

# SIMPLIFIED APPROACH TO MODELING NON-LINEAR BEHAVIOR OF TYPICAL CHILEAN REINFORCED CONCRETE SHEAR WALLS

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## Abstract

In this study, a simple shear model is developed that can be used with the Multiple-Vertical-Line-Element-Model (MVLEM) to represent a shear wall in a reinforced concrete (RC) building. The MVLEM considers the inelastic axial and flexural responses of the wall represented by several vertical-parallel uniaxial elements with infinitely rigid beams at the top and bottom of the wall element, and the inelastic shear response is simulated by a single horizontal spring. One of the major problems of the MVLEM is the difficulty to estimate the shear properties when experimental results are not available. To address this issue, detailed finite element models of RC walls are used in an extensive parametric study to develop phenomenological models for characterizing shear behavior of squat and intermediate walls. This parametric study is based on the statistical data of Chilean RC walls built before and after the 1985 earthquake, but prior to the 2010 Chilean earthquake. Values of web reinforcement ratio, wall length to wall thickness ratio, aspect ratio, and level of axial load are taken as the main wall web parameters. The nonlinear response of 3960 finite element wall panel models were simulated so as to have an adequate number of data points to develop the shear model. These panels only consider the wall web, without the boundary elements, since the shear response is controlled by the properties of the web section. The results of the pushover simulations are then evaluated in order to develop shear model parameters based on the above-mentioned characteristics of the wall panel. Finally the developed shear model is validated by comparing the predicted shear response to finite element simulations for a set of representative wall panels. Based on these comparisons, it was observed that the proposed shear model works well for the majority of cases, but also exhibits a loss of accuracy for cases with a significant post-peak descending branch. Findings from the study will be useful in advancing nonlinear simulation models for analysis of shear wall structures.

Keywords: RC shear-wall, non-linear behavior, shear behavior, finite element simulation



## 1. Introduction

Reinforced concrete shear-walls are the preferred seismic resistant system used in Chile. Therefore, the need to accurately model shear wall behavior is of major importance if the results of nonlinear building simulations are to be used as part of seismic assessment studies. Currently, several models to assess the non-linear behavior of shear-walls can be found in the literature. However, high computational cost or the reliability of the results render many of these models to be difficult to implement or use.

The main objective of this work is to develop a simple shear model to be used as part of the MVLE Model (MVLEM) for the simulations of RC walls. It is desirable that the constitutive shear behavior be based on geometrical and material properties of the wall to be modeled, as well as on the applied axial load. The MVLEM was chosen from the other macro-models due to the simplicity in its implementation and its ability to provide an understanding of the overall shear behavior [1]. Although this macro-model does not consider interaction of the shear and flexure behavior or the interaction between shear and the axial forces, considering the effects of axial load in the shear modeling could help to overcome part of this problem. Recent modifications [2, 3, 4] to the MVLEM incorporated panel behavior, facilitating shear-flexure interaction, but the use of this updated model requires deeper understanding of the concrete material behavior and membrane model. Additionally, in some of these improvements loss in accuracy can be found in the modeling of some intermediate and squat walls.

As with other structural elements, the final response of RC walls will depend greatly on its geometry. Slender walls will respond in a flexural manner so that that the shear behavior is not important to the final performance. The behavior of squat walls will be dominated by its shear component; therefore axial-shear interaction should be included in the model. Finally, the response of intermediate walls will be a combination of their flexure and shear behaviors, which the MVLEM is capable of predicting if the correct shear properties is specified for the lateral shear spring, even though no flexure-shear interaction is included in the model.

In this study finite element models of RC wall panels are used in a parametric study to develop a suitable model for describing the shear behavior of squat and intermediate walls. The finite element platform used in this research work is the commercial software LS-DYNA [5], which includes refined model (finite element) and phenomenological component model (frame element) representations. Concrete is modeled using solid elements, where confined and unconfined concrete can be considered. The modeling of steel reinforcement distributed along the wall cross-section and height is facilitated through structural beam elements.

The shear model presented in this paper is based on the geometric features of RC walls, along with the axial load level applied to the wall and its material properties. Values of thickness, height, length, and web steel reinforcement are the key parameters used in this study and in the model generation. The ensuing parametric study of different rectangular wall characteristics will form the basis of the development of the empirical shear model representing the shear component in the MVLE model to be used later in OpenSEES [6] simulations of building frames.

## 2. Simulation program for developing the shear model for RC walls

Since the development of the shear model is based on the characteristics and performance of Chilean RC walls, two different Chilean shear-wall cross sections were extracted from typical wall sections constructed in Chile. The selected RC wall panels were analyzed under monotonic loading. Considering that the shear behavior is mainly governed by the web of the wall, the shear model will be calibrated from the simulation of these wall panels. The two different cross-sections selected were the same ones used in a previous study to examine the behavior of Chilean walls [7] - the goal of that study was to examine how the changes in the wall cross-sectional layout through the years affected the final performance when subjected to lateral loads. The same configurations were maintained to enable additional studies on the observed behavior after the 2010 Maule earthquake [8]. A schematic drawing of the wall cross-section is shown in Fig.1, where some geometric characteristics are shown for a typical Chilean rectangular wall.



Fig. 1 - Details of selected wall cross-section

## 2.1 Parametric Study

In order to develop the shear model based on the geometric and reinforcement characteristics of typical walls, a parametric study was carried out. This parametric study is based on the statistical information gathered in a previous phase of this study [8]. The selected wall cross-sections were chosen based on typical dimensions and material properties of RC buildings constructed before and after 1985, but prior to 2010. From here, four cross-sections, two pre-1985 and two post-1985, were created. The database used to obtain these characteristics considered 57 walls from pre-1985 buildings and 55 walls from post-1985 buildings, and included damage and undamaged walls located between the first basement and the second floor. Averages of the cross-sectional properties were computed with the available data, from which two types of walls were chosen according to their lengths ( $l_w$ ): Type 1 for a wall length equal to 300 cm, and Type 2 for  $l_w$  equal to 750 cm. Table 1 shows a summary of the values extracted from the mentioned statistical analysis of real squat and intermediate Chilean RC walls divided in two time periods. From Table 1, several sections for each type and year of construction were created based on the average and standard deviation values for each of the parameters, and they are listed in Table 2. Since the cross-sections must represent realistic values that can be found in the field, the generated cross-section information was suitably modified to match a real design of the wall element.

		$l_w/t_w$	$\rho_{w}$ (%)			$l_w/t_w$	$\rho_w$ (%)
	Average	12	0.37		Average	14	0.39
Pre-1985 T1	Min	7	0.17	Post-1985 T1	Min	10	0.20
$l_w = 300 \text{ cm}$	Max	18	1.83	$l_w = 300 \text{ cm}$	Max	19	1.45
	Std	3	0.38		Std	3	0.29
	Average	24	0.24		Average	32	0.38
Pre-1985 T2	Min	16	0.16	Post-1985 T2	Min	22	0.23
$l_w = 750 \text{ cm}$	Max	34	0.37	$l_w = 750 \text{ cm}$	Max	52	1.06
	Std	6	0.06		Std	8	0.24

Table 1 - Statistical data of typical squat and intermediate Chilean walls

With the information illustrated in Table 2, models of several wall panels were built in LS-DYNA to have an adequate number of data points to develop the shear model. All the wall panels were modeled using a concrete strength of 25 MPa and a steel yield strength of 420 MPa. Additionally, values for different thicknesses were established according to the average values of  $l_w/t_w$  from the statistical analysis, and heights ( $h_w$ ) following the different aspect ratios to be investigated. Therefore, the cross-sections of the two types of walls were modeled using eight different heights, representing eight different aspect ratios ( $AR=h_w/l_w$ ) for each era: 0.3, 0.5, 0.7, 0.9, 1.0, 1.25, 1.5, and 1.75. Furthermore, in order to consider the axial load as a variable included in the model, 11 levels of axial load were considered in the study:  $\eta = 5$ , 10, 12, 15, 17, 20, 22, 25, 27, 30, and 35% of  $A_g f'_c$ . These levels were selected according to the axial load levels estimated in RC walls in buildings constructed prior to and after the 1985 Chilean earthquake, but before the 2010 Chilean earthquake [8]. Considering all cross-sections, aspect ratios and axial load levels, a total of 3960 wall panels were modeled in LS-DYNA. The



results of the pushover simulations are then classified in order to develop shear model performance points based on certain characteristics of the RC wall panel.

	Pre-1985 T1		Pre-1985 T2			Post-1985 T1		Post-1985 T2				
	$l_w = 300 \text{ cm}$		$l_w = 750 \text{ cm}$			$l_w = 300 \text{ cm}$			$l_w = 750 \text{ cm}$			
	$l_w/t_w$	$t_w$ (cm)	$\rho_{\rm w}$ (%)	$l_w/t_w$	$t_w$ (cm)	$\rho_{\rm w}$ (%)	$l_w/t_w$	$t_w$ (cm)	$\rho_{\rm w}$ (%)	$l_w/t_w$	$t_w$ (cm)	$\rho_w$ (%)
Section 1	15	20	0.75	30	25	0.30	18	17	0.70	38	20	0.60
Section 2	15	20	0.60	30	25	0.25	18	17	0.50	38	20	0.50
Section 3	15	20	0.40	30	25	0.20	18	17	0.40	38	20	0.40
Section 4	15	20	0.20	25	30	0.30	18	17	0.25	38	20	0.25
Section 5	12	25	0.75	25	30	0.25	15	20	0.70	30	25	0.60
Section 6	12	25	0.60	25	30	0.20	15	20	0.50	30	25	0.50
Section 7	12	25	0.40	20	38	0.30	15	20	0.40	30	25	0.40
Section 8	12	25	0.20	20	38	0.25	15	20	0.25	30	25	0.25
Section 9	10	30	0.75	20	38	0.20	12	25	0.70	25	30	0.60
Section 10	10	30	0.60	-	-	-	12	25	0.50	25	30	0.50
Section 11	10	30	0.40	-	-	-	12	25	0.40	25	30	0.40
Section 12	10	30	0.20	-	-	-	12	25	0.25	25	30	0.25

Table 2 - Data of wall cross-sections considered in LS-DYNA simulations

2.2 Finite element modeling of wall panels in LS-DYNA

As mentioned before, solids elements were used for the concrete part and beam elements for the wall reinforcement. Both elements are connected along the reinforcement lengths at every node of the solid elements thereby assuming full bond. Fig. 2 shows a representation of a wall panel. At the top of the wall rigid solids elements were included to ensure the correct transfer of the lateral displacement imposed at the top of the wall, and avoid localized effects.



Fig. 2 - Finite element model of RC wall panel. a) Concrete, b) Web reinforcing steel

The solids are 8-node elements that employ constant stress solid element formulation, and the beams are Hughes-Liu beam type elements with cross section integration. More information on these formulations can be found in the theory manual of LS-DYNA [9]. With respect to the material models, unconfined concrete was



modeled using the material "MAT\_CSCM" (Continuous Surface Cap Model) included in the LS-DYNA framework. With this material, the program is capable of considering failure of solid elements and removal of the elements when the specified failure criterion is reached. The failure threshold is defined by the user and represents the maximum attainable strain prior to failure of the material. More details about this material can be found elsewhere [10]. The material model used to represent the reinforcement steel is called "MAT\_PIECEWISE\_LINEAR\_PLASTICITY". This material is an elastic-plastic material with failure based on a plastic strain and a stress-strain curve that can be treated as a bilinear curve by specifying a tangent modulus.

For all the simulations, the wall panels were modeled as isolated walls fixed only at the base. A lateral displacement at the top of the wall was imposed until 2.5% drift was attained. The lateral displacement was applied monotonically, and the axial load was applied at the beginning of the simulations and remained constant during application of the monotonic lateral displacement. From the analyses performed in LS-DYNA two different files were generated: nodal displacements along the mid-section of the thickness at the top of each wall, and the reaction forces at the base nodes. From this data pushover curves were generated for each wall.

## 2.3 Results from the finite element wall panel simulations

The pushover curves resulting from the analysis were classified according to year of construction, according to the wall length (Type), aspect ratio, and  $l_w/t_w$  values. Additionally they were normalized by the wall height on the abscissas and  $A_g f'_c$  on the ordinates. Some of the pushover curves obtained from the simulations are shown in Fig. 3 and 4 for two selected aspect ratios. In general, the pushover curves tend to have a descending branch when the axial load level and the aspect ratio increase, independent of the year of construction. Moreover, when the descending branch is not observed and the axial load level is low, the simulations show a plateau after the peak strength has been reached. Finally, in all the simulation an increase of the shear peak strength was obtained for higher axial load levels, this incremental increase tends to decrease as the axial load increases. Additionally, Fig. 5 shows the deformed mesh for the two selected aspect ratios illustrated in Fig. 4 (post-1985) with an axial level of 0.20  $A_g f'_c$  at final stage. Fig. 5a presents the final step of the simulation program for an AR of 0.3, and Fig.5b show final step before a complete loss of shear strength for an AR of 1.0 wall is achieved.



Fig. 3 – Pushover curves Type 1, Pre-1985 wall panels,  $l_w/t_w=15$ : (a) AR 0.3, (b) AR 1.0



Fig. 4 – Pushover curves Type 1, Post-1985 wall panels,  $l_w/t_w=15$ : (a) AR 0.3, (b) AR 1.0



Fig. 5 – Deformed mesh for Type 1, Post-1985 wall panels ( $l_w/t_w$ =15 and  $A_g f'_c = 0.2$ ) before complete loss of shear strength (a) AR 0.3 (b) AR 1.0

## 3. Proposed model for estimation of the shear behavior of RC walls

The generated pushover curves from the finite element simulations were classified and analyzed to find statistical correlation between selected parameters and the shear behavior of the wall. From the critical points identified in the curves, the shear behavior was represented by a four-point multi- linear relationship as illustrated in Fig. 6. Each one of the points described below are computed from the geometric properties of the wall defined by the aspect ratio  $(h_w/l_w)$  and the parameter  $l_w/t_w$ , along with the axial load level, and the reinforcement ratio of the wall web. The points described in Fig. 6 were estimated based on observed trends in the generated pushover curves. The final models presented in this work relate base shear normalized by  $A_g f'_c$  with the displacement at the top normalized by the height of the wall panel (drift) as a percentage (%).



Fig. 6 - Proposed force vs displacement shear model

## 3.1 Cracking point Figura 4.1 – Modelo de corte propuesto (Fuerza vs drift)

The base shear at cracking is estimated from:  $V_{cr} = \beta \cdot V_y$ , where  $\beta$  is obtained by taking the average value of  $V_{cr}/V_y$  for the different panels simulations. The corresponding displacement was calculated from the cracking base shear ( $V_{cr}$ ) and considering an initial stiffness of  $0.5GA_g$ , where  $A_g$  is the gross section of the wall, and G is the shear modulus.

## 3.2 Peak shear strength

In the simulations results, this point is defined at the peak of the response before a visible plateau or a descending branch can be observed in the response. The normalized peak base shear and the corresponding drift levels are estimated from the simulations using the following form of a regression model:

$$V_{y}/(A_{g}f_{c}) \text{ or } \delta_{y} = b_{1} \cdot (l_{w}/t_{w})^{b_{2}} \cdot (\rho_{w})^{b_{3}} \cdot (AR)^{b_{4}} \cdot (\eta)^{b_{5}}$$
(1)

where,  $l_w$  is the length of the wall,  $t_w$  is the thickness of the wall,  $\rho_w$  is the reinforcement ratio of the wall web measured as a percentage value, AR is the aspect ratio of the wall, and  $\eta$  is the axial load level presented as a percentage of the  $A_g f'_c$  value, with  $f'_c$  the concrete peak strength. A unique set of  $b_i$  factors were obtained for the estimation of the normalized peak shear and the corresponding drift, respectively.

## 3.3 Descending branch

This point delimits the post-peak section of the shear curve. This part is characterized by a descending branch, which usually starts from the peak shear strength point for some cases. Depending on the properties of the wall, the peak shear  $(V_y)$  could be greater or equal to the ultimate shear  $(V_u)$ . The descending branch point is defined as a fraction of the peak base shear  $(V_u = \gamma \cdot V_y)$ , where  $\gamma$  is obtained from the same procedure to estimate  $\beta$  from the cracking point. The corresponding drift value at ultimate point is computed from the same regression model (with different b<sub>i</sub> factors) as the one shown in Eq. (1).

#### 3.4 Ultimate or failure point

This point defines the residual shear strength at ultimate deformation capacity. The strength associated with this threshold is defined as the 20%  $V_u$ . Therefore, this point marks the degradation of the shear strength to a residual value, although in some simulations the shear strength showed a complete loss of the shear strength. The ultimate drift is estimated from:

$$\delta_f = \left( \left( V_f / (A_g f'_c) - V_u / (A_g f'_c) \right) + \alpha \cdot \delta_u \right) / \alpha$$
<sup>(2)</sup>



where  $\alpha$  is the slope of the shear vs. deformation curve from the beginning of the descending branch to the failure point (Fig. 6), and is estimated from the regression equation shown in Eq. (1).

Table 3 and Table 4 summarize the results of the model formulation. Table 3 lists the  $b_i$  factors of the regression model of the four different variables estimated through this approach. Table 4 presents the factors that multiply  $V_y$  to obtain  $V_{cr}$  and  $V_u$ . Additionally, Table 5 specifies the different equations to estimate the base shear and drift pairs for each point of the shear vs. displacement model described in Fig. 6.

Variable	<b>b</b> 1	<b>b</b> <sub>2</sub>	<b>b</b> 3	<b>b</b> <sub>4</sub>	<b>b</b> 5	$R^2$
$\delta_y$	0.007	-0.128	0.500	-0.105	-0.114	0.502
$V_y/(A_g f'_c)$	0.036	-0.003	0.205	-0.796	0.323	0.966
$\delta_u$	0.036	-0.007	-0.048	-0.075	-0.244	0.293
α	-0.001	0.755	0.004	-0.406	1.856	0.143

Table 3 – Computed  $b_i$  factors for regression models

Table 4 - Mean and standard deviation values for estimated variables

Variable	Mean	Std.
β	0.64	0.175
γ	0.92	0.109

Table 5 – Shear model parameters

Variable	Model
$\delta_{cr}$	$V_{cr}/(A_g f'_c) / (0.5 G A_g)$
$V_{cr}/(A_g f'_c)$	$0.64 \cdot V_y / (A_g f'_c)$
$\delta_y$	$0.007 \cdot (l_w/t_w)^{-0.128} \cdot (\rho_w)^{0.5} \cdot (AR)^{-0.105} \cdot (\eta)^{-0.114}$
$V_y/(A_g f'_c)$	$0.036 \cdot (l_w/t_w)^{-0.003} \cdot (\rho_w)^{0.205} \cdot (AR)^{-0.796} \cdot (\eta)^{0.323}$
$\delta_u$	$0.036 \cdot (l_{w}/t_{w})^{-0.007} \cdot (\rho_{w})^{-0.048} \cdot (AR)^{-0.075} \cdot (\eta)^{-0.244}$
$V_u/(A_g f'_c)$	$0.92 \cdot V_y / (A_g f'_c)$
α	$-0.001 \cdot (l_w/t_w)^{0.755} \cdot (\rho_w)^{0.004} \cdot (AR)^{-0.406} \cdot (\eta)^{1.856}$

From Table 3, the estimations of the peak shear strength point ( $\delta_y$  and  $V_y$ ) show reasonable R<sup>2</sup> values, which means a good estimate of this point was possible for the majority of the simulation curves. On the other hand, from estimated values of  $\delta_u$  and  $\alpha$ , the R<sup>2</sup> value tends to decrease, which is expected given the variability of the responses obtained after the peak shear. This variability makes the shear model difficult to calibrate, and might suggests developing a model with different branches, according to the axial load level and aspect ratio of the wall inspected.



## 4. Validating the proposed shear model

From the model formulation presented in Table 5, several random wall panels from the LS- DYNA simulations were chosen to assess the accuracy of the proposed shear model. In Fig. 7, four comparative simulations are presented consisting of Type 1 ( $l_w$ =300 cm) pre-1985 walls with two different aspect ratios (0.3, and 1.0), and two different axial load levels ( $A_{e}f'_{c} = 5\%$ , 20%).

In most of the cases shown in Fig. 7, the proposed shear model reproduces reasonably the shear response obtained in the detailed finite element simulations. The estimates of the cracking and peak shear point are close to the values obtained in the simulations. Additionally, when the simulation contains a post-yield plateau, the shear model predicts very well this behavior, and when a steep descending branch is observed, it is capable of representing the initiation of the softening response. Although the model predicts the existence of this branch, additional adjustments are needed to obtain a better fit to the FE simulations. As observed from Fig. 3 and 4, the results of the finite element simulations show a large variability in the responses after the peak shear strength point is reached, therefore, as seen in Fig. 7, the prediction of this behavior with only one model for all the cases may be challenging, and an enhancement of the proposed shear model is recommended.



Fig. 7 – Comparison of proposed shear model (continuous line) versus FE simulation (dotted line): pre-1985 Type 1 RC walls: a) AR=0.3; b) AR=1.0

## 5. Conclusions

The main objective of the study was the development of an efficient yet reliable shear model to be used as input into a macro-model representation of a shear wall element. From the statistical data gathered on existing Chilean buildings, wall parameters were varied in order to generate different sets of Chilean wall features. The specific characteristics considered in this study were: wall thickness, wall length, web reinforcement ratios, axial load applied, and aspect ratio (height-to-length). Based on these control parameters, several wall panel cross-sections (without considering boundary elements), and subsequently modeled as three-dimensional continuum elements in LS-DYNA. Findings from the study are summarized below.

1. From the analysis of the simulation curves presented in section 2, the responses show certain patterns of behavior depending on the value of the considered parameters. When the level of axial load applied over the wall is inspected, a higher load level results in an increase of shear strength along with more brittle behavior after the peak strength is reached. Additionally, when the aspect ratio of the wall decreases, a slight increase in shear strength and a visible plateau in the final response of the wall panel was observed.



On the other hand, when the aspect ratio increases, the shear strength tends to decrease, and the post-peak plateau changes to a descending branch when the axial load level also increases.

- 2. With respect to the proposed approximate shear model, it can be observed that the  $R^2$  values for some estimated parameters are lower than desired (Table 3), especially for estimations  $\delta_u$  and  $\alpha$ . In some wall panels a plateau can be observed after the maximum strength is attained, but in other cases, a steep descending branch characterizes the last part of the shear deformation envelop.
- 3. One of the major advantages of the proposed shear model is the ability to consider axial-load interaction in an implicit manner, which is observed from the finite element simulations. A variation of the axial load causes a variation of the shear strength and post-yield behavior. This means that the critical axial load level during seismic loading needs to be established prior to determining the properties of the shear spring when the model is to be used as part of the MVLEM definition.
- 4. Based on comparisons between the empirical models and the simulation results from detailed finite element analyses of wall panels using LS-DYNA, it was observed that the proposed shear model works well for the majority of cases, but also exhibits a loss of accuracy for cases with a significant post-peak softening.
- 5. Given the variability of the responses in the post-peak phase, this study suggests a re-evaluation of the proposed shear model by generating different branches based on the axial load level applied and the aspect ratio of the wall considered in the analysis.

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