



## SENSITIVITY ANALYSIS OF IN-PLANE STIFFNESS OF CLT WALLS WITH OPENINGS

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### Abstract

Cross-laminated timber (CLT) is a relatively new product that is gaining popularity in residential and non-residential applications in the North American construction market. As an engineered wood product categorized as “mass timber”, CLT provides many benefits when compared to either light-frame wood construction or concrete and steel construction. The cross-lamination itself provides improved dimensional stability to the product that allows for prefabrication of long floor slabs and single storey walls. The good thermal insulation and a fairly good behaviour in case of fire are added benefits resulting from the massiveness of the wood structure. Furthermore, it is a clean product to work with resulting in much less waste and dust produced on site which is better in terms of health and safety.

An accurate quantification of in-plane stiffness of the CLT panels and shearwalls is required to design a CLT structure subjected to lateral loads. Nevertheless, till today, no general approach is available for the design of CLT-members loaded in plane. In addition, there are no standardized methods for determining the stiffness of CLT shearwalls in the respective material design standards: the CSA O86 in Canada, and the NDS in the US. Therefore, the present study aims to quantify the stiffness of CLT walls under in-plane loading. A finite element analysis (FEA) model of CLT walls was developed in ANSYS. Wood was modelled using orthotropic elastic material, where the glue-line in between panels was modelled using non-linear contact element. The material properties were adopted from experiments. The FEA model was verified from test results of CLT panels under in-plane loading. The stiffness was calculated from the FEA and compared against existing analytical models. Results showed that the FE analyses were sufficiently accurate in predicting elastic stiffness of CLT panels. A parametric study was performed to evaluate the change in stiffness of CLT walls with the variation of opening size and shape. A simplified equation to predict the in-plane stiffness of CLT walls with openings was proposed. Subsequently, a sensitivity analysis was performed using Meta-model of Optimal Prognosis (MOP) to evaluate the contribution of each parameter on the model response.

**Keywords:** Cross-laminated timber (CLT); stiffness; shearwalls; finite element; sensitivity analysis.



## 1. Introduction

Cross-laminated timber (CLT) is an engineered wood product categorized as “mass-timber”. CLT is gaining popularity because of its benefits when compared to concrete, steel, or light-frame wood construction: it has low carbon footprint due to low embodied greenhouse gas emissions and carbon storage capacity of wood. The cross lamination provides dimensional stability, strength, and rigidity. The good thermal insulation and a fairly good behaviour in case of fire are added benefits. CLT panels consist of several layers of boards stacked crosswise and glued together. An uneven number of layers of boards glued orthogonally to form a solid CLT panel.

CLT offers large opportunities to be applied in lateral load-resisting systems (LLRS) in seismic zones. Understanding of the mechanical properties of CLT panels and connectors connecting them is needed to design CLT shear walls. Several studies investigated the mechanical properties of CLT panels loaded in plane. Blass and Fellmoser (2004) developed a method for the design of CLT panels under in-plane loading based on the composite theory. The k-factors were proposed to account for the strength and stiffness of CLT panels in various directions, based on single layer properties. The effective bending stiffness under in-plane loading can be calculated using Eq. (1) and Eq. (2).

$$E_{\text{eff}} = E_0 \cdot k_3 \quad (1)$$

$$k_3 = 1 - \left( 1 - \frac{E_{90}}{E_0} \right) \frac{a_{m-2} - a_{m-4} + \dots \pm a_1}{a_m} \quad (2)$$

where,  $E_{\text{eff}}$  is the effective stiffness of CLT panel,  $E_0$  is the stiffness of a homogeneous cross-section,  $k_3$  is the stiffness ratio, and  $a_m$  is the thickness of  $m$  layers.

Moosbrugger et al. (2006) proposed a model based on the regular periodic internal geometric structure of CLT elements, considering uniform shear loading on wall boundaries. They defined the complex internal structure of CLT elements with a unit cell called Representative Volume Element (RVE). The RVE extends over the plate thickness and is sub-divided into Representative Volume Sub-Elements (RVSE). Total shear loading was decomposed into two basic mechanisms: pure shear in a single board (mechanism I), and torsional-like behaviour in the glue interface between two boards (mechanism II) and proposed a simplified analytical model:

$$\frac{G^*}{G} = \frac{1}{G(1/G_1 + 1/G_2)} = \frac{1}{1 + 3(G/G_{\text{eff}})(t_i/a)^2} \quad (3)$$

where,  $G^*$  is the equivalent shear modulus,  $G$  is the shear modulus parallel to the grain, and  $t_i/a$  is the board thickness-to-width ratio

Bogensperger et al. (2007) carried out experiments on CLT panels with a 3-point bending/shear test configuration according to EN 384 (2004). The results were verified with FEA. A discrepancy in stresses and strains between an ideal CLT-structure and the test configuration under shear load was observed. Jobstl et al. (2008) argued that the Common Understanding of Assessment Procedure (CUAP) that uses 4-point bending test to determine the in-plane shear strength of CLT panels, does not lead to shear failure in most cases and further investigated the in-plane behaviour of CLT panels. Ninety CLT specimens consisting of 3 and 5 layers of boards were tested using CUAP test configuration. It was observed that none of them failed in shear parallel to the grain with few exceptions that failed in rolling shear. Consequently, a new symmetric test configuration was proposed and subsequently used for determining the shear properties. Bogensperger et al. (2010) also performed FE analysis and further verified the previous studies by Moosbrugger et al. (2006) and Jobstl et al. (2008). They investigated the effect of boundary conditions on the shear stiffness of the CLT elements. They introduced a correction factor to calculate the shear modulus for three, five, and seven layers of CLT panels. Their FE model accurately predicted the effective shear of CLT panels.

Brandner et al. (2013) investigated the influence of test configuration and other parameters affecting the shear strength of CLT panels loaded in-plane. They proposed a new test configuration rotating the test specimen by 14° from its original configuration, so that the resultant loading and support remain in-plane. The test setup



and specimen configuration allow for only one failure plane to develop compared to two shear planes in the previous setup. The study showed that parameters that significantly affect the shear strength of CLT elements are the core lamella thickness, the annual ring orientation, and the width of the gap between the boards. Flaig and Blass (2013) developed analytical equations for shear stress and stiffness and verified them with test results. It was observed that the effective shear strength of CLT beams strongly depends on cross sectional arrangement, thickness ratio of longitudinal and transversal layers, and on the width of lamellae. The proposed equations were successfully verified with tests performed on prismatic beams, notched beams, and beams with holes.

When using CLT as LLRS in structures, openings for doors and windows are very common in the CLT wall panels. The areas around an opening experience stress concentrations that can reduce in-plane stiffness and load bearing capacity of the panel. In case of light-frame wood shear walls, often only the full wall segments are taken into consideration when determining the wall resistance. While this approach may be appropriate for light-frame wood shear walls, it can lead to significant underestimation of the stiffness and resistance of CLT walls.

Moosbrugger et al. (2006) performed FEA to quantify the stiffness of a CLT panel with a quadratic opening at the center and determined the reduced stiffness of the CLT wall with opening by taking the ratio of the effective wall area ( $A_{\text{wall}}$ ) to total area ( $A_{\text{total}}$ ) where,  $A_{\text{wall}} = A_{\text{total}} - A_{\text{opening}}$ . The results overestimated the reduced stiffness when compared tests. Dujic et al. (2007) experimentally investigated the behavior of CLT wall panels with different opening locations. Four cyclic tests were performed and it was observed that for a wall with an opening equal to 30% of the wall area, the strength of the wall did not change. The stiffness, however, was reduced by about 50%. A FE parametric study determined the influence of the size and layout of openings on the strength and the stiffness of CLT walls. Eq. (4) was proposed to calculate the reduced stiffness of the CLT wall panels:

$$K_{\text{opening}} = K_{\text{full}} \frac{r}{2-r} \quad (4)$$

where,  $K_{\text{opening}}$  is the stiffness of CLT walls with opening, and  $r$  is the panel area ratio given in Eq. 5.

$$r = \frac{H \sum L_i}{H \sum L_i + \sum A_i} \quad (5)$$

where,  $H$  is the height of wall,  $\sum L_i$  is the summation of length of full height wall segments (excluding length of openings from total length), and  $\sum A_i$  is the summation of openings area.

Ashtari (2012) investigated the in-plane stiffness of CLT floor diaphragms using FEA. Models of four different CLT floor diaphragm configurations with and without opening were analyzed. A smeared panel-to-panel connection model was developed by calibrating it with test results from Yawalata and Lam (2011). However, the applicability of FEA was limited as these tests were conducted with a special connection with self-tapping wood screws. Pai (2014) investigated the mechanism of force transfer around openings in CLT shearwalls and reinforcement requirements for the corners of the opening. Four different models from literature calculating the transfer forces in wood frame shearwalls were evaluated. Among them Diekmann's model (1995) was found to be the most suitable for CLT shearwalls with opening.

Accurate quantification of the in-plane stiffness of the shear-walls is required to design a CLT structure subjected to lateral loads. Nevertheless, till today, no general approach is available for the design of CLT-members loaded in plane. In fact, the strength and the stiffness properties reported in different technical approvals for verification of in plane resistance of CLT walls vary significantly (Flaig and Blass 2013). In addition, there are no standardized methods for determining the stiffness and resistance of CLT shearwalls in the respective material design standards: the CSA O86 (2014) in Canada, and the NDS (2015) in the US. The objectives of this research are: i) to calculate the in-plane stiffness of CLT walls with and without openings, and ii) to perform the sensitivity analysis of the proposed analytical models to evaluate the contribution of each parameter affecting the in-plane stiffness.

## 2. Numerical and Analytical Investigation on CLT walls with openings

### 2.1 Model development and validation

A 3D Finite Element Analysis (FEA) model was developed in ANSYS. The CLT panel was modelled using 20-node solid elements where each node has three degrees of freedom. Elastic material properties were assigned in each orthogonal direction of the lamella as provided by the manufacturers' specifications. The glue-line between lamellas was modelled using contact elements with a friction coefficient of 1.0. Two sets of experiments were used for model validation. The first set consisted of four-point bending tests on CLT panels at FPInnovations, Quebec, Canada. Three series of CLT panels (2 replicates of each type) were tested with a span length of 3.5m, 5.9m, and 8.4m, see Fig. 1. The specimens were 1.2m high and laterally supported. The 5-ply boards with a thickness of 175mm were from Canadian S-P-F. The second set of tests consisted of quasi-static monotonic tests on CLT walls at FPInnovations, Vancouver, Canada (Popovski et al. 2010). The CLT panels were 3-ply with 2.3m x 2.3m panel size and thickness of 94mm made of European spruce. Several types of connectors (hold-downs and steel brackets) and fasteners (annular ring nails, spiral nails, screws, and timber rivets) were used for the connections (Popovski et al. 2010).

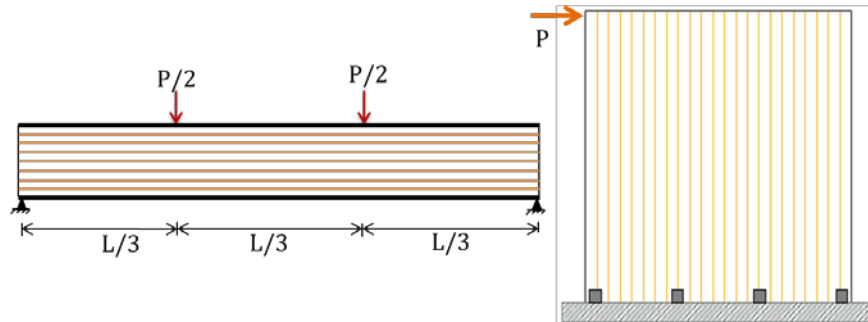


Fig. 1 – CLT beam (left) and wall (right) test configurations

The FEA was validated using the elastic part of load- as seen in Fig. 2, the elastic part of the load-deflection curves matched closely to the test results. The FEA better capture the elastic part of the CLT beam's load-deflection curves as they undergo little non-linear deformation compared to the CLT walls.

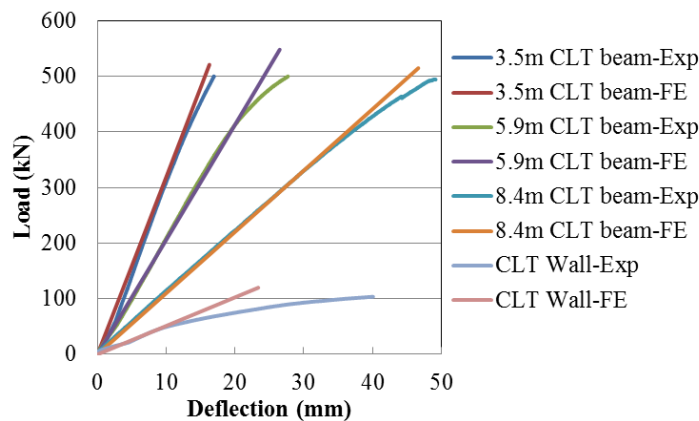


Fig. 2 – Experimental vs FE load-deflection curves

### 2.2 Parametric study

A parametric study was performed with the variation of the shape and location of openings. The stiffness reduction of CLT walls with different number, size, shape and location of the opening was investigated. Typical openings like, a window and/or a door as shown in Fig. 3 were considered. A maximum of 50% of the total wall area was removed in the FE analysis. It was found that with a removal of half of the wall area, the stiffness of the wall reduced more than 60%.

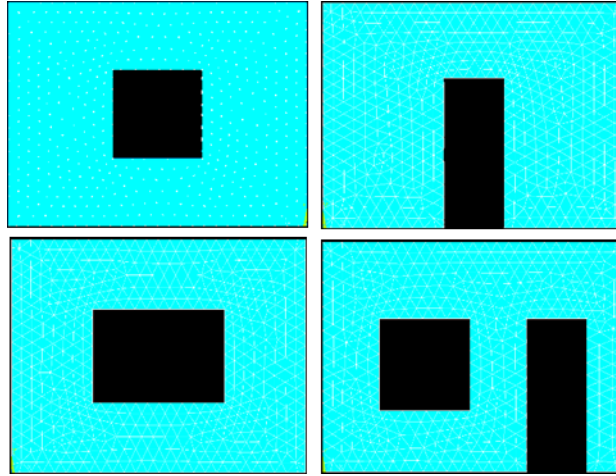


Fig. 3 – Typical openings in CLT Walls

An analytical model to calculate the in-plane stiffness of CLT walls with openings was proposed based on the parameters that affect the stiffness (Eq. 6). From FEA, it was found that the ratio of opening to wall area,  $A_o/A_w$ , the aspect ratio of opening,  $r_o$ , and the aspect ratio of opening to wall,  $r_{o/w}$  affect reduction in wall area. Therefore, these parameters are considered:

$$K_{opening} = K_{full} \left[ 1 - \frac{r_{o/w} \left( \frac{A_o}{A_w} \right)}{\sqrt{r_{o/w} + r_o \left( \frac{A_o}{A_w} \right)}} \right] \quad (6)$$

where,  $K_{opening}$  and  $K_{full}$  is the stiffness of walls with and without opening, respectively;  $A_o$  and  $A_w$  is the area of walls with and without opening, respectively;  $r_o$  is the aspect ratio of opening (smaller to larger dimension); and  $r_{o/w}$  is the maximum aspect ratio of opening to wall dimension (max of  $l_o/L$  or  $h_o/H$ , where,  $L$  and  $H$  is the wall length and height, respectively, and  $l_o$  and  $h_o$  is the opening length and height, respectively). The equation was more accurate in predicting stiffness of CLT walls with openings when compared to a previously proposed model by Dujic et al. (2007), see Fig. 4.

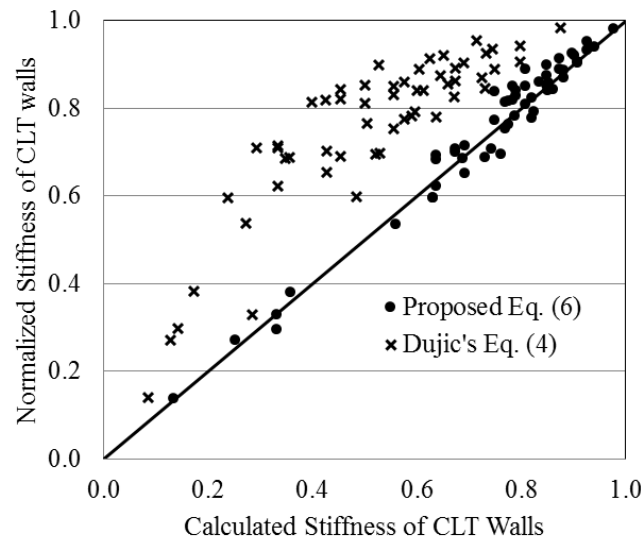


Fig. 4 – Comparison of FE vs calculated stiffness of CLT walls from this study compared to previous studies



### 3. Sensitivity analysis

#### 3.1 Methods

A sensitivity analysis quantifies the uncertainty in the output of a model qualitatively or quantitatively, to different sources of variation in the input of a model. In addition, it also analyzes the contribution of each input variable to the model response. In this research, a sensitivity analysis was performed in the commercial software package optiSLang (Most and Will 2008). The design of experiments (DOE) for random sampling for sensitivity analysis utilizes Advanced Latin Hypercube Sampling (ALHS) technique. ALHS is effective to represent the non-linearity of the model in a reduced space. Meta-models were used to represent the model responses of surrogate functions in terms of the model inputs. A surrogate model is often advantageous due to the inherent complexity of many engineering problems to approximate the problem and to solve other design configurations in a smooth sub-domain (Sacks et al. 1989, Simpson et al. 2001).

However, most meta-models i.e., Moving Least Square (MLS) approximation, Kriging or Neural Networks require a high number of samples to represent high-dimensional problems with sufficient accuracy. To overcome these limitations, the Meta-model of Optimal Prognosis (MOP) approach was developed for the optimal filter meta-model configurations (Most and Will 2008). By doing this, a surrogate model of the original physical problem can be used to perform various possible design configurations without computing any further analyses. To develop an automatic approach requires defining a measure for the characterization of the approximation quality. The MOP uses the generalized coefficient of determination (CoD) which results for the special case of pure polynomial regression. The CoD assesses the approximation quality of a polynomial regression by measuring the relative amount of variation explained by the approximation as follows:

$$R^2 = \frac{SS_R}{SS_T} = 1 - \frac{SS_E}{SS_T}; \quad 0 \leq R^2 \leq 1 \quad (7)$$

where,  $R^2$  is the CoD,  $SS$  is the sum of square,  $SS_T$  is the equivalent to the total variation,  $SS_R$  represents the variation due to the regression, and  $SS_E$  quantifies the unexplained variation as follows:

$$SS_T = \sum_{i=1}^N (y_i - \mu_Y)^2, SS_R = \sum_{i=1}^N (\hat{y}_i - \mu_{\hat{Y}})^2, SS_E = \sum_{i=1}^N (y_i - \hat{y}_i)^2 \quad (8)$$

However, in order to penalize the over-fitting, Montgomery and Runger (2003) also introduced the adjusted CoD, as shown in Eq. 9:

$$R_{adj}^2 = 1 - \frac{N-1}{N-p} (1 - R^2) \quad (9)$$

where,  $N$  is the number of sample points and  $p$  is the number of regression co-efficient. The quality of an approximation was evaluated in terms of the prognosis quality by using an additional test data set. The agreement between this real test data and the meta-model estimates is measured by the coefficient of prognosis,  $CoP$  (Most and Will 2008) as defined in Eq. 10:

$$CoP = \left( \frac{\mathbf{E} \left[ Y_{Test} \cdot \hat{Y}_{Test} \right]}{\sigma_{Y_{Test}} \sigma_{\hat{Y}_{Test}}} \right)^2; \quad 0 \leq CoP \leq 1 \quad (10)$$

An optimal meta-model can be searched with a defined  $CoP$ . Each meta-model is investigated for all possible significance by varying the significance quantile from 99% to a given minimal value. A polynomial regression is developed thereafter and the coefficients of importance ( $CoI$ ) are calculated for each variable following Eq. 11:





$$CoI_{Y,X_i} = R_{Y,X}^2 - R_{Y,X \setminus i}^2 \quad (11)$$

where,  $R_{Y,X}^2$  is the CoD of the full model including all terms of the variables in  $X$  and  $R_{Y,X \setminus i}^2$  is the CoD of the reduced model, where all linear, quadratic and interactions terms belonging to  $X_i$  are removed from the polynomial basis. The threshold  $CoI_{min}$  is varied from 1% to a given value. Based on the CoI of each variable the meta-model is built up and the coefficient of prognosis is computed. The optimal meta-model is chosen from the maximum  $CoP$  configuration. The training data set is used for the construction of meta-model, while the test data set is used for the calculation of the  $CoP$ . On the contrary, a merge data set from training and test data is used for the correlations for the significance filter and the regression for the importance filters. However, if no additional test data set is available, the initial data set is split into training and test data in a way that each data set the response ranges are represented with maximum conformity to the entire data set. The sensitivity analysis in optiSLang involves following steps:

- i. A solver chain was created in optiSLang for the sensitivity analysis.
- ii. The range of the input parameters and their types (i.e., deterministic and/or stochastic) were defined.
- iii. ALHS sampling technique was used for DOE and total 1,000 samples were created randomly with the defined input parameters range.
- iv. An input file created for the proposed equation which link to the ALHS for sampling.
- v. Python script was used as a solver and calculates the output.
- vi. The solver chain was run for  $n$  times (1000 times) to generate all output.
- vii. The MOP was created to quantify the contribution of each parameter on the proposed model.
- viii. The algorithm for the sensitivity analysis is presented in Fig. 5. The parameters for the sensitivity analysis are given in Table 1.

Table 1 – Parameters for Sensitivity analysis in optiSLang

|                        |   |
|------------------------|---|
| Sampling method        | ALHS                                      |
| Number of samples      | 1,000                                     |
| Meta-models            | Polynomial, Clasic MLS, Interpolating MLS |
| Solver                 | Python script                             |
| Sample splitting ratio | 50%                                       |
| Number of steps        | 10  |
| CoI limit              | 0-5%                                      |
| Rank criteria          | $CoD_{adj}$                               |
| Parameters (range):    |   |
| $A_o/A_w(x_1)$         | 0.05-1.2                                  |
| $r_{o/w}(x_2)$         | 0.1-0.8                                   |
| $r_o(x_3)$             | 0.3-2.0                                   |

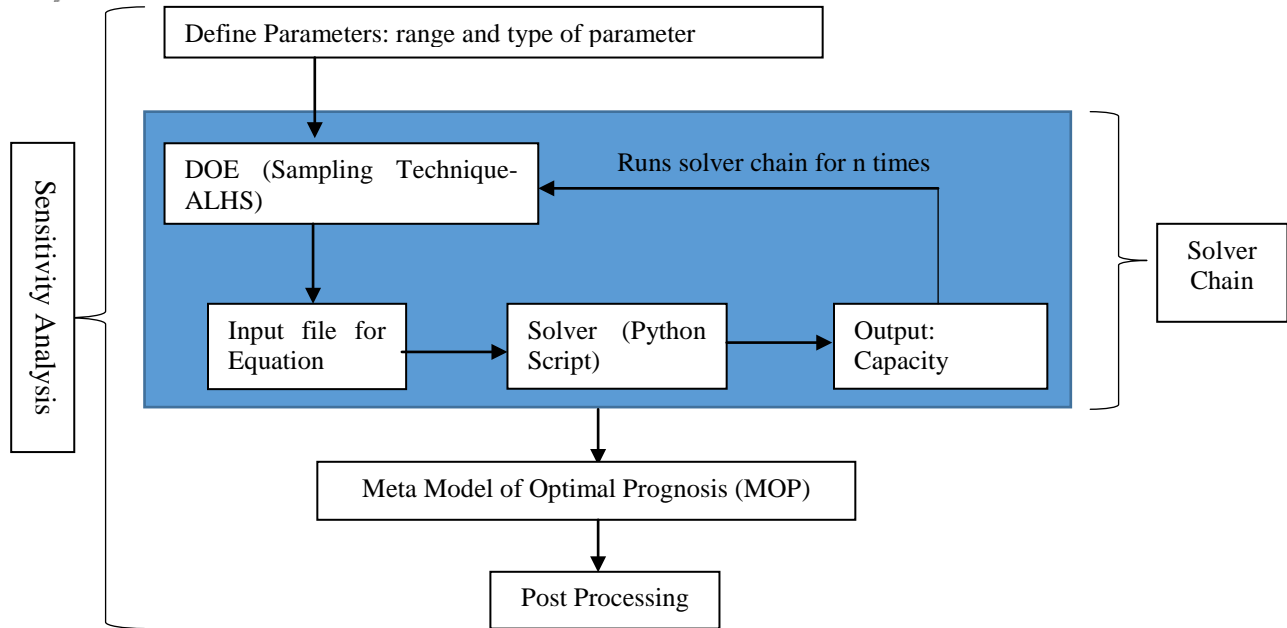


Fig. 5 – Algorithm for sensitivity Analysis in optiSLang

### 3.2 Results of Sensitivity Analyses

The impact of each input parameter on the output along with the models three dimensional space is illustrated in Fig. 6 showing the MLS approximation of the CLT wall stiffness with opening with respect to the two model input parameters  $A_o/A_w(x_1)$  and  $r_{o/w}(x_2)$ . The MLS approximation exhibited the variation of the output in the predicted model space. A smooth surface of the output (see Fig. 6) indicated a good approximation of the proposed model. Using 1,000 samples, a *CoP* of 100% was achieved for the proposed models which indicated a perfect approximation using MOP. By comparing the *CoP* value of each parameter, it was found that both parameters  $A_o/A_w(x_1)$  and  $r_{o/w}(x_2)$  showed the highest influenced on stiffness as compared to  $r_o(x_3)$ .

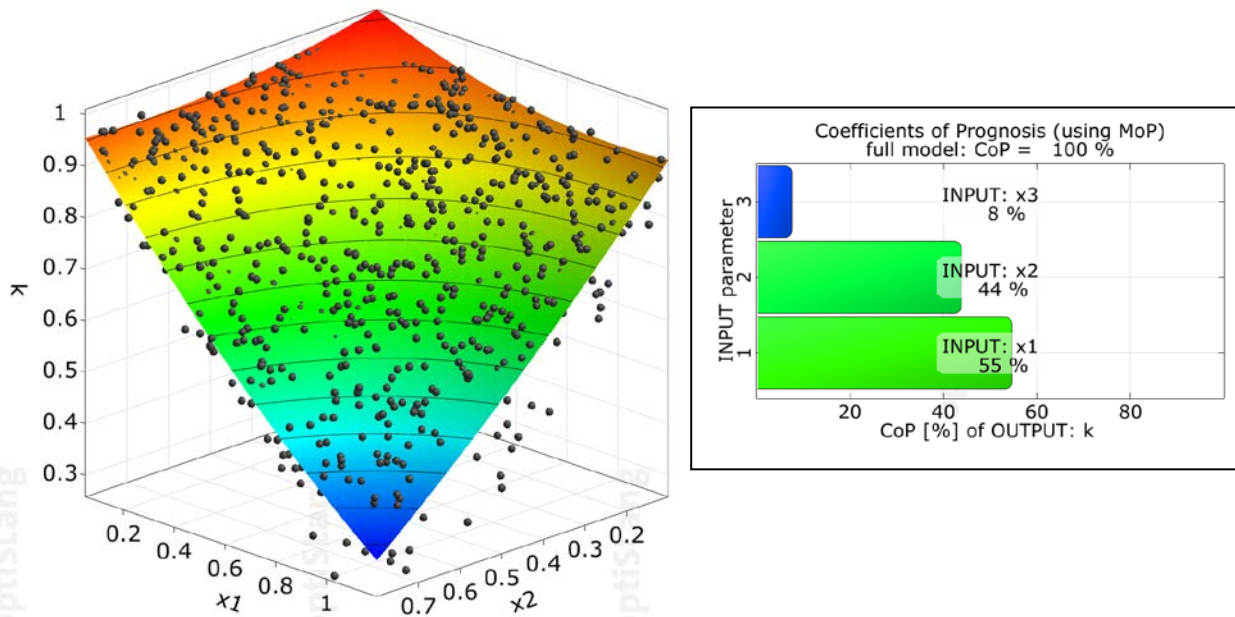


Fig. 6 – MLS approximation of CLT wall stiffness,  $k$  with respect to the two input parameters,  $A_o/A_w(x_1)$  and  $r_{o/w}(x_2)$  (left) and *CoP* values of each parameter (right).





## 4. Conclusion

FE models were developed that accurately predicted the in-plane behaviour of CLT panels with openings. The effect of opening size and shape on the stiffness of CLT walls was investigated. The experimental, numerical, and analytical investigation on the CLT panels allows the following conclusions to be drawn:

- The FE model accurately predicted the load-deformation of CLT beams and walls.
- With the removal of half of the wall area, stiffness was reduced by more 60%.
- The proposed equation (Eq. 6) better predicted the reduced stiffness of the CLT walls compared to previous equations.
- The sensitivity analysis using MOP and ALHS sampling technique showed a smooth variation of the proposed model with respect to input parameters.
- The sensitivity analysis showed that the ratio of opening to wall area,  $A_o/A_w$  and the aspect ratio of opening to wall,  $r_{o/w}$  had the highest influence on the proposed model to calculate the in-plane stiffness.

Ongoing work investigates the in-plane stiffness and strength of CLT walls with different connections under static and cyclic loading.

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