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PARK-ANG DAMAGE MODEL UNCERTAINTY QUANTIFICATION USING EVIDENCE THEORY

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Abstract

The Park-Ang damage model has been widely used due to the consideration of first exceedance failure and cumulative damage failure in terms of defining the structural damage under earthquake. However, performance evaluation and design of civil facilities against earthquakes using the Park-Ang model is a challenge to engineers because of the large uncertainty in this damage model. Some of these uncertainties stem from factors that are inherently random (or aleatory) in engineering or scientific analysis (e.g, material properties such as, Young's modulus of steel; compression strength of concrete). Others arise from a lack of knowledge, ignorance, or modeling (or epistemic) (e.g, simplification of mathematical model of buildings for structural analysis purposes). Uncertainties in damage models and their influence on structural behavior are important considerations in performance evaluation and design of structures against earthquakes. In this article, a methodology based on the evidence theory is presented for uncertainty quantification of Park-Ang damage models. The proposed methodology is applied to Park-Ang models while considering various sources of uncertainty emanating from experimental force-displacement data of reinforced concrete column. The Park-Ang damage model uncertainties are propagated through evaluation the damage level of the RC columns. A global optimization technique is used for efficient calculation of the propagated belief structure of the structural response and aggregation rule is used for multi-models consideration. Finally, damage evaluation example of a column is investigated to demonstrate the effectiveness of the proposed method.

Keywords: Epistemic uncertainty; Park-Ang damage model; evidence theory.

1. Introduction

With the rapid development of performance based earthquake engineering (PBEE), the studies on effective evaluation measures for structure damage have become the hot spot of earthquake engineering. In consistence with the different performance assessment criteria, the computational approach of damage value for a structure has been classified as displacement-based approach, energy-based measure as well as the combination of both. With the simplicity and convenience of observation and description for structural damage, the displacement-based approach and corresponding damage index (e.g. inelastic displacement, maximum inter story drift ratio, and ductility demand, etc.) has been widely documented in the building seismic evaluation and retrofit of existing building guidelines [1]. Notwithstanding the displacement method based damage assessment has been wide spread, the defect of lacking the influence of low cyclic fatigue of structural components is obvious. The hysteretic energy dissipation is considered as a more reasonable indicator for seismic structural damage, because it is a cumulative parameter involved cyclic–plastic deformations in a structure during earthquakes [2]. Although, the hysteretic energy is relatively simple and effective, some observations demonstrated the expression of energy would be significantly affected by the exceedance plastic deformation [3]. And the cumulative laboratory experimental data on structural members and structures indicate the fact that the structure is damaged by a combination of the excessive deformation and hysteretic energy. Park–Ang damage model [4], which takes into account the effects of both the first exceedance failure and cumulative damage failure in low-cycle-fatigue for a structural component during seismic load is served as a baseline for many researches. Due to intrinsic simplicity as well as calibrations against a significant amount of observed seismic damages, the Park-Ang model and modified version have been widely used in performance evaluation of structures [5-7].

Although the applicability and practicability of using the Park–Ang model and its modified versions have been supported by many researchers [8-9], it should be noted that the Park-Ang-damage-index-based performance evaluation is a very challenging task due to the large uncertainties associated with the damage model parameters [10]. With the influence of these uncertainties [11-12], the evaluation results of structural damage state are always represented with the empirical interval value (e.g. the minor damage state is represented by $0.25<D<0.4$ or $0.11<D<0.4$ etc. [13]). Some of these uncertainties stem from factors that are inherently random (or aleatory) in engineering or scientific analysis (e.g. material properties such as, Young's modulus of steel; compression strength of concrete). Others arise from a lack of knowledge, ignorance, or modeling (or epistemic) (e.g. simplification of mathematical model of buildings for structural analysis purposes). The large uncertainties associated with the Park-Ang damage model is due to the fact that the model parameters derived from sparse experimental data and approximate modeling (lack of knowledge)[2-4-5-10]. Considering the importance of damage model in assessment of damage state for a structure, the epistemic uncertainty shall be taken into account in seismic damage state assessment with great care. Hence, it is significant to present a comprehensive uncertainty analysis methodology to quantify the epistemic uncertainty and obtain more reliable results.

The traditional probability theory, based on the sufficient experimental data, is used to represent the objective uncertainty (random) which is inherent in physical variability of materials and environment. However, the limited number of experimental data set cannot support the strong assumption of probability theory and the process of collecting data is always costly and time consuming. These shortcomings lead the assessment result of damage state of components and structures are not random but epistemic. In the past decades, several alternative approaches have been developed to deal with epistemic uncertainty. Some of the potential uncertainty theories are the theory of fuzzy set [14], possibility theory [15], the theory of interval analysis [16], imprecise probability theory [17] and evidence theory [18-19]. Among these promising uncertainty representation models, evidence theory with the ability of handling aleatory and epistemic uncertainty is used to uncertainty quantification, risk assessment and reliability analysis.

With two complementary measures of uncertainty: belief and plausibility, using evidence theory to uncertainty quantification (UQ) is very flexible and effective. In comparison with the calculation of single probability density function (PDF) in probability theory, the computationally intensive problem involves computing the bound values over all possible discontinuous sets is a main shackle of wide application for evidence theory. In order to alleviate the computational costs in the evidence theory based uncertainty quantification analysis, the differential-evolution-based interval optimization is employed to enhance the computational efficiency as described previously by the authors are introduced [20].

The main theme of current paper is to investigate uncertainty of Park-Ang damage model using sparse experimental data and explore the feasibility of the proposed approach. Firstly, the frame work of uncertainty quantification, the empirical prediction model of Park-Ang damage model constants and the calibration results with database are introduced. Then, the basic concept of evidence theory and evidence-based UQ frame work for Park-Ang damage model are presented. For studying the effectiveness of proposed methodology, the UQ of Park-Ang damage index for a column load test is applied.

2. Calibration the constants of Park-Ang model

2.1 Park-Ang model and empirical expression of its constants

The Park-Ang damage model [4] combines the first exceedance failure and cumulative damage failure with a linear expression as:

$$D = \delta_{\mathrm{m}} / \delta_{\mathrm{u}} + \beta \int dE \Big/ F_{\mathrm{y}} \delta_{\mathrm{u}} \tag{1}$$

where δ_{m} is the maximum deformation under earthquake, δ_{u} is the ultimate deformation under monotonic load, $\int dE$ is the cumulative energy under earthquake, F_{y} is the yield strength. In order to simplify the analysis procedure, the value of F_{y}, δ_{u} and β are always assumed as the constants and have nothing to do with the loads pattern. Following above assumption, the value of damage index D for per-load stage can be computed by only using the current value of δ_{m} and $\int dE$. Furthermore, the damage evolution of structures and components can be described and this evolution index is supported to estimate the true damage stage of structure and components.

From the last two decades of the twentieth century, a set of experimental results were conducted and some illuminate empirical or mechanical based expression of F_{y}, δ_{u} and β were successively generated. Park and co-workers [4] computed the value of β as Eq. (2):

$$\beta = \left(-0.447 + 0.073 l/d + 0.24 n_0 + 0.314 \rho_t\right) \times 0.7^{\rho_\omega} \tag{2}$$

where l and d denote the length span and effective height of cross section, n_0 is the axial load ratio, ρ_{t} is the longitude tension steel ratio (%) and ρ_ω is confinement ratio (%). Kunnath et al. [5] used 260 beams and columns data to fit the value of β as Eq. (3):

$$\beta = \left[0.37 n_0 + 0.36\left(k_p - 0.2\right)^2\right] 0.9^{p_w} \tag{3}$$

where $k_p = \rho_t f_{\mathrm{y}} / 0.85 f_{\mathrm{c}}$ is normalized steel ratio and p_w is confinement ratio. Similarly, δ_{u} can be determined with statistical approach or fundamental method using the mechanics of concrete and steel. As a typical statistical measure, Park [6] determined the ultimate displacement as:

$$\delta_u = 0.52\left(l/d\right)^{0.93} \rho^{-0.27} \rho_\omega^{0.48} n_0^{-0.48} f_c^{-0.15} \times \delta_y \tag{4}$$

where ρ is normalized steel ratio and δ_{y} is the yield displacement of components which can be computed with reference [4] and other factors are same as above. Compared to this statistical calculation model, EU 8 [21] and Fardis with his co-workers [22] presented two different models with the mechanics of concrete and steel:

$$\delta_{\mathrm{u}} = \frac{1}{\gamma_{el}} 0.016(0.3)^{n_0} \left[\frac{\max(0.01, \omega')}{\max(0.01, \omega)} f_c \right]^{0.225} \left[\min\left(9, \frac{l}{h}\right) \right]^{0.35} 25^{\left(\alpha \rho_{sx} \frac{f_{yw}}{f_c}\right)} \times l \tag{5}$$

$$\delta_u = \alpha_{st}(1-0.4\alpha_{cyc})(1+0.5\alpha_{sl})(0.3)^{n_0} \left[\frac{\max(0.01, \omega')}{\max(0.01, \omega)} f_c \right]^{0.175} \left(\frac{l}{h}\right)^{0.4} 25^{\left(\alpha \rho_s \frac{f_{yw}}{f_c}\right)} \times l \tag{6}$$

where γ_{el} is coefficient of primary and secondary elements, $\acute{\omega}$ and ω are mechanical steel ratio of compression and tension reinforcement, h is cross section height, α is confinement effective factor, ρ_{sx} is confinement steel ratio, f_{yw} is yield strength of traversal steel, α_{st}, α_{cyc} and α_{sl} are coefficients for type of steel, loading and anchorage slip. For the yield strength of concrete components, the expression is given by Panagiotakis [23]:

$$F_y = \frac{bd^3}{l} \phi_y \left\{ E_c \frac{k_y^2}{2} \left(0.5(1+\delta') - \frac{k_y}{3} \right) + \frac{E_s}{2} \left[(1-k_y)\rho + (k_y - \delta')\rho' + \frac{\rho_v}{6}(1-\delta') \right] (1-\delta') \right\} \tag{7}$$

Conventionally, the damage index D can be obtained by using above expressions to compute the nominal value of Park-Ang constants. Owing to using limited statistical data and incomplete knowledge of mathematical model to predict these constants, the large convergence are reported as references [4, 5, 6, 22, 23]. Furthermore, these uncertainty will influence the quantification result of Park-Ang damage index. In order to verify the impact of damage quantification result derived from uncertainty of Park-Ang model constants, we present the PEER structural performance database [24] to calibrate these constants and determine the uncertainty fluctuation range of each constant.

2.2 Comparison between the calibration results and empirical results

In this article, the calibration set is selected from PEER structural performance database and the selection criteria are (1) the cross section of column is rectangle; (2) the column is loaded cyclically until failure and the corresponding failure model is dominated by flexure; (3) the longitude bars in column should not be spliced and the column should experience more than two hysteretic cycles. In conformity with these criteria, 185 specimens are selected. Using these column load-displacement data, the performance points on the backbone curve of column under cyclic load are calibrated.

Similar to the most studies [22], the ultimate deformation under monotonic load δ_{u} is defined as a distinct reduction on the negative stiffness slope of backbone curve and 80% of maximum strength which is always assumed as F_{u}. Unfortunately, the number of monotonic load experiments is so scarce that we have to employ the statistical relationship of ultimate displacement under cyclic load and monotonic load to characterize the ultimate displacement. Herein, the failure displacement under typical load histories is assumed as 60% of their ultimate deformation capacity, which is firstly observed by Panagiotakis [23]. For yield force, we defined the value is the 75% of the maximum force. Following above definitions, the energy coefficient β is computed with the assumption that damage index D is 1 at the ultimate state. In light with above definitions, the performance point is marked on the backbone of columns as depicted in Fig.1.

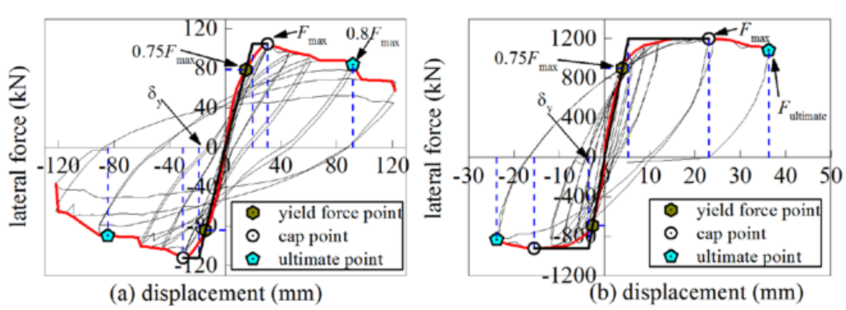


Fig. 1 – Performance point of backbone curve

As shown in Fig. 1, the column backbone curves are divided into two categories: one with obvious ultimate state point (the 80% maximum force) point like subfigure (a), the other with the largest displacement in backbone curve (e.g. subfigure (b)). In order to obtain the uncertainty distribution of empirical model, the attention is concentrated on the first category. Using the selected force-displacement data, the comparison of empirical model results and calibration results are given in Fig. 2.

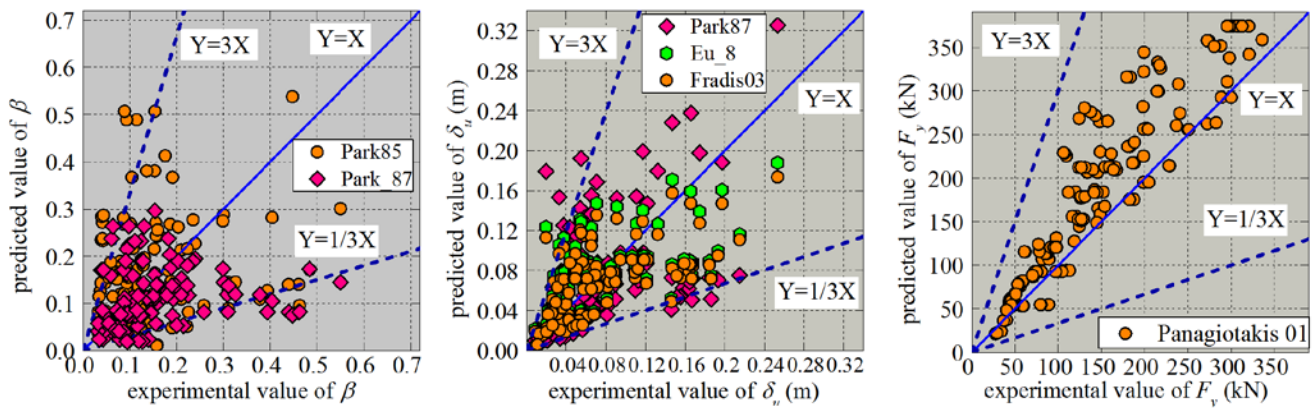


Fig. 2 – Comparison of predicted results and experimental results of β, δ_u and F_y

As shown in Fig. 2, the predicted and experimental values are scattered in a wide range, and this means the researchers should carefully handle the uncertainty derived from the empirical model in the process of evaluating damage state with Park-Ang model. Employing the parameter ε to represent the variability of predicted model deviation, the experimental value V_{exp} can be expressed as: $V_{exp} = V_{pre} \times \varepsilon$. Take into account the major fluctuation range of ε and the number of experimental samples, the ε which located in the interval [1/3, 3] are selected and the range of data points which located less than 1/3 or more than 3 are discarded. In the light of above rules, the uncertainty source of β, δ_u and F_y consist of 83, 111 and 173 specimen, respectively. Along with classical concept, probability theory acts the key role in the uncertainty quantification (UQ) of physical model, and the distribution type is determined by hypothesis test and related parameter are calibrated by enough experimental data. However, the limited data of experimental set and large variation restricted the ability of probability theory. As a generalized uncertainty quantification measure, evidence theory is compatible with both aleatory and epistemic uncertainties. So, the evidence theory is adopted in this article to handle the epistemic uncertainty rooted in parameters of Park-Ang damage model.

3. Uncertainty quantification using evidence theory

3.1 Basic concept of evidence theory

Evidence theory so-called Dempster and Shafer Theory (DST) was proposed by Dempster [18] and Shafer [19]. Given a measureable sample space (Ω, F), the basic belief assignment (BBA) on F, m is a mapping $F \rightarrow [0, 1]$ that satisfies the following axioms:

$$m(A) \geq 0 \quad m(\varnothing) = 0 \quad \sum m(A) = 1 \quad \text{for each } A \subseteq \Omega \tag{8}$$

Different from traditional probability distribution function, the BBA is defined on the power set $p(\Omega)$, whereas the former is defined on the probability space Ω. In evidence theory, the original information is represented with a nonempty subset A ($m(A)>0$), the subset A is named focal element. Corresponding to a single measure in probability theory, belief and plausibility measures are used in evidence theory to characterize uncertainty by indicating the confident degree to believe that event are true and not false, respectively. Similar to additive rule in probability, belief and plausibility measures of proposition B can be calculated from following:

$$Bel(B) = \sum_{A \subseteq B} m(A) \quad \text{for all } B \subseteq \Omega \tag{9}$$

$$Pl(B) = \sum_{A \cap B \neq \varphi} m(A) \quad \text{for all } B \subseteq \Omega \tag{10}$$

where A represents different elements in $p(\Omega)$. In terms of two complementary sets A and $\tilde{A}$, the sum of belief function and plausible function are not required to be one. But the weaker rule $Pl(A)+Bel(\tilde{A})=1$ is satisfied, and this expression is completely different from probability distribution function p in probability theory, that is $p(A)+p(\tilde{A})=1$. Fig. 3 is used to illustrate this weaker relationship.

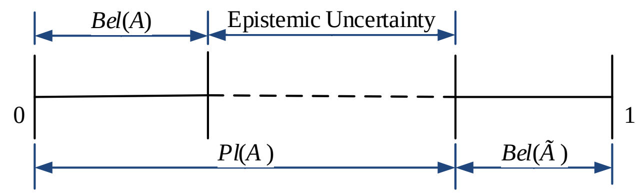


Fig. 3 - Uncertainty description of proposition

As the most remarkable distinction from probability theory, evidence theory allows evidence stemming from different sources and employs the rules of combination to aggregate. One of most important combination rules is Dempster's rule which has following formulation. Given two independent BBA $m_1(A)$ and $m_2(C)$, the Dempster's rule can be expressed as:

$$m(B) = \frac{\sum_{A \cap C = B} m_1(A) m_2(C)}{(1-K)} \quad \text{for all } B \neq \varnothing \tag{11}$$

where $K=\sum_{A \cap C=\varnothing} m_1(A)m_2(C)$ can be viewed as contradict or conflict among the information given by the independent knowledge sources and $m(B)$ denotes the supported evidence.

3.2 Evidence based UQ frame work for Park-Ang damage model

Following the brief description of evidence theory, the uncertainty quantification frame work for Park-Ang damage model using evidence theory is presented. Similar to the traditional probability theory, the BBA value of each focal element can be derived from statistics of experimental data. Herein, the evidence construct rule from finite experimental data is provided by Salehghaffari et al. [25]. To illustrate this rule, two adjacent bins I_1 and I_2 which consist of A and B number data points ($B<A$) are used to denote the adjacent bins in its histogram. Relying on the ratio of the B and A the relationship of I_1 and I_2 can be assorted as ignorance, agreement and conflict, and the corresponding BBA values of bins are listed in Table 1:

Table 1 – BBA value with three relationships

BBA	**Ignorance $B/A<0.5$**	**Agreement B/A>0.8**	**Conflict $0.5\leq B/A\leq 0.8$**
$m(\{I_1\})$	$A/(A+B)$	Two adjacent intervals can be combined into one	$A/(A+B)$
$m(\{I_2\})$	0		$B/(A+B)$
$m(\{I_1,I_2\})$	$B/(A+B)$		0

Employing this strategy, the uncertainty of Park-Ang model parameters can be properly represented with evidence theory. In Fig. 4, we use $\varepsilon_A(\beta)$, $\varepsilon_B(\beta)$ and $\varepsilon(F_y)$to denote the variability of the predicted models in reference [4], [5] for energy constant β and the one in [23] for yield force F_y of columns. The $\varepsilon_C(\delta_u)$, $\varepsilon_D(\delta_u)$ and $\varepsilon_E(\delta_u)$ in Fig. 5 represent the fluctuation of the empirical model for ultimate displacement under monotonic loading in articles [6], [21] and [22], respectively.

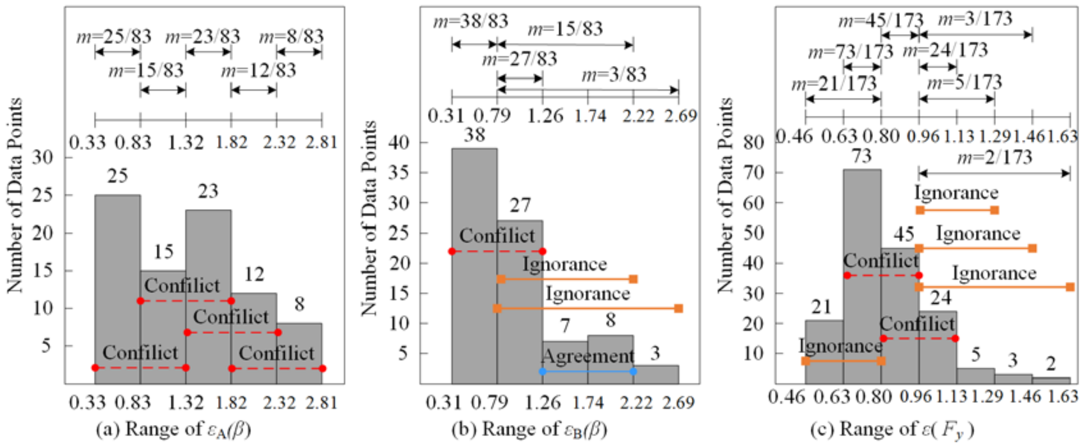


Fig. 4 - Uncertainty description of proposition

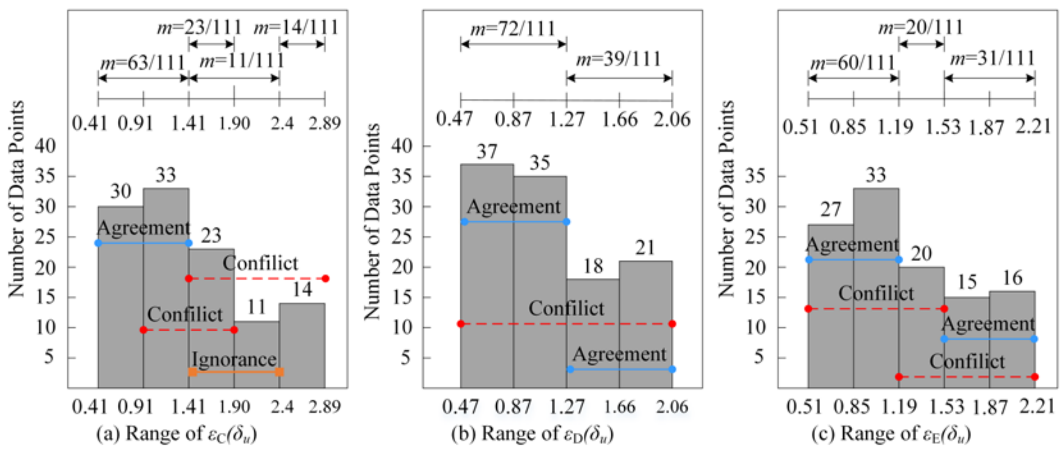


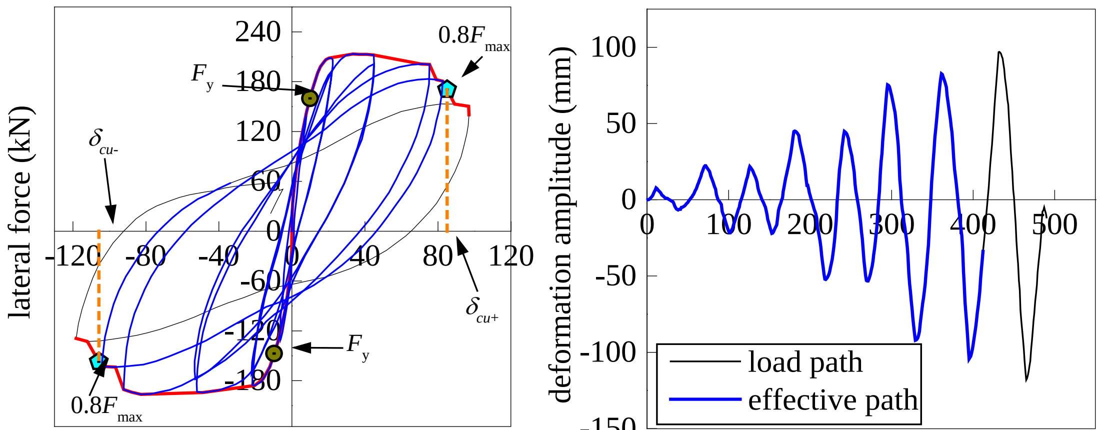
Fig. 5 - Uncertainty description of proposition

After representing the uncertainty with evidence theory, the joint BBA structure is constructed with Cartesian product of all variables. Compares to the single point value in probability theory, intervals are used in evidence theory to denote the uncertainty fluctuation. And this interval representation brings about the intensive computational cost for uncertainty propagation which involves obtaining the boundary response of each joint interval. Some strategies, like Monte-Carlo sampling and its modified versions, interval analysis and optimization algorithm are applied to solve this problem. It is well known that the accuracy of Monte-Carlo sample is based on the numbers of population and the interval analysis unable to handle complex problems, especially to calculate a black-box. As an intelligent optimization algorithm, differential evolution algorithm (DE) [26] is adopted herein not only to obtain accurate system response but to overcome the barricade of computation cost.

As a stochastic direct search method with advanced strategies, general selection and fast convergence, DE is always used to solve non-differentiable, non-linear, high-dimensional and other complex computational optimization problems. In this paper, we use differential evolution global optimization to calculate the boundary responses of each joint interval. After completing the computation of each joint interval, the cumulative belief function (CBF) and cumulative plausibility function (CPF) are attained by sorting the uncertainty response of each joint focal element and staking the BBA of each joint focal element.

4. Case study

In order to investigate the effectiveness and feasibility of the proposed UQ measures, the column "zahn86u7" [27] is selected to compute the Park-Ang damage index in its load step. The backbone curve and load history are shown in Fig.6.



(a) displacement (mm) (b) load path

Fig.6 – backbone curve and load path of columns

As shown in Fig. 6 (a), the cyclic ultimate displacement is calibrated by using the average value of 80% maximum force point on the force capacity reduction slope of positive and negative direction. The effective path in Fig. 6 (b) denotes the load path from initial state to ultimate state and the load path is the global displacement history. Using the properties of column, listed in the webpage of PEER, the nominal value of Park-Ang damage model constants β, δ_u and F_y can be calculated with the empirical expressions from Eq. (2) to Eq. (7), respectively. In consistent with section 3.2, the uncertainty distribution of model constants can be depicted as the nominal value multiply the factor ε. Taking the computed results into the evidence representation process, the BBA structures of β, δ_u and F_y with different models are listed in Table 2 and Table 3. Using above information, the evidence theory based uncertainty quantification results for each load step as shown in Fig. 7.

Table 2 – the BBA structure for multi-source of β and F_y

$\boldsymbol{\beta}$				$\boldsymbol{F_y}$	
Model A		**Model B**			
Range	BBA	Range	BBA	Range	BBA
[0.0345, 0.087]	0.301	[0.0266, 0.067]	0.458	[77.40, 133.19]	0.121
[0.0873, 0.139]	0.181	[0.0672, 0.108]	0.325	[105.22, 133.19]	0.422
[0.139, 0.192]	0.277	[0.0672, 0.189]	0.181	[133.19, 161.01]	0.26
[0.192, 0.244]	0.145	[0.0672, 0.230]	0.036	[161.01, 188.82]	0.139
[0.244, 0.296]	0.096			[161.0, 216.63]	0.029
				[161.01, 244.44]	0.017
				[161.01, 272.42]	0.012

Table 3 – the BBA structure for multi-source of δ_u

Model C		**Model D**		**Model E**	
Range	BBA	Range	BBA	Range	BBA
[0.034,0.115]	0.568	[0.043,0.116]	0.649	[0.0442, 0.104]	0.541
[0.115, 0.156]	0.207	[0.116, 0.188]	0.351	[0.104, 0.133]	0.180
[0.115, 0.196]	0.01			[0.133, 0.193]	0.279
[0.196, 0.237]	0.125				

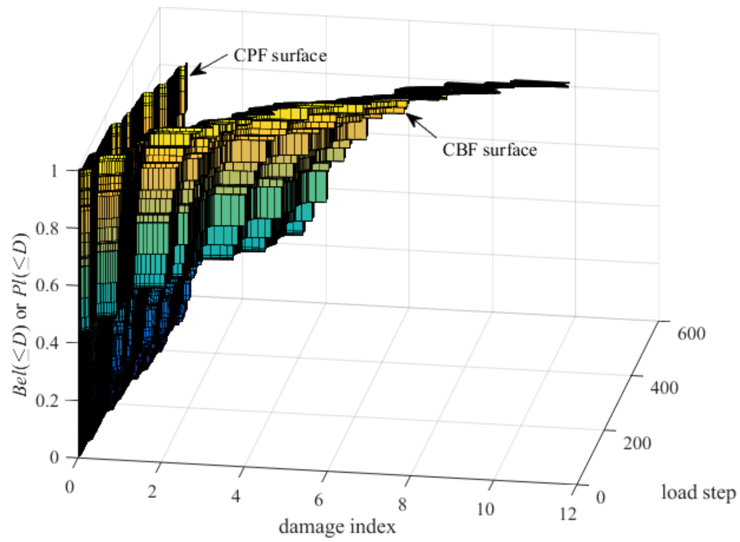


Fig. 7 – The uncertainty propagation results using evidence theory

To validate the generality of evidence theory, the variability of Park-Ang model parameters is also represented by probability theory. The goodness of fit test is applied to test the distribution type and determine the related distribution parameters. The uncertainty distribution information of model B for β, model C for δ_u and F_y are presented in Table 4.

Table 4 – The distribution information of Park-Ang constants

constants	distribution type	m_u	σ
$\varepsilon_B(\beta)$	normal	0.963	0.529
	lognormal	-0.171	0.514
$\varepsilon_C(\delta_u)$	normal	1.404	0.697
	lognormal	0.206	0.537
$\varepsilon(F_y)$	lognormal	-0.272	0.225

From Table 4, the value of $\varepsilon_B(\beta)$ and $\varepsilon_C(\delta_u)$ do not refuse the normal and lognormal distribution. We use two strategies to construct the probability input of variables. In first strategy, the lognormal distribution is applied to fit the all uncertainty input and the cumulative distribution function of uncertainty response which is indicated as CDF1. In other strategy, the probability distribution of $\varepsilon_B(\beta)$ and $\varepsilon_C(\delta_u)$ are assigned as normal distribution, while the distribution of $\varepsilon(F_y)$ is lognormal and corresponding cumulative distribution function of uncertainty result is represented as CDF2. To compare the quantification results of probability theory and evidence theory, the Fig. 8 is presented to describe the damage index evolution in load steps 70, 140, 210, 280, 350 and 412, respectively. To make a further illustration for the damage state evolution in each load step, the point 0.25, 0.5, 0.75 and 1 are used to represent the minor, moderate, severe and collapse damage state, respectively.

As illustrated in Fig. 8, the probability theory based uncertainty quantification results CDF1 and CDF2 are located in the range of curves CPF and CBF, this indicates that evidence theory is compatible to probability theory. The discrepancy of CDF1 and CDF2 demonstrates that probability theory may not be suitable to handle the epistemic uncertainty which stem from limited experimental data. In other words, the probability-theory-based uncertainty quantification result is ambiguous due to epistemic uncertainty and the choice of distribution type have a great impact on the quantification result. However, evidence-theory-based uncertainty quantification strategy demonstrates its power to quantify the epistemic uncertainty because of its two uncertainty measures belief function and plausibility function.

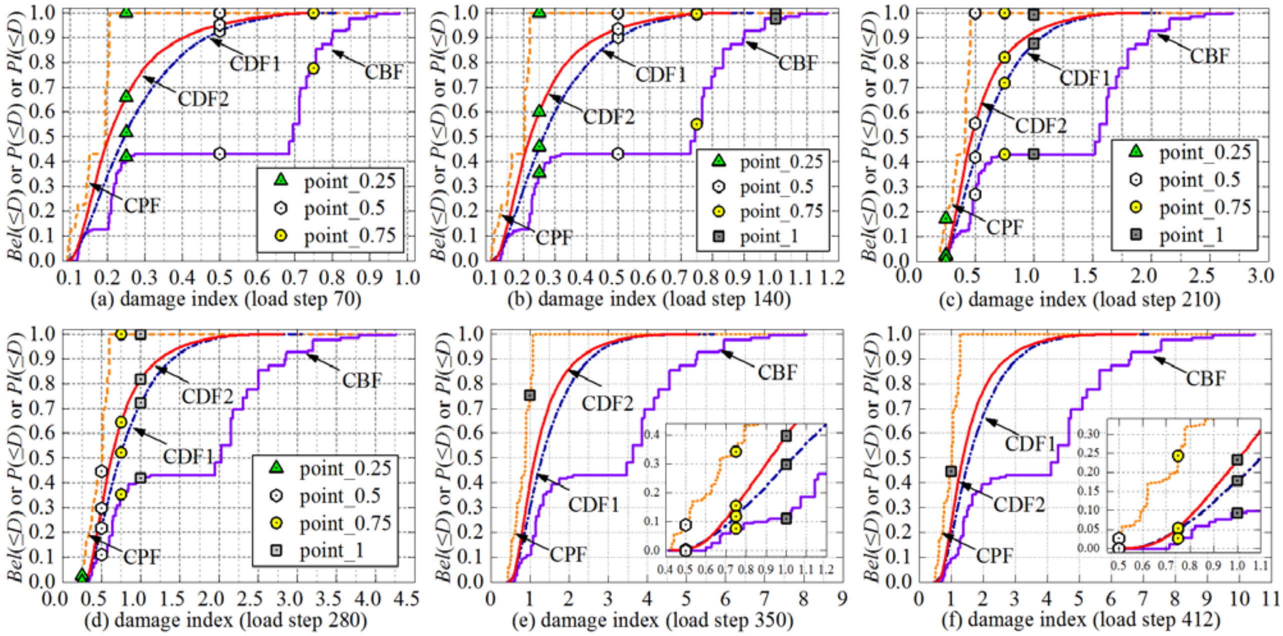


Fig. 8 – Comparison of propagation results using evidence theory and probability theory

In order to further clarify the influence of epistemic uncertainty, the quantitative results of damage index in subfigure (d) and (f) of Fig. 8 are reported in Table 5. As shown in Table 5, the belief interval of moderate damage state in steps 280 and 412 are [0.11, 0.447] and [0, 0.026], respectively. This means the exceeding probability of moderate damage state are [0.553, 0.89] and [0.974, 1] in steps 280 and 412, respectively. Table 5 also displays the cumulative distribution value for moderate damage state for probability-theory-based quantification results. Using the first probability strategy CDF1, the cumulative distribution for moderate damage state are 0.217 and 0 corresponding to steps 280 and step 412. This means the exceeding probability of moderate damage state are 0.783 and 1 in steps 280 and 412, respectively. Analogously, the cumulative distribution value of CDF2 for moderate damage state are 0.298 and 0 in steps 280 and 412, respectively. It is worth noting the divergence of the cumulative distribution values of CDF1 and CDF2 in step 280. Furthermore, the divergence of two kinds probability-based quantification results provides the evidence that probability theory is not able to handle the epistemic uncertainty. Comparing the quantification results of collapse damage state, the similar conclusion can be obtained. Especially, the cumulative distribution value for collapse damage state in step 412, the evidence result is [0.094, 0.447], this means the value of damage index larger than 1 is located in the interval [0.453, 0.906]. While the cumulative probability of CDF1 and CDF2 are 0.178 and 0.233 and this illustrates the exceedance probability of collapse state is 0.822 for CDF1 and 0.767 for CDF2. From the view of risk assessment, the evidence theory will give decision maker a more robust UQ results, but the probability can't.

Table 5 – The cumulative distribution value of Park-Ang constants in step 280 and 410

damage index	**cumulative distribution curve in step 280**				**damage index**	**cumulative distribution curve in step412**			
	CPF	CDF1	CDF2	CBF		CPF	CDF1	CDF2	CBF
0.25	0.026	0	0	0	0.25	0	0	0	0
0.5	0.447	0.217	0.298	0.11	0.5	0.026	0	0	0
0.75	1	0.522	0.644	0.354	0.75	0.244	0.050	0.053	0.026
1	1	0.722	0.818	0.419	1	0.447	0.178	0.233	0.094

With the incomplete knowledge of prediction model under the various operation conditions, different expert evidence conflicts are inevitable. To reconcile this task challenge, evidence combination rules is proposed to combine the evidences from multi-source. Herein, the Dempster's rule is applied to aggregate the different source of evidence for β, δ_u and F_y as Table 6.

Table 6 – The BBA structure for β, δ_u and F_y

β		δ_u		F_y	
Range	BBA	Range	BBA	Range	BBA
[0.035, 0.067]	0.297	[0.044, 0.104]	0.568	[77.40, 133.19]	0.121
[0.067, 0.087]	0.351	[0.104, 0.115]	0.189	[105.22, 133.19]	0.422
[0.087, 0.108]	0.127	[0.115, 0.116]	0.102	[133.19, 161.01]	0.26
[0.087, 0.139]	0.085	[0.116, 0.133]	0.055	[161.01, 188.82]	0.139
[0.139, 0.189]	0.108	[0.133, 0.156]	0.058	[161.01, 216.63]	0.029
[0.139, 0.192]	0.021	[0.133, 0.188]	0.028	[161.01, 244.44]	0.017
[0.192, 0.230]	0.011			[161.01, 272.42]	0.012

Using the aggregated BBA structures of these three uncertain parameters, the system uncertain response CPF2 and CBF2 are shown in Fig. 9. To clarify the effectiveness of combination rule, the uncertainty propagation results CPF1 and CBF1 from the model B of β, model C of δ_u and F_y are also listed in Fig. 9.

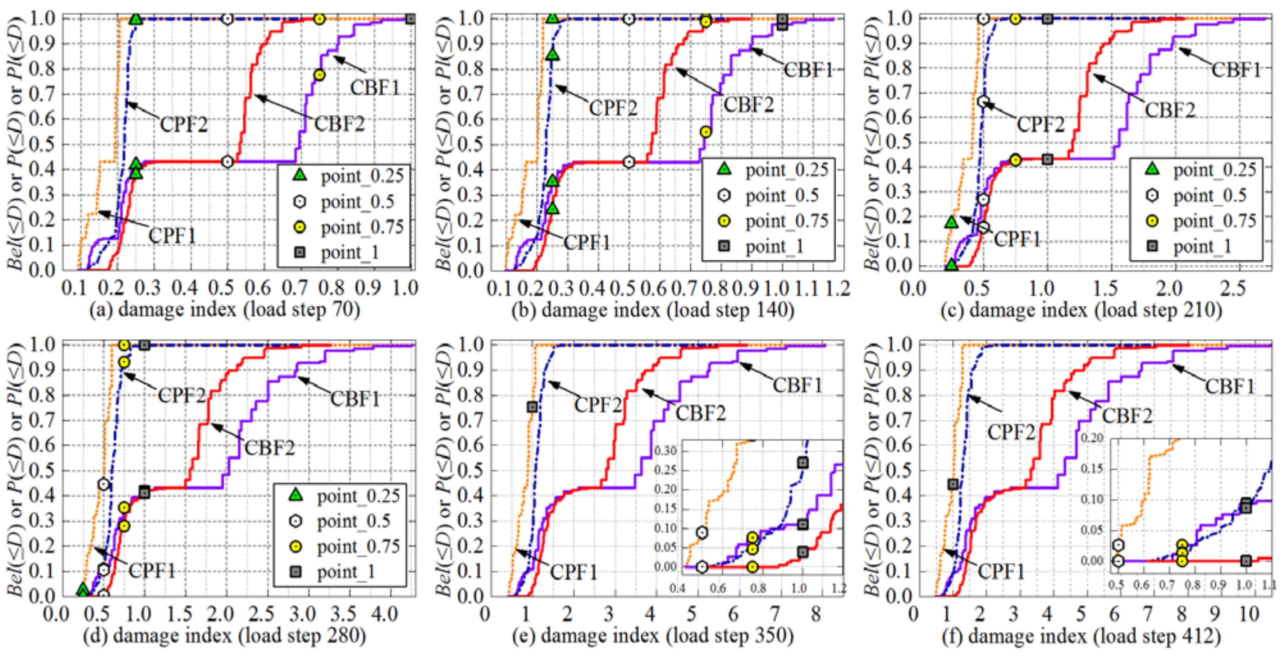


Fig. 9 – Comparison of propagation results with uncombined and combined BBA input

As shown in Fig. 9, the uncertainty quantification results of Park-Ang damage index variates in a large range. The distance of CBF and CPF denotes to the epistemic uncertainty that derived from the limited experimental data and lack of knowledge for complicated composited materials (e.g. parameters model hypothesis, material properties) or incomplete knowledge of empirical model. In comparison with the distance of CPF1 and CBF1 for uncombined BBA, the distance of CPF2 and CBF2 for combined BBA is much narrower, and this can be explained as the high conflict information of multi-sources are discarded by aggregating the multi-sources evidence. However, the aggregation rule is not established in probability theory. From this point of view, the evidence theory has great potential to quantify the uncertainty from multi-sources which are great existence in civil engineering.

5. Conclusion

Uncertainty quantification of seismic damage model is important for performance based seismic design and performance based seismic assessment. In this paper, the epistemic uncertainty of the constants of Park-Ang model is taken into account. The Park-Ang damage model constants are calibrated with column set, selected from PEER column performance database, to construct the uncertainty source. To effectively represent the uncertainty inherent in Park-Ang model constants with limited experimental data, the uncertainty quantification measurement that combines evidence theory and differential evolution is presented. In order to further investigate the feasibility and effectiveness of presented uncertainty quantification measure, the Monte-Carlo sampling method combined with classical probability distribution, which is fitted with given data, are used. Comparing the propagation results of evidence theory and classical probability theory, we can conclude that the evidence theory is flexible to handle the epistemic uncertainty which stem from lack knowledge or sparse experimental data, while the classical probability theory may be limited by the selection of distribution type and the determination of value for distribution parameters. Using the aggregation rule of evidence theory demonstrate that evidence theory is very capable to handle the uncertainty from multi-sources.

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