



RESEARCH ON THE INFLUENCE OF THE STIFFNESS RATIO AND STRENGTH RATIO ON THE FAILURE MODE OF FRAME STRUCTURE

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Abstract

The failure mode of the frame structure is influenced by the stiffness and strength ratio of the beam and column. The stiffness ratio determines the distribution of internal force in structural members and the strength ratio influences the failure process and final failure degree of structural members. Based on the assumption that the floor and frame structure are all working together, the paper establishes an example of plane frame, which includes 6 different linear stiffness ratios and 3 different bending capacity ratios between the beam and column. The influence of linear stiffness ratio and bending strength ratio on seismic performance and failure mode of frame structure is investigated by ABAQUS nonlinear analysis. The research shows that the accurate analysis of the seismic performance of the structure is based on accurate consideration of cooperative working degree of the floor and structure under horizontal earthquake. To ensure that the structure appears more reasonable "beam column mixed hinge" failure mode, the stiffness and strength matching should be reasonable. The actual bending strength ratio of the column and beam should be more than 1.1 when the linear stiffness ratio of the beam and column is 1~4; the actual bending strength ratio of the column and beam should be more than 1.3 when the linear stiffness ratio of the beam and the column is less than 1 or more than 4.

Key words: frame structure; stiffness ratio of the beam and column; strength ratio of the column and beam; strong column and weak beam; failure mode

1. Introduction

Frame structure is one of the most common structure types, which is widely used in multi-story or high-rise industrial and civil buildings. Traditional design methods, mainly uses strength ratio of the column and beam (The actual bending capacity ratio of all beams and columns, which are determined by the actual reinforcement condition at the same frame joint, $\sum M_c = \eta_c \sum M_{bua}$, In this paper it - strength ratio η_c) to ensure frame structure forming a reasonable failure mode under the action of strong earthquake^[1]. Some research results have shown that the linear stiffness ratio of the beam and column influences the moment values on the end of beam and column (In this paper it - stiffness ratio k is defined as $k = \frac{\sum i_b}{\sum i_c}$ at a same joint. In which, the linear stiffness of beam is $i_b = \frac{E_c I_b}{l_b}$, and the linear stiffness of column is $i_c = \frac{E_c I_c}{l_c}$). Matching degree between η_c and k has an important influence on seismic performance and failure mode of the structure, but it is often ignored^[2-4].

In this paper, the plain frame is adopted as the analysis object. The influence of strength ratio and the control effect of stiffness ratio on failure mode of frame structure are also been investigated. The numerical simulation method was used to analysis the influence of strength ratio and stiffness ratio on bearing capacity,

deformation, energy dissipation, yield mechanism and distribution of plastic hinges. According to comparatively analysis on 18 two-story plain frame examples, the reasonable matching relationship between strength ratio and stiffness ratio are proposed in this paper.

2. Finite element simulation of RC plain frame

In order to accurately simulate the mechanical behavior of the structure under horizontal cyclic loading, linear reduction integration unit (C3D8R) is used to simulate concrete, and linear truss element (T3D2) is used to simulate steel bar. The interaction between concrete and steel bar is established by "Embedded" technique. The plastic damage model was used in the constitutive model of concrete. The mechanical behavior of concrete can be simulated by the model under cycling loading^[5]. The constitutive model of reinforcement uses the Usteel-02 model in PQ-Fiber developed by Tsinghua University.

2.1 Validation of finite element simulation

In order to verify the validity of finite element simulation results, the simulation is carried on the quasi-static collapse test of a three-story RC plain frame with the scale ratio of 1: 2 completed by Tsinghua University in 2011. The specific parameters and details of test model is shown in reference [6]. The analysis results and the experiment comparison are shown in Fig.1. The conclusion is obvious that the analysis method used in this paper is in good agreement with experimental results.

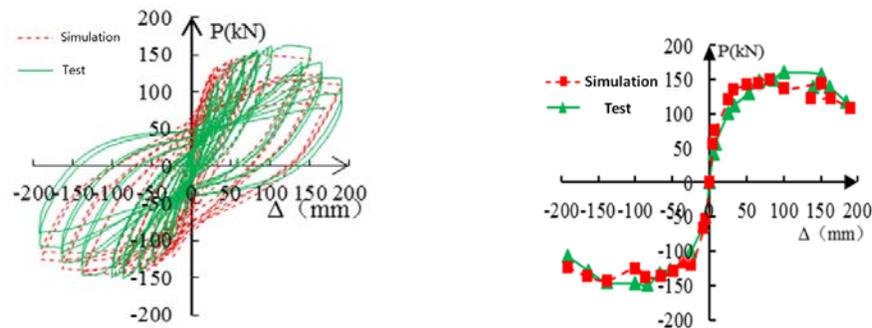


Fig.1 – Hysteretic curve and skeleton curve

2.2 Example design

The example is settled as two-story and two-span plain frame, the story height is 3.6 m and the span is 6 m. The column section is 400mm×400mm, the floor thickness is 150 mm, the concrete strength defined as C30 in Chinese code. The floor reinforcing bar is HPB300, the longitudinal bars is HRB400, and the stirrups is HPB300. The floor dead load is 3.5kN/m², the live load is 2.0kN/m², line load of beam is taken as 11kN/m, the axial compression of the mid column and side column on the top floor are 1464kN and 732kN, the actual axial compression ratio of mid column on the bottom floor is 0.433.

The horizontal loading is inverted triangular distribution and is controlled by the displacement. The load ratio of the first layer and the second layer is 1:2, as shown in Fig.2. Load displacement is: ±3.6mm→±7.2mm→±13.1mm→±22mm→±44mm→±66mm→±88mm→±110mm→±132mm→±144mm→±180mm. Considering the strength and stiffness degradation of the structure in elastic-plastic state, each stage displacement is cycled twice after displacement amplitude reach 22mm. Finite element mesh dividing of the model and schematic diagram of horizontal loading are shown in Fig.2. The contribution of the floor which is in an effective flange width of beam side to the strength and stiffness of the beam is considered.

The example design was completed by adjusting sectional size and reinforcement of beam and column. The adjustment principles are as follows:(1) The floor of eight times floor thickness on one side of the beam is taken as equivalent flange of beam. The strength and stiffness increase of frame beams caused by the floor slab concrete and bars is considered. (2) The column reinforcement is adjusted according to the actual bending capacity of the beam end, so that the strength ratio of the column and beam are 1.1,1.3,1.5.(3) The minimum value of the beam section size (200mm × 400mm) is chosen as the reference. The stiffness ratio of beam and

column are 0.7,1.2,1.7,2.6,3.4,4.3 respectively by increasing the beam section size. Cross matching the two parameters, 18 examples are formed. In order to obtain models with different strength ratio, according to the Chinese design code , fixed beam section size and reinforcement, adjusting column section reinforcement according to the actual bending strength of the beam end. The reinforcement of column with different strength ratio are shown in Table 1; In order to obtain models with different stiffness ratio, according to the Chinese design code, starting from the beam section size (200mm × 400mm), increasing the beam section size step by step to improve the beam and column linear stiffness ratio K. The reinforcement of beams with different stiffness ratio are shown in Table 2. According to uniform manner the models are numbered, such as M-07-11denotes that stiffness ratio is 0.7, strength ratio is 1.1.

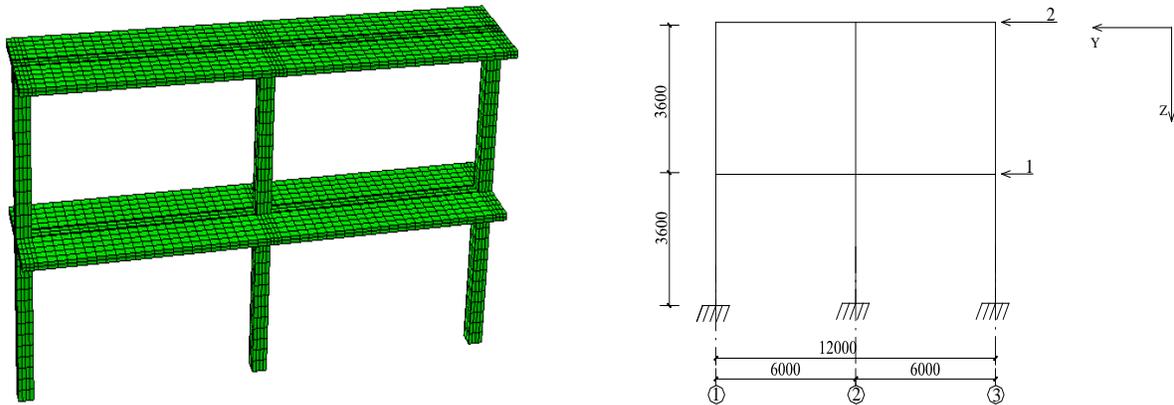


Fig. 2 – Finite element model and horizontal loading.

Table 1 The reinforcement of column of models with different flexural strength ratio

strength ratio η_c	Model numbering	Middle column reinforcement	Side column reinforcement
1.1	M-07-11 M-(12 ~ 43)-11	12 Φ 22	8 Φ 22+4 Φ 20
1.3	M-07-13 M-(12 ~ 43)-13	12 Φ 25	8 Φ 25+4 Φ 22
1.5	M-07-15 M-(12 ~ 43)-15	12 Φ 28	8 Φ 28+4 Φ 25

Table.2 The reinforcement of beams of models with different flexural stiffness ratio

Stiffness Ratio k	Model numbering	beam section size (mm)	beam section reinforcement
0.7	M-07-11 (M-07-13) (M-07-15)	200×400	upper steel bar : 3 Φ 18+2 Φ 16 lower steel bar : 3 Φ 20

1.2	M-12-11 (M-12-13) (M-12-15)	250×450	upper steel bar : 2Φ18+1Φ14 lower steel bar : 3Φ20
1.7	M-17-11 (M-17-13) (M-17-15)	250×500	upper steel bar : 2Φ18+1Φ14 lower steel bar : 3Φ18
2.6	M-26-11 (M-26-13) (M-26-15)	300×550	upper steel bar : 3Φ18 lower steel bar : 2Φ18
3.4	M-34-11 (M-34-13) (M-34-15)	300×600	upper steel bar : 2Φ16+1Φ14 lower steel bar : 2Φ18
4.3	M-43-11 (M-43-13) (M-43-15)	300×650	upper steel bar : 2Φ16 lower steel bar : 2Φ18

3. The results of analysis

3.1 hysteretic curve and skeleton curve

Due to limited space, only two groups of model with η_c (strength ratio) = 1.1 and k (stiffness ratio) = 0.7 are studied. The influence of stiffness ratio and strength ratio on structural hysteresis curves and skeleton curves are obtained.

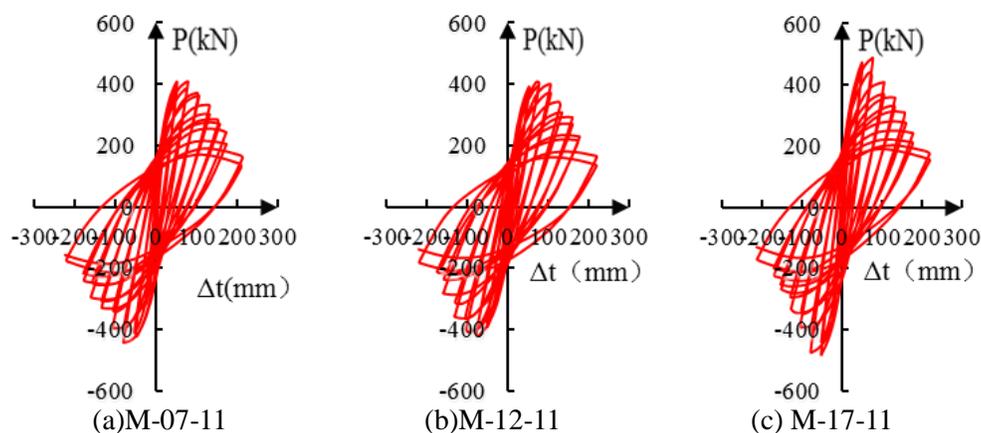


Fig.3 – The hysteretic curves of models with different stiffness ratio when the strength ratio is 1.1

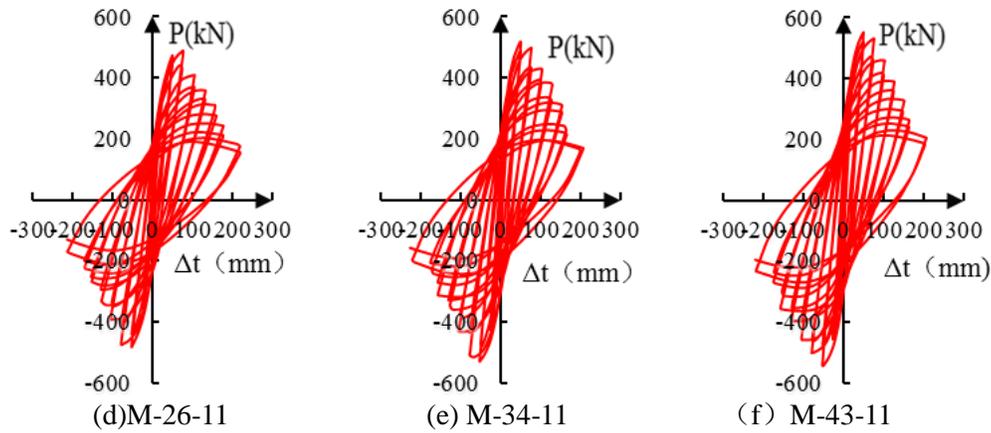


Fig.3 – The hysteretic curves of models with different stiffness ratio when the strength ratio is 1.1 (continuous)

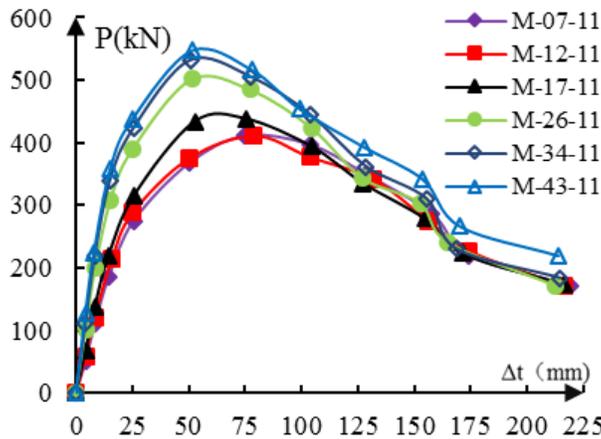


Fig.4 – The skeleton curves of models with different stiffness ratio when the strength ratio is 1.1

Hysteretic curves and skeleton curves when $\eta_c = 1.1$, $k = 0.7 \sim 4.3$ are plotted in figure 3 and figure 4. Conclusions can be seen from Fig.3 and Fig.4:(1) With the increase of the stiffness ratio, the structural stiffness is obviously increased and the peak load is significantly increased, but the peak load and the stiffness ratio are not linear proportional. (2) After the peak load, with the stiffness ratio increasing, the slope of the falling segment is increased. (3) The envelope area of hysteresis curve showed that with stiffness ratio increase, energy dissipation capacity of the structure can be enhanced.

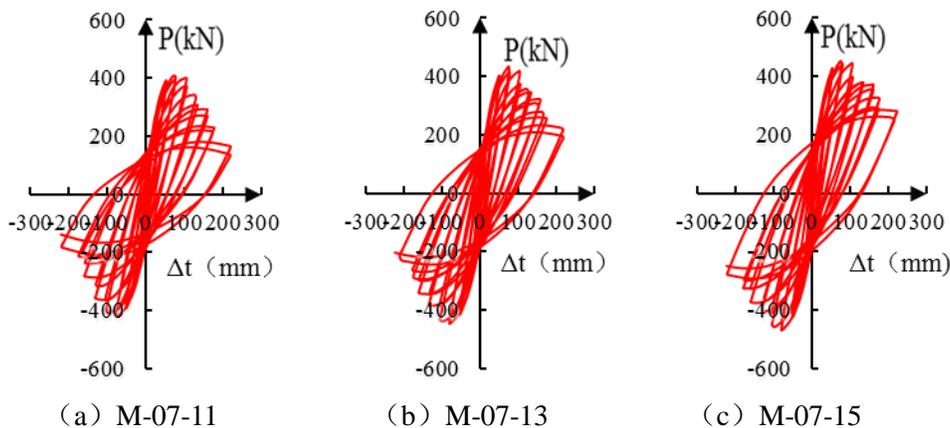


Fig.5 – The hysteretic curves of models with different strength ratios when stiffness ratio is 0.7

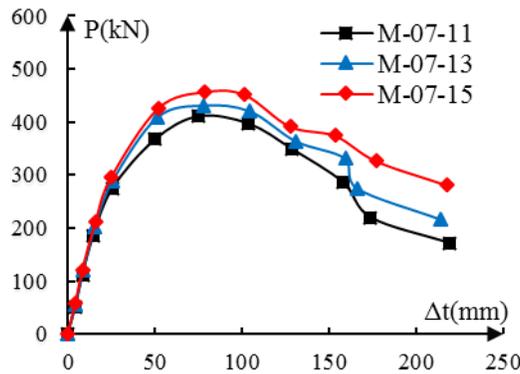


Fig.6 – The P- Δ_t hysteretic skeleton curves of models when the stiffness ratio is 0.7

The hysteresis curves and skeleton curves of different strength ratio when $k = 0.7$ are shown in Fig.5 and Fig. 6. Conclusions can be obtained from the Fig.5 and Fig. 6: (1) Raising strength ratio have no significant effect on the structural stiffness, but have a certain influence on the peak load and deformation capacity after yielding. (2) With the increase of strength ratio, the peak load is basically increase proportionately, descending segment are basically parallel after the peak load, the curve envelope area increased. The results showed that strength ratio has a certain effect on improving energy dissipation capacity of the structure.

3.2 Characteristic load and characteristic displacement

According to the skeleton curve, the yield load and yield displacement are determined by the equal energy method. Characteristic load of different stiffness ratio are shown in Table 3. When the stiffness ratio increased from 0.7 to 1.2, the peak load (P_{max}) of structure increased less. When the stiffness ratio increased from 1.2 to 1.7 and 2.6, the peak load (P_{max}) of structure increased about 6% and 14%. When the stiffness ratio increased from 2.6 to 3.4 and 4.3, the peak load of structure increased about 6% and 3%. The influence of stiffness ratio on the structural peak load shows that the increase of the bearing capacity of structure is not linear correlation with the increase of the stiffness ratio.

Table 3 – The average value of characteristic load corresponding to different stiffness ratios

Model numbering	Strength ratio η_c	Stiffness ratio k	Yield load P_y (kN)	Peak load P_{max} (kN)	Ultimate load P_u (kN)
M-07-11	1.1	0.7	274.8	410.5	348.7
M-12-11	1.1	1.2	287.7	411.7	342.3
M-17-11	1.1	1.7	315.9	438.6	335.5
M-26-11	1.1	2.6	388.5	502.0	424.0
M-34-11	1.1	3.4	422.9	530.8	444.5
M-43-11	1.1	4.3	437.1	547.3	453.7

Peak displacement ductility coefficient $\mu_{max} = \frac{\Delta_{max}}{\Delta_y}$ and ultimate displacement ductility coefficient

$\mu_u = \frac{\Delta_u}{\Delta_y}$ corresponding to different stiffness ratio of beam and column can be calculated according to the



skeleton curve (the ultimate displacement Δ_u is corresponding to $0.85P_{max}$ on the skeleton curve descending segment). Conclusions can be seen from Table 2. (1) stiffness ratio have little effect on the yield displacement Δ_y of structure. (2) The peak displacement and ultimate displacement of structure which stiffness ratio is lower or equal 1.7 is significantly greater than that of stiffness ratio is more than 1.7. Ductility coefficient are shown in

Table 4. When stiffness ratio is 1.7 or less, the ductility coefficient of structure $\mu_u = \frac{\Delta_u}{\Delta_y}$ is significantly greater

than that of stiffness ratio is 2.6 or more. The result indicating that ductility of frame structure is better when stiffness ratio is relatively small. The main reason is that when stiffness ratio of frame is relatively small, beam end can not restraint column end effectively. Beam end have better rotation capacity, so the ductility of the whole structure increased. When the stiffness ratio of beam and column increased, the additional deformation of the structure should be provided by the column end. Due to the deformation capacity of the column end is smaller than that of the beam end, so the ductility of the whole structure decreases when the stiffness ratio is increased.

Table 4 – Characteristic displacement and ductility coefficient of structures corresponding to different stiffness ratios

Model numbering	Stiffness ratio k	Yield point	Peak point		Ultimate point	
		Δ_y^t (mm)	Δ_{max}^t (mm)	μ_{max}	Δ_u^t (mm)	μ_u
M-07-11	0.7	25.6	75.0	2.92	128.7	5.02
M-12-11	1.2	24.9	78.9	3.16	131.8	5.28
M-17-11	1.7	25.8	75.2	2.92	127.2	4.93
M-26-11	2.6	24.8	51.2	2.07	104.6	4.22
M-34-11	3.4	25.6	50.6	1.97	103.8	4.06
M-43-11	4.3	24.9	51.4	2.06	98.8	3.97

Characteristic load corresponding to different strength ratios are listed in Table 5. Conclusions can be seen from the data in the Table 3. With the increase of strength ratio, the characteristic load of each stage is increased. Enhanced the bending capacity of column, peak load of structure can be effectively improved. The strength ratio increased one grade, bending capacity of the structure almost increased in the same degree. For instance, M-07-11 and M-07-13, with the strength ratio increasing from 1.1 to 1.3, yield load, peak load and ultimate load all increased about 5%.

Table 5 – The average value of characteristic load corresponding to different strength ratios

Model numbering	Strength ratio η_c	Stiffness ratio k	Yield load P_y (kN)	Peak load P_{max} (kN)	Ultimate load P_u (kN)
M-07-11	1.1	0.7	274.8	410.5	348.7
M-07-13	1.3	0.7	288.8	430.9	363.6
M-07-15	1.5	0.7	294.8	457.3	392.1
M-17-11	1.1	1.7	315.9	438.6	335.5
M-17-13	1.3	1.7	370.0	495.9	433.9
M-17-15	1.5	1.7	379.3	523.8	443.8
M-43-11	1.1	4.3	437.1	547.3	454.1



M-43-13	1.3	4.3	451.7	571.7	474.9
M-43-15	1.5	4.3	482.5	609.1	510.2

Characteristic displacement and ductility coefficient of structures corresponding to different strength ratios are shown in Table 6. When stiffness ratio is 0.7, with the strength ratio increasing from 1.1 to 1.5, ductility coefficient of structure increased from 5.02 to 5.15. The results indicating that when the stiffness ratio is small, increase strength ratio have a certain effect on improving ductile deformation capacity of structure. But the ductility coefficient of structure only increased 1.03 times. When the stiffness ratio is 1.7 and 4.3, the ductility coefficient increased 1.07 and 1.05 times respectively. These results indicate that, the stiffness ratio and strength ratio reasonable matching is beneficial to improve ductility capacity of frame structure. But due to the mutual coupling, more specific interactions between stiffness ratio and strength ratio remains to be further studied.

Table 6 – Characteristic displacement and ductility coefficient of structures corresponding to different strength ratios

Model Coding	Strength ratio η_c	Stiffness ratio k	Yield point	Peak point		Ultimate point	
			Δ_y^t (mm)	Δ_{max}^t (mm)	μ_{max}	Δ_u^t (mm)	μ_u
M-07-11	1.1	0.7	25.6	75.0	2.92	128.7	5.02
M-07-13	1.3	0.7	25.8	78.2	3.02	131.0	5.07
M-07-15	1.5	0.7	24.8	78.6	3.17	127.7	5.15
M-17-11	1.1	1.7	25.8	75.2	2.92	127.2	4.93
M-17-13	1.3	1.7	25.8	77.3	3.00	131.5	5.10
M-17-15	1.5	1.7	25.1	79.6	3.17	132.3	5.27
M-43-11	1.1	4.3	24.9	51.4	2.06	98.8	3.97
M-43-13	1.3	4.3	24.9	51.2	2.06	101.9	4.09
M-43-15	1.5	4.3	24.8	51.7	2.08	102.9	4.15

3.3 distribution of plastic hinges

The distribution of plastic hinges can intuitively indicate the degree of structural damage in limit state. As shown in Fig.7, the distribution of plastic hinge of models with strength ratio is 1.1 at limit state. Circle size represents the value of curvature of plastic hinge section. The hollow circle represents plastic hinge of one-way yielded, solid circle represents the plastic hinge of two-way yielded. With the stiffness ratio increased from 1.2 to 3.4, conclusions can be seen from Fig.7: (1) Deformation of plastic hinge section increased gradually. (2) The number of plastic hinge increased gradually. (3) The degree of damage has been improved, failure mode basically belongs to " beam column mixed hinge ". (4) When the stiffness ratio is too small ($k=0.7$), there is a trend of the layer lateral displacement on the top floor of the structure. (5) When stiffness ratio is too large, the damage of whole structure increased, and in the bottom floor formed lateral displacement mechanism, failure mode is the most disadvantageous.

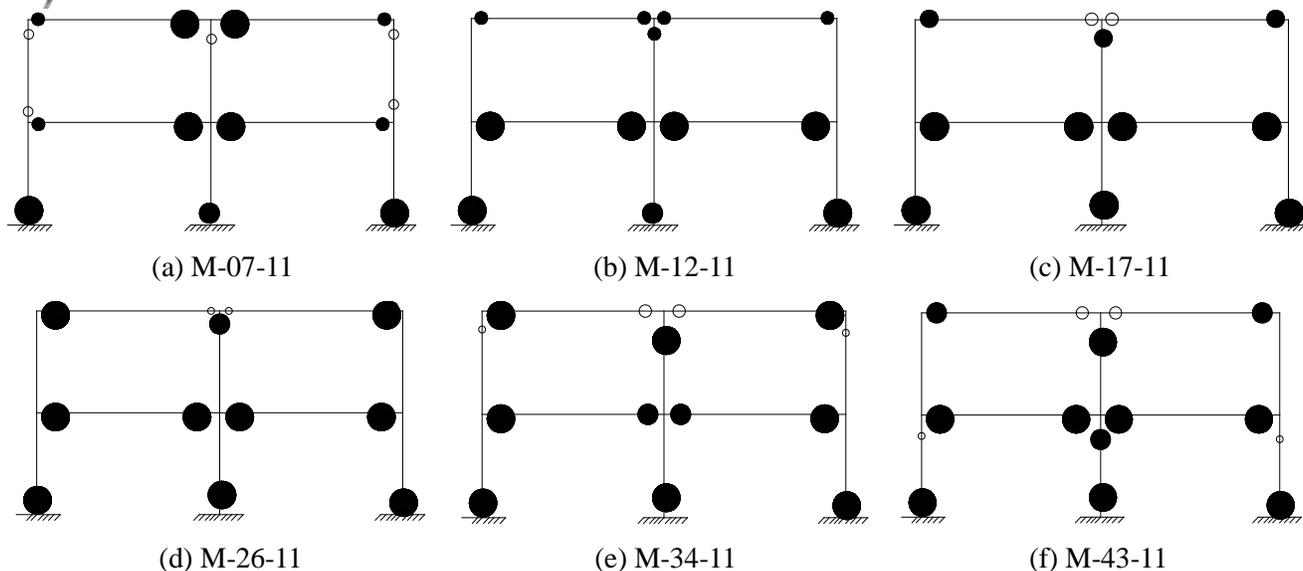


Fig.7 – The distribution of plastic hinge of models with strength ratio is 1.1 at limit state

As shown in Figure 8, the distribution of plastic hinge of models with different strength ratios which stiffness ratio is 0.7, 1.7 and 4.3. $K = 1.7$ represents the medium stiffness ratio ($k = 1.2 \sim 3.4$). Some conclusions can be obtained from Fig. 8: (1) Structures of medium stiffness ratio even if strength ratio is only 1.1, upper and lower ends of the column in examples do not emerge plastic hinges at the same time. The phenomenon indicating that reasonable stiffness ratio is the premise of emergence good distribution of plastic hinges. (2) All of upper and lower end of the column of top and bottom layers have emerged plastic hinges, when stiffness ratio is relatively small ($k = 0.7$) or relatively large ($k = 4.3$). (3) If the strength ratio is small ($\eta_c=1.1$), only when strength ratio increased to 1.3 or even above 1.5, the number of plastic hinges of column is significantly reduced. Only in this case can effectively avoid upper and lower end of the column emerge plastic hinges at the same time.

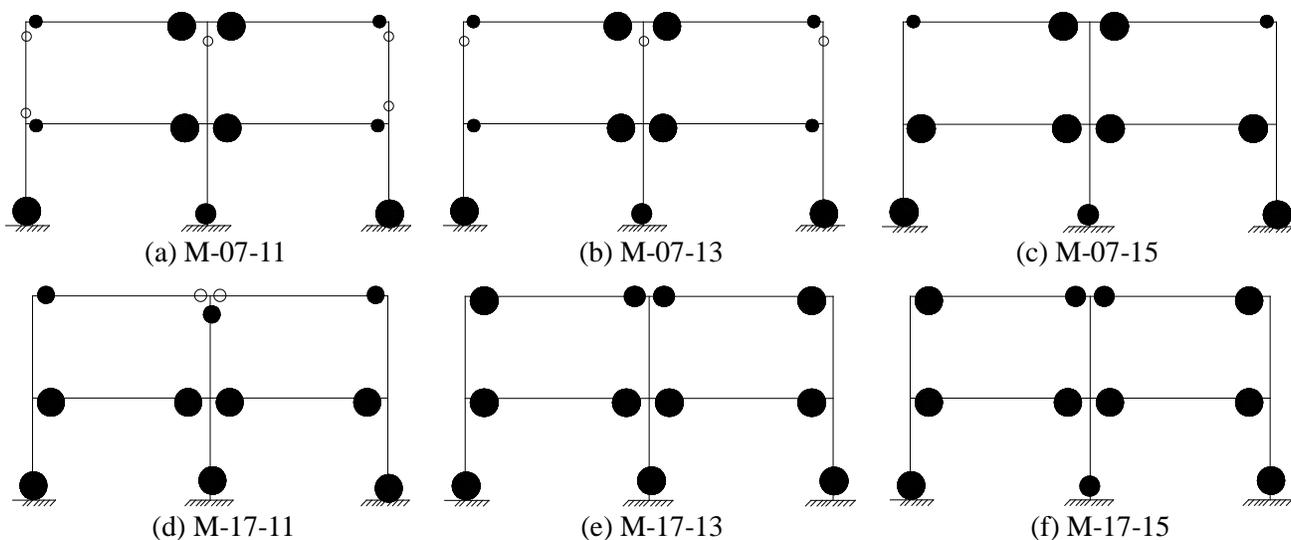


Fig.8 – The distribution of plastic hinge of models with different strength ratios as stiffness ratio is 0.7, 1.7 and 4.3 at ultimate limit state

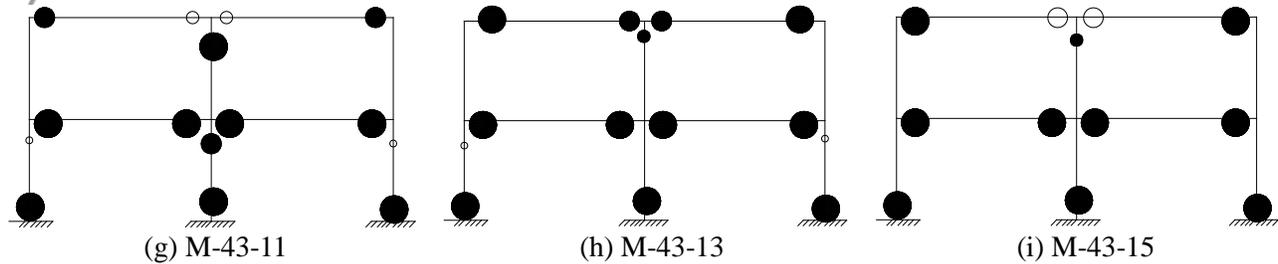


Fig.8 – The distribution of plastic hinge of models with different strength ratios as stiffness ratio is 0.7, 1.7 and 4.3 at ultimate limit state (continuous)

4 Conclusions

In this paper, based on the finite example analysis, the influence of stiffness ratio and strength ratio on frame structure seismic performance have been studied. The main results are as following:

(1) Increasing the stiffness ratio of the beam and column or the strength ratio of the column and beam, both of the horizontal bearing capacity and energy dissipation capacity of the frame structure can be improved. In order to improve the peak load, increasing the stiffness ratio is more effective than improving the strength ratio. From the angle of improving the ability of deformation, increasing the strength ratio is more effective for the deformation capacity after the peak load.

(2) In the design of frame structure, the contribution of the floor to the stiffness and strength of frame structure beams should be considered. When the stiffness ratio of the beam and column in the middle stiffness ratio ($k=1.2\sim 3.4$), the actual bending strength ratio of the column and beam should be controlled above 1.1. When the stiffness ratio of the beam and column is less than 1 or greater than 4, the actual bending strength ratio of the column and beam should be increased to 1.3 or more than 1.5.

5. References

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