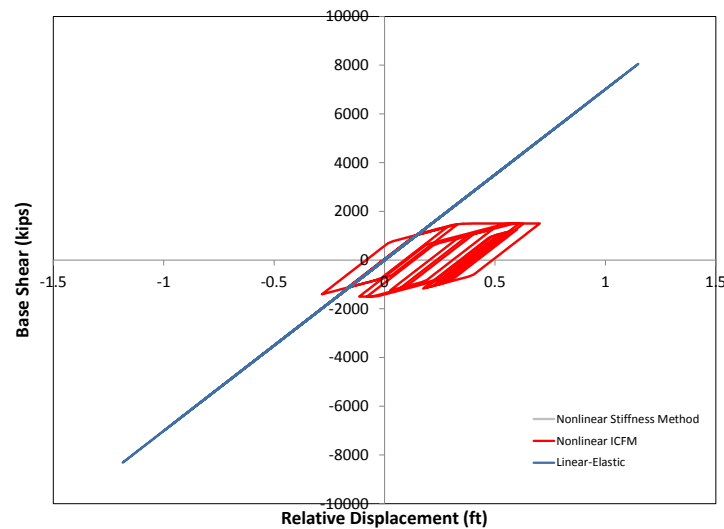


Fig. 11 – Comparison of nonlinear frame response to linear-elastic behavior with same initial stiffness

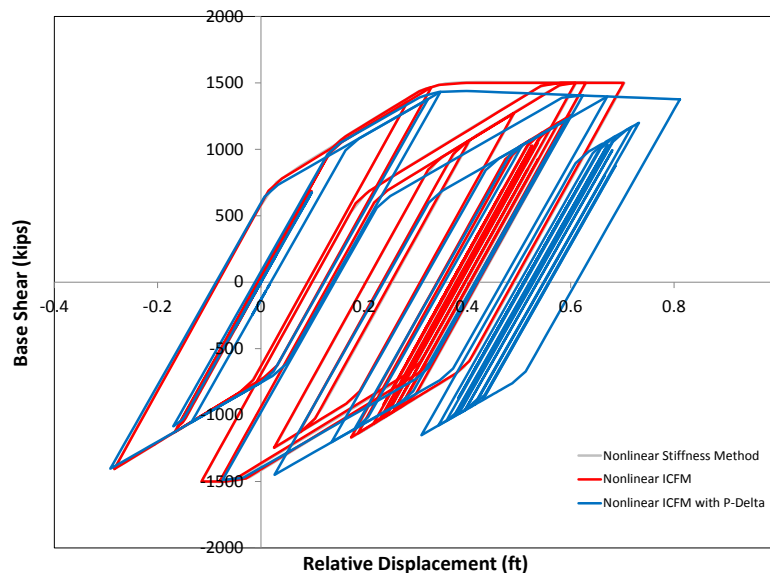
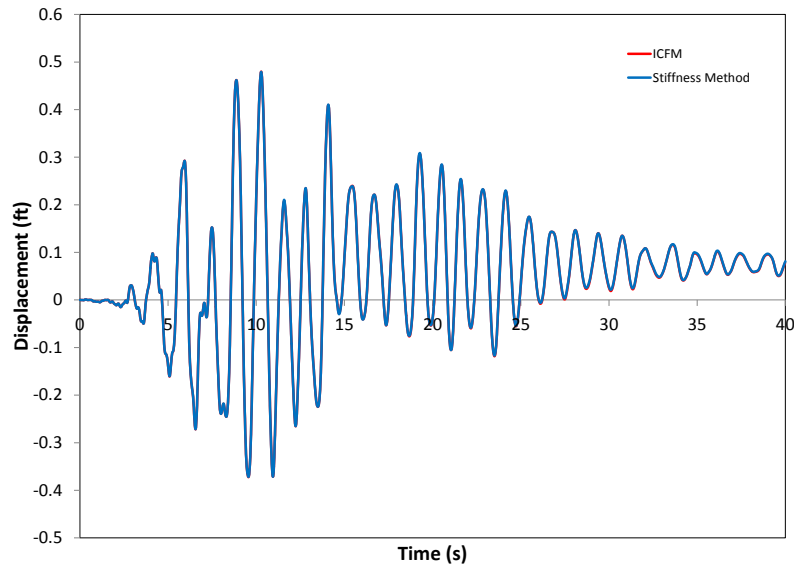


Fig. 12 – Response of bridge frame with P-Delta effects included (Stiffness Method did not converge)

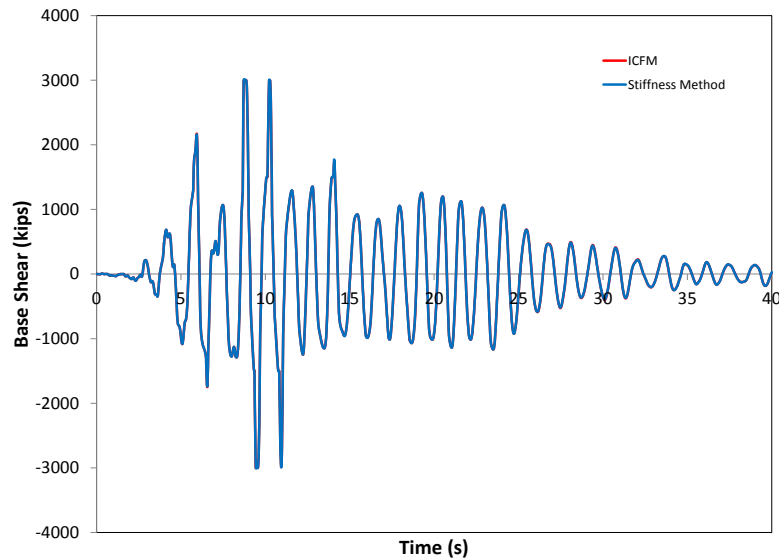
Seat-type bridge abutments are included in the analysis presented in Figs. 13 through 15. As shown in Table 1, for this analysis the ICFM was more than 25,000 times faster than the stiffness method, resulting in less than one tenth of a second for ICFM versus over 42 minutes of running time for the stiffness method. Furthermore, while the ICFM provides exact results for all time increments, the stiffness method gave approximate results since it was not possible to include infinite stiffness for the compression-only gap elements that activate the abutments. However, as seen in the frame relative displacement (Fig. 13a) and restoring force (Fig. 13b) the results from the two methods follow each other very close over time. It is only when plotting the frame hysteretic force-deformation response (Fig. 14) that a slight difference is seen in the results from the two methods, with the stiffness method providing approximate values as mentioned above. Prior to closing the initial gap in each direction the response follows the behavior with no abutments included. But once this gap is exceeded the initial stiffness of the soil is superactivated and then the soil crushes at 1500 kips. Since only one abutment is activated at a time (when the superstructure is moving into it and away from the other abutment), the total force capacity is 3000 kips, which is the longitudinal frame capacity with all plastic hinges activated plus the plastic capacity of the soil behind one abutment. Plotting the abutment force versus time - at both abutments



(A1 and A2) - shows that only a few impacts occur (Fig. 15). This is because after plastic crushing of the soil behind the backwall a larger displacement than the prior maximum plastic displacement is required to close the new gap that has opened up. In essence, the original gap between the superstructure and backwall has increased, making it more difficult for the superstructure to reach the backwall in future cycles. Abutment force results from the ICFM and stiffness method match well for the duration of loading.



(a) Relative displacement



(b) Base shear

Fig. 13 – Time-history responses of bridge frame with abutments included from ICFM and the Stiffness Method

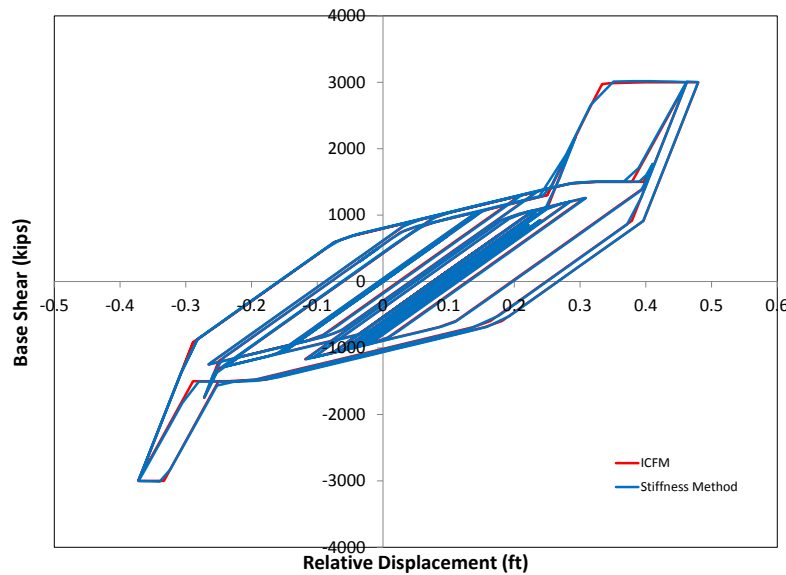


Fig. 14 – Hysteretic force-displacement time-history responses of bridge frame including abutments from the ICFM and the Stiffness Method

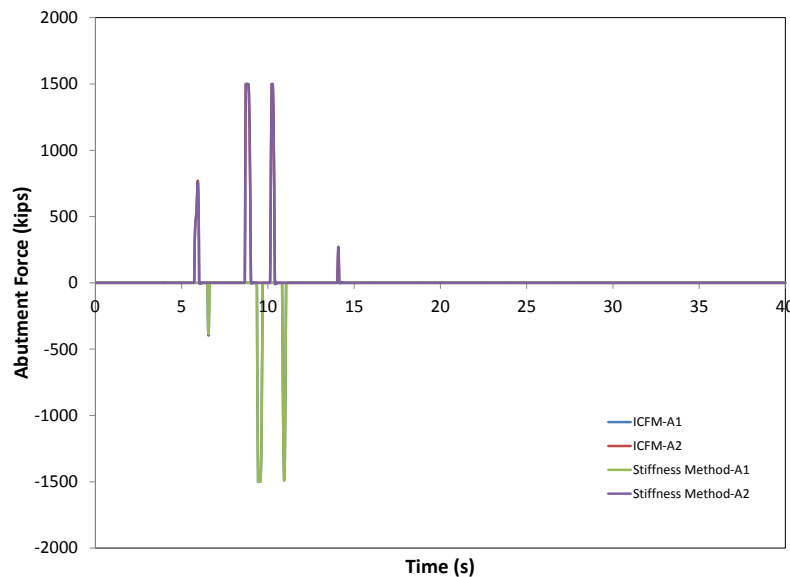


Fig. 15 – Force time-history responses of abutments from the ICFM and the Stiffness Method

3.2 Convergence Verification

Like most nonlinear analysis, the question of convergence and verification comes up. In the computer program using the ICFM developed by the author, a plot is provided that is intended to demonstrate that the nonlinear analysis for the entire frame has converged. The idea is that energy principles must not be violated; energy is dissipated by the nonlinear response of the bridge frame and not created. This is shown in Fig. 16 where total dissipated energy of the bridge frame per hysteretic loop is plotted versus loop number. A loop is defined by displacement reversals. The energy dissipated in any loop (area under curve between displacement reversals) must be positive or exactly zero. If any of the reversal loops violate this then it is clear that the solution has lost its way and did not converge. Results in Fig. 16 demonstrate that the ICFM has perfect convergence and is stable. It also shows that there were close to 100 reversals.

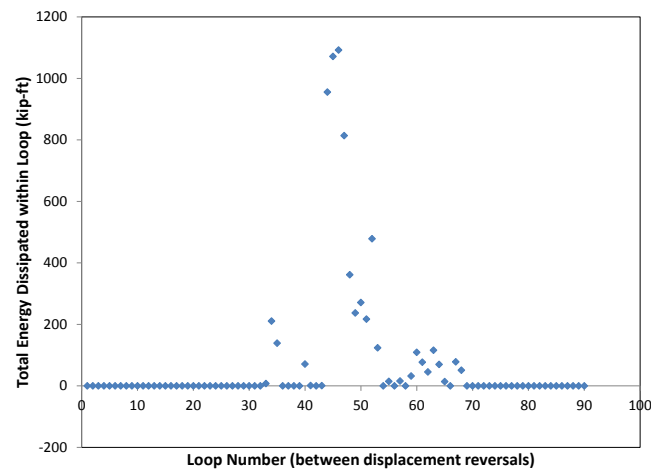


Fig. 16 – Dissipated energy from hysteretic behavior of bridge frame

4. Conclusions

A new and novel method has been developed to determine the nonlinear time-history response of a bridge frame under seismic attack. The progress of this method (and associated computer program) is on-going with various additions planned or underway. Shear deformations have been included in modified closed-form equations [9] - originally derived based on flexure only – but shear flexibility has not yet been added to the computer program. Inclusion of axial deformations is a current hot topic in the closed-form approach, with extensive derivation work being carried out to modify the existing equations so that exact member-end-moment equations that recognize flexure, shear and axial deformations for all members of a bridge frame structure are found. Prior to adding these modifications to the computer program the equations must be fully developed.

As discussed in the body of the paper, the ICFM gives identical results to the stiffness method but is 1000s of times faster, and stable, making it a convenient and realistic tool for everyday seismic bridge design. The results are identical when the ICFM and the stiffness method include only flexural deformations. With the addition of shear deformations [9], the results are also identical between the two analysis methods. This is why there is so much desire at this time to add axial deformations to the closed-form method, allowing the ICFM to give identical results to the stiffness method with all deformation modes included.

5. References

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