

# RELIABILITY OF SIMPLIFIED MODELING STRATEGIES FOR IN-PLANE WALLS IN URM BUILDINGS USING ASCE 41-13 PROCEDURES

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#### Abstract

The current ASCE 41-13 standard does not provide recommendations or include references regarding computational modeling strategies for performance-based seismic assessment of in-plane walls in unreinforced masonry buildings. This paper is attempting to fill this research gap through an evaluation of the reliability of three suitable simplified nonlinear modeling approaches for in-plane walls and providing a comprehensive list of references on more complex modeling strategies for masonry buildings. The models -namely nonlinear spring macro-models, equivalent frames and a layered shells- were validated under general monotonic, cyclic quasi-static and earthquake loading on two full-scale wall tests reported in the literature. To demonstrate their practical use, the models were developed in the commercial software SAP2000. The numerical simulations showed that the pushover capacity curve and hysteretic loops for base shear vs. roof displacements were generally in acceptable correlation with the experimental results.

Keywords: Unreinforced masonry; nonlinear analysis; reliability; in-plane walls; seismic evaluation.



## 1. Introduction

The high vulnerability of Unreinforced Masonry (URM) buildings to in-plane damage and out-of-plane collapse has been consistently observed in past earthquakes. In an effort to conduct reliable nonlinear seismic analysis of multi-story URM buildings, several simplified numerical methods have been proposed in the last twenty years; and most of them have been validated through experimental data. Table 1 summarizes some of the most relevant approaches and corresponding authors. These modeling strategies can represent, with certain degree of accuracy, the strength, stiffness, and deformation capacity of piers and spandrels. Also, they can account for the combination of highly nonlinear materials, complex geometric configurations and static and dynamic loading. Nonetheless, in despite of these advances, current provisions in Chapter 11 of ASCE 41-13 [1] do not include recommendations or references regarding computational modeling strategies for URM buildings.

In this paper, three simplified and robust non-linear numerical macro-models for in-plane walls were developed and validated using previous experimental data found in the literature. The models were selected from studies already published in the literature. Also, the models can be easily deployed to analyze a full multi-story URM building subjected to seismic loads following the ASCE 41-13 procedures. They are suitable for pushover and/or nonlinear response history analysis, since peak strength, failure modes such as rocking or sliding and sequence of failure can be obtained with minimal computational time. Even more, the models were created in SAP2000 [2] to demonstrate their practical use for practicing engineers.

Global approach	Specific strategy	Authors and applied software				
Macro-scale models	Rigid body and springs	[3; 4]				
	Homogenization	[5; 6] in ABAQUS				
	Equivalent frame	[7-13] in SAP2000				
	Fiber or macro-model elements	[14-18] in TREMURI				
	Zero-length springs	[19-21] in DRAIN2DX and OpenSees				
	Rigid body models	[22-24] in ELS				

 Table 1 – Summary of simplified computer methods in structural masonry

# 2. ASCE 41-13 Seismic Evaluation Procedures for In-plane Walls

Since the models were selected so that they can implemented together with the ASCE 41-13 procedures, the inplane behavior of URM walls and assumptions to estimate its stiffness, strength, and force-deformation curves as outlined in Chapter 11 of ASCE 41-13 will be summarized below.

## 2.1 Seismic Behavior

Typically, URM walls show elastic behavior during the initial stage of in-plane seismic loading. This behavior can be described effectively using plane-stress models. Then, as the seismic loading increases, flexural or shear cracking (or both simultaneously) will take place and the behavior become highly nonlinear as large deformations occur in comparison with the applied forces. Small components of the wall such as piers will behave similarly and, therefore, can be modeled under the same principles [25].

# 2.2 Strength Capacity

ASCE 41-13 considers five kinds of in-plane failure modes for URM walls which can be classified either as ductile (displacement-controlled) or brittle (force-controlled). The strength is calculated according to the dominant failure mode in a solid wall, when an in-plane force is applied along its top boundary. Those failures mechanisms are summarized as follows:



- *Rocking failure*: This deformation-controlled failure has the ability to absorb relatively high amount of energy after initial cracking at the base of the wall or pier. As the seismic load increases, rocking about the vertical axis of the element is produced and the final failure is obtained by overturning of the wall.
- *Toe crushing*: This happens when the base of the wall crushes under the combined action of shear force and overturning moment. This failure mode, which is considered force-controlled, tends to be brittle in nature and typically the wall undergoes a sudden failure.
- *Bed-joint sliding*: This failure mode is more frequent in walls with low aspect ratio, low compressive force and high shear loads. Potential sliding planes will form along the cracked bed joints when subjected to reverse seismic actions.
- *Diagonal Tension:* This failure mode occurs when diagonal cracks forms through the masonry units. It is considered a brittle failure mode that quickly endanger the vertical load capacity of the piers.
- *Compressive Strength*: This happens when the compression strength of the piers is exceeded under the additional axial load induced by seismic forces. Thus, the compressive strength of existing masonry walls should be limited by the lower bound masonry compressive stress.

### 2.3 Elastic Stiffness

Since ASCE 41-13 assessment methods are mainly based on allowable drifts, it is necessary to calculate the initial in-plane stiffness of the wall. Conventional principles of mechanics for homogeneous materials are used, and both, flexural and shear deformations are taken into account. The standard recommends two equations to calculate the stiffness depending on boundary conditions (cantilevered or fully restrained against rotation at its top and bottom), although it is recognized that these boundary conditions are hardly found in practice. Values for the elastic and shear modulus can be found in the standard or obtained through experimental test. Also, it is important to highlight that depending on the wall and pier geometry, the effective height used to estimate pier stiffness may vary in the same wall assembly.

### 2.4 Hysteretic Behavior

Knowledge of the nonlinear force-deflection relations for individual wall, pier or spandrel members is needed to perform a nonlinear static and dynamic analysis. ASCE 41-13 prescribes a simplified multilinear force-deformation relation, for rocking and sliding mechanisms of primary elements in the lateral-force resisting system. This backbone envelope is not valid for force-controlled elements as they experience brittle failure and do not show nonlinear behavior. The multilinear curve is defined by the elastic stiffness and the component strength and acceptable drifts limits which are also stated within ASCE 41-13.

## 3. Selected Simplified Modeling Strategies

Three modelling techniques for in-plane URM walls were selected in this research. The first approach, based on nonlinear spring macro-models its simple, robust and computationally inexpensive. The model is suitable for probabilistic studies, which might require a significantly large number of analyses. The second selected approach is similar to the Equivalent Frame method referenced in Table 1. However, this model has more general applications and can be used for pushover, cyclic pushover and 2D response history analysis. This is a suitable methodology for performance-based assessment in that it balances complexity, efficiency and accuracy as it requires less computational time than current micro-scale finite element models but is more complete than the nonlinear spring macro-model. The third and final selected modeling strategy adopt nonlinear layered shell elements. This is an even more comprehensive approach that accounts for axial force-bending moment interaction and in-plane shear, in both, pier and spandrel elements. All modeling strategies can be implemented in SAP2000 [2]. The following sections describe these selected modeling strategies in more detail.

### 3.1 Nonlinear Springs Macro-Model (NSM)

The nonlinear spring modeling strategy is illustrated in Fig.1. This modelling strategy is based on the macromodel approach proposed in [20]. In this approach, masonry piers, spandrels and diaphragm are modeled in



each analysis direction using nonlinear springs with one active degree of freedom. Each spring represents a particular failure mode (i.e. rocking, joint shear sliding, toe-crushing or diagonal shear) for which the force-displacement backbone curve is predicted in advance using equations provided by ASCE 41-13.

The multi-linear plastic Pivot hysteresis rule shown in Fig.2, is used in combination with the previously defined backbone curve for cyclic quasi-static and dynamic analysis, as recommended in [12]. The cyclic behavior of piers and spandrels is characterized by stiffness and strength degradation. These characteristics are only partially captured by the multi-linear plastic Pivot hysteretic property in SAP2000. This hysteretic model was originally proposed by [26] to represent nonlinear behavior of reinforced concrete members. It is based on the assumption that unloading and reversed loading are directed toward specific points ("pivot" points) in the force-displacement plane. Three parameters define the cyclic behavior in this model namely, 1) alpha, which locates the pivot point for unloading to zero from positive or negative force, 2) beta, which locates the pivot point for reversed loading from zero toward positive or negative force and 3) nu, which determines the amount of degradation of the elastic slopes after plastic deformation [27]. These hysteretic parameters were defined as alpha=0.45, Beta=0.45 and nu=0.45.



Fig. 1 – Nonlinear Spring Macro-model

The overturning moment and P-delta effects were not addressed in this research due to inherent limitations in the composite spring macro-model. In particular, overturning moment effect could potentially been incorporated in the analytical model through modifying the soil-structure interface with a global rotational spring but vertical constraints must be added to the model to avoid evident instability of the system. Even without these considerations, it is demonstrated in the following sections that the seismic response of the wall could yet be captured with acceptable accuracy.



P.

Fig. 2 – Multi-linear Pivot plasticity property for uniaxial deformation [27]

α,Fy,

## 3.2 Equivalent Frame Method (EFM)

This approach has been extensively used by other researchers and practitioners in Europe and New Zealand. In this paper, a strategy similar to that followed in [13] is implemented. As shown in Fig.3, masonry piers can be effectively replaced by a beam element with discrete non-linear springs representing a particular failure mode for which force-displacement curve is computed in advance using equations given by ASCE 41-13.

With reference to Fig. 3, the beam element has dimensions H, L and t, and Young's modulus of the masonry Em. The combination of rocking and toe-crushing behavior is modeled using a rotational spring located at the top and the base of the beam element, whereas the bed joint and diagonal shear is simulated through placing one spring in the middle of the element. Note that the shear force-displacement relationship needs to be converted to bending moment-rotation relationship. As recommended by [13], spandrels can be modeled as beam elements with a shear spring in the middle, to somehow capture the maximum shear capacity. This may not introduce a high extent of precision as, in reality, the spandrel global capacity is in reality dependent on the interaction between axial and moment capacity. Nevertheless, most masonry walls are controlled by the shear or rocking behavior of the piers, rather than spandrels and this assumption yields to acceptable level of approximation.

The hysteretic behavior of piers and spandrels are characterized by stiffness degradation and pinching behavior. This behavior has been previously simulated using different proposed hysteresis rules and currently, there is no agreement on which rule provides a better representation. For instance, [12] used a Takeda rule, whereas [25] adopted a Bilinear Elastic rule to simulate the rocking mode of failure associated with piers and a Bilinear Inelastic rule to represent the bed joint shear sliding collapse mode of spandrels. A Pivot hysteresis rule for both piers and spandrels was used in [13]. In this study, the Pivot model utilized in the nonlinear spring macro-model was also selected to simulate the hysteretic behavior of piers and spandrels.

It must be noted that some major limitations are associated with this modeling approach. First of all, SAP2000 does not account for the axial force-bending moment interaction when using Nlink elements to simulate rocking or sliding behavior. In a real-life building, the axial force distribution in the piers changes significantly as the wall enters in the nonlinear range of material behavior and experiences global overturning. This could affect the ultimate failure mode of the pier and ultimate response of the building.

Another shortcoming associated with the adopted modeling approach comes from neglecting the contribution of flanges in the force-displacement capacity of the piers. It has been previously demonstrated that flanges of return wall have a major impact on the initial dominant failure mode of the piers, shifting it from rocking to shear sliding depending of the location of the flange. Moreover, the model does not account for axial force-bending moment interaction in spandrels, which could lead to over conservative estimates of the global seismic response. Moreover, the bidirectional simultaneous behavior of URM wall piers under seismic loads could potentially have major impact in stiffness and strength degradation, and should be incorporated in the model.



Fig. 3 – Modified Equivalent Frame Method

### 3.3 Layered Shell Method (LSM)

The seismic nonlinear behavior of unreinforced masonry walls can also be simulated using layered shell elements. The approach implemented by [5] is followed in this study. The anisotropic behavior of masonry is modelled using two different-strain curves, each simulating the compressive stress-strain behavior and the shear stress-strain properties. The compressive behavior is determined using the equations provided by [28] which defines a stress-strain behavior as shown in Fig.4. On the other hand, the shear stress-strain material properties is simulated using the approach TREMURI software, accounting only for cohesive behavior. An advantage of this method is that only one shell element is required to define both, piers and spandrels. Also, there is no need to estimate in advance the potential failure mode of the walls. Geometric nonlinearity and out-of-plane behavior are difficult to account for and were not considered in this study. The model formulation is illustrated in Fig.5.



Fig. 4 – Compressive stress-strain curve for masonry prisms [28].



Fig. 5 - Nonlinear Layered Shell Elements

## 4. Reliability of Simple Modeling Strategies for Nonlinear Static Procedures

The selected simple nonlinear strategies were all implemented in SAP2000 and validated through pushover and cyclic pushover analysis. To this end, experiments conducted by [29] on single pier elements and [30] on a full scale two-story URM building were used for comparison purposes.

#### 4.1 Pushover Analysis

The wall dimensions of the experiment carried out by [29] are shown in Fig.6 (a). The thickness of this wall is approximately 13" and the modulus of elasticity was estimated as 1250 ksi. This wall was subjected to a compressive stress of 70 psi on each pier and then subjected to increasing monotonic loading.

The nonlinear spring approach for the specimen is shown in Fig.6 (b). Note that there is only a shear spring located at the middle of the piers to represent rocking behavior. Other connecting elements are rigid beam elements with zero mass Fig.7 shows the actual force-displacement parameters calculated for Pier 2, and implemented in SAP2000. To run this simulation was also important to provide enough restrictions in the XY plane, so that the system was stable and deformed only in the in-plane direction.

Fig.6 (c) illustrates the implementation of the equivalent frame method. Herein, piers and spandrels were simulated using beam elements connected by rigid link elements. Note that two rotational springs were used for the EFM, one at the top, one at the bottom of each pier. Similarly to the nonlinear spring macro-model, all parameters of the multi-linear plastic Pivot hysteresis rule were set equal to 0.45. The wall was modeled in 2D with fixed joint restraints at the base and a unit load was applied on top of the wall to carry out the pushover analysis.

The corresponding layered shell model for the specimen is depicted in Fig.6 (d). Also, the nonlinear stress-strain relationship for compression and shear are shown in Fig.8 (a) a and Fig.8 (b), respectively. Note that no tension was allowed in the concrete material. For this simulation, an Elastic modulus of 700 psi was considered and a lower bound capacity in shear of 40 psi was assumed. The layer definitions is shown in Fig. 9.

The force-displacement relationship obtained from both the experiment and the analysis are plotted in Fig.10. Note that no attempt was made to calibrate the models and match the experimental results, since the intent is to show the reliability without manipulating material properties. Overall, the models reproduce fairly well the initial stiffness and maximum strength and closely reproduced the strength (about 40 kips). However,



the NSM gives an initial lower bound capacity of 33 kips whereas the LSM yields an upper bound strength of 50 kips. The greater difference between the models is more evident at ultimate limit state, where a significant residual capacity is overestimated by the layered shell model.



(c)

(d)

Fig. 6 - a) Experimental wall tested by [29], b) NSM, c) EFM, d) LSM

Link/Support Type MultiLinear Plastic		Identification		Hysteresis Type And Parameters					
Property Name PIER2 Set Default Name		Property Name PIER2		Hysteresis Type Pivot •					
Property Notes Modify/Show Total Mass and Weight		Direction	U2	α45	β	.45	η .45	45	
			MultiLinear Plastic	α45 2	β2	.45			
Aass 0.	Rotational Inertia 1	0.	NonLinear	Yes					
eight 0. Rotational Inertia 2 0.		Properties Used For Linear Analysis Cases		Hysteresis Definition Sketch					
	Rotational Inertia 3	0.	Effective Stiffness	2908.3114	Multilinear Plas	tic - Pivot			
Fartors For Line Area and Solid Springs		Effective Damping	Effective Damping 0.			2 Fy 2	P4 P3		
Property is Defined for This Length	in a Line Spring	0.0394	Shear Deformation Location	n			/		
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V R1 V	Modify/Show for R1		Order Rows	Add Row 10		P2 P1	α 1 Fy 1		
V R2 V	Modify/Show for R2	OK			,				
V R3 V 0		Cancel		-					

Fig. 7 - Nonlinear force displacement hysteresis defined for NSM



Fig. 8 - Nonlinear Layered Shell Elements properties for a) Compression and b) Shear



Fig. 9 - Nonlinear Layered Shell Elements Definition in SAP2000



Fig. 10 – Force-displacement curve for the wall: a) Numerical and b) Experimental [29]

### 4.2 Cyclic Quasi-static Behavior

In 2002, the Mid-America Earthquake (MAE) Center conducted a research project that focused on evaluating the procedures specified in FEMA 356 to assess the seismic performance of unreinforced masonry buildings. The research program included a full-scale quasi-static test of a two-story masonry building and is used in this study to validate the proposed nonlinear macro-models at the global response level. Fig. 11 (a) shows detailed dimensions of the two-story masonry test model. In this study, only Wall 1 is shown for comparison purposes, but additional hysteresis loops were also simulated of walls A, B, and 2 to illustrate the robustness of this approach.



The material properties and axial loads from walls and floor as described in [30]. The cyclic loading protocol was applied at the floor and the roof of the model, as it was done in the real experiment. A displacement controlled simulation was conducted, therefore, the DOF associated to the point of loading were constrained in the same direction of the applied loading. For the NSM and the EFM, the hysteretic behavior was also simulated using the Pivot rule available in SAP2000 with all parameters set equals to 0.45. The LSM used the formulation described in Section 3.3 but considering an Elastic modulus of 1000ksi. Again, the shear strength capacity was set to a lower bound of 40 psi.

Fig. 12 shows a comparison between the numerical and experimental cyclic force- displacement curve obtained for Wall 1. Overall, a good agreement can be seen in terms of maximum force and cyclic behavior. The NSM and the EFM yield similar results and capture stiffness and strength degradations. The LSM captures the asymmetric behavior of the cyclic response but overestimates the energy dissipated on each cycle and failed to capture stiffness degradations. All models predicted initial stiffness and peak strength accurately.



Fig. 11 - a) Experimental wall tested by [30] b) NSM, c) EFM, d) LSM



Fig. 12 – Force displacement relationship of Wall 1: a) Numerical and b) Experimental [30]

#### 5. Conclusions

This paper presented practical modeling techniques for nonlinear analysis of in-plane URM walls, which could be implemented in SAP2000 and potentially extended to simulate the seismic behavior of multi-story URM buildings. The absence of recommendations and/or references in ASCE 41-13 highlighted the need for a study to promote the implementation of simplified numerical models can properly estimate the seismic response of in-plane walls in URM buildings.

The rocking and shear sliding behavior of URM wall piers was simulated using three different simplified modeling techniques, namely the Nonlinear Spring Macro-models, the Equivalent Frame Method, and the Layered Shell Method. In the NSM and EFM, the force displacement or bending moment-rotation relationships are constructed from the equations provided in ASCE 41-13 and further implemented in SAP2000, using rotational or shear nonlinear springs. The LSM make use of simple constitutive material models and is aimed to estimate the compression and shear stress-strain relationships. Monotonic and cyclic pushover analysis results revealed good agreement with previously conducted experiments on in-plane walls and demonstrated the reliability of the selected modeling techniques.

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