

# $V_{S30}$ - $\kappa_0$ RELATIONSHIP IMPLIED BY GROUND MOTION MODELS?

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#### Abstract

For deep soil profiles, measurement of the in-situ dynamic soil properties particularly damping is not practical. While the shear-wave velocity can be obtained from surface measurement or other existing deep geophysical measurements, the soil or rock material damping is often assumed for the deep layers. Seismologists have characterized the high-frequency attenuation of very low intensity recorded ground motions using the site attenuation parameter ( $\kappa_0$ ) as proposed by Anderson and Hough (1984). For 1D wave propagation, the  $\kappa_0$  is related to characteristics of the site profile, specifically the wave travel time (i.e., shear-wave velocity) and the small-strain damping, and can be used to constrain the damping in the site profile. By constraining the damping with  $\kappa_0$ , the uncertainty is reduced and the confidence in the results is increased. However, developing a site-specific estimate of  $\kappa_0$  is not feasible for sites without recorded broadband ground motions. Instead,  $\kappa_0$  may be estimated using published  $V_{s30}$ - $\kappa_0$  relationships. This paper examines the  $V_{s30}$ - $\kappa_0$  relationships implicit in several modern ground motion models (GMMs). The comparison provides insight how studies targeted on assessment of the  $V_{s30}$ - $\kappa_0$  relationship compare with development GMMs that do not explicitly consider the  $V_{s30}$ - $\kappa_0$  relationship. The results show that GMMs generally consistent with published  $V_{s30}$ - $\kappa_0$  scaling relationships. Additional comparisons indicate that nonlinearity is not influential to the calculated  $\kappa$  except for  $V_{s30}$  of 200 m/s the lowest  $V_{s30}$  value considered in this study.

Keywords: site amplification; site attenuation; site kappa; ground motion



# **1** Introduction

For deep soil profiles, measurement of the in-situ dynamic soil properties particularly damping is not practical. While the shear-wave velocity can be obtained from surface measurement or other existing deep geophysical measurements, the soil or rock material damping is often assumed for the deep layers. Seismologists have characterized the high-frequency attenuation of very low intensity recorded ground motions using the site attenuation parameter ( $\kappa_0$ ) as proposed by Anderson and Hough (1984) [1]. The subscript zero is indicative of zero distance.

For 1D wave propagation, the  $\kappa_0$  is related to characteristics of the site profile, specifically the wave travel time (i.e., shear-wave velocity) and the small-strain damping, and can be used to constrain the damping in the site profile. By constraining the damping with  $\kappa_0$ , the uncertainty is reduced and the confidence in the results is increased. However, developing a site-specific estimate of  $\kappa_0$  is not feasible for sites without recorded broadband ground motions. Instead,  $\kappa_0$  may be estimated using published  $V_{s30}$ - $\kappa_0$  relationships.

The functional form of a site amplification term of a GMM is developed using analytical site response analysis, empirical data, or a combination of the empirical and analytical sources. The change in the site amplification with  $V_{s30}$  implies a  $V_{s30}$ - $\kappa_0$  trend. This paper examines the  $V_{s30}$ - $\kappa_0$  relationships implicit in several recent ground motion models (GMMs) with published relationships that explicitly consider  $V_{s30}$ - $\kappa_0$  scaling.

# 2 Published V<sub>s30</sub>-κ<sub>0</sub> Relationships

A number of published studies have documented the relationship between  $V_{s30}$  and  $\kappa_0$ . The models considered in this study are:

- Silva et al. (1999) [2] abbreviated herein as Sea99,
- Chandler, Lam, and Tsang (2006) [3] with modification by Campbell et al. (2013) [4] abbreviated herein as CLT06,
- Van Houtte, Douet, and Cotton (2011) [5] abbreviated herein as VDC11, and
- Edwards (2012) [6] abbreviated herein as E12.

These models use data from different regions and use different methods for quantifying  $\kappa_0$ . The empirical models are compared in Fig. 1. The general trend of these models suggests that site attenuation ( $\kappa_0$ ) decreases with increasing site velocity ( $V_{s30}$ ). The E12 [6] model is based solely on the Swiss data and considers three relationships: (a) linear  $V_{s30}$  and logarithmic  $\kappa_0$ , (b) logarithmic  $V_{s30}$  and logarithmic  $\kappa_0$ , and (c) linear  $V_{s30}$  and linear  $\kappa_0$ . All of three of the E12 models show low sensitivity of  $\kappa_0$  to  $V_{s30}$ .



Fig. 1 – Relationship between  $V_{s30}$  and the site attenuation parameter ( $\kappa_0$ ) suggested by published models.



# **3** Considered Ground Motion Models

This study considers the following ground motion models (GMMs):

- Abrahamson, Silva, and Kamai (2014) [7] abbreviated herein as ASK14,
- Akkar, Sandıkkaya, and Bommer, (2014) [8] abbreviated herein as ASB14,
- Boore, Stewart, Seyhan, and Atkinson (2014) [9] abbreviated herein as BSSA14,
- Campbell and Bozorgnia (2014) [10] abbreviated herein as CB14,
- Chiou and Youngs (2014) [11] abbreviated herein as CY14,
- Derras, Bard, and Cotton (2014) [12] abbreviated herein as DBC13,
- Hermkes, Kuehn, Riggelsen (2014) [13] abbreviated herein as HKR14, and
- Idriss (2014) [14] abbreviated herein as I14.

Development of a traditional GMM (e.g., ASK14, ASB14, BSSA14, CB14, and CY14) relies on a functional form that is based on supplemental studies or expected behavior, and some of the model parameters may be fixed to restrict/limit the behavior of the GMM. Empirical data is then used to calibrate the remaining free model parameters. Non-parametric models (e.g., DBC14 and HKR14) are based solely on empirical data. These models were selected because they include a range of approaches for inclusion the site response term.

The following is a brief summary of the consideration of site terms in these models:

- ASK14 used 1D site response analyses documented in [15] to constrain nonlinear and log-period dependence of the site amplification,
- ASB14 used the nonlinear site amplification proposed by [16],
- BSSA14 used semi-empirical site term from [17] with empirical data used for both linear and nonlinear terms,
- CY14 used an empirical site term with a constrained functional form,
- I14 linear site term using NGA-West2 [18] dataset,
- DBC14 and HKR14 used non-parametric regression and did not specify a functional form.

The ground motion models used in this study have been made available via the Python Library pyGMM, which is publicly available here: <u>https://github.com/arkottke/pygmm</u>.

The influence of  $V_{s30}$  on the spectral shape of the considered GMMs is illustrated by computing the response spectra of a vertical strike-slip M 6 earthquake at a distance of 20 km and model default top of rupture. The acceleration response spectra are then normalized by the spectral acceleration at a period of 0.01 sec, shown in Fig. 2. As the  $V_{s30}$  decreases, the peak in the spectrum generally shifts to the longer period and increases in amplitude. While this behavior is generally observed in all considered ground motion models the specific characteristics of the change in shape vary between models.



Fig. 2 – Influence of  $V_{s30}$  on the spectral shape of the considered ground motions.

#### 4 Calculation of the Site Attenuation Parameter (κ)

The shift in the spectral shape observed in the previous section is related to the site attenuation parameter  $\kappa_0$ . In order to quantify  $\kappa_0$ , the Fourier amplitude spectrum (FAS) associated with a response spectrum must be computed. Al Atik et al. (2014) [19] introduced a procedure for computing  $\kappa_0$  from GMMs using inverse random vibration theory (RVT). In this procedure, an FAS that is compatible with a target response spectrum is computed using the methodology proposed by Vanmarcke (1976) [20]. Next, frequency range over which the high-frequency decay is linear in log-linear space is selected. In this study, the selected frequency range is done visually for each considered scenario and was typically selected in the frequency range of 5 of 20 Hz. The slope over the selected frequency range is related to  $\kappa$  by [1]:

$$a(f) = A_0 \cdot \exp(\pi \kappa f) \tag{1}$$

This process is illustrated for ASK14 GMM at with  $V_{s30}$  value of 200, 400, and 800 m/s. The  $\kappa$  value computed using this manner does not reflect the zero distance of the original definition by Anderson and Hough (1984) [1]. Instead, Al Atik et al. (2014) [19] denote the average of 5, 10, and 20 km as  $\kappa_1$  and treat it as a proxy for  $\kappa_0$  due to the limited influence of distance attenuation at such short distances. The results in the subsequent sections use no subscript for  $\kappa$ , because the three considered distances (5, 10, and 20 km) are considered independently.

The calculation of the compatible FAS was performed with the Python Library pyRVT, which is publicly available here: <u>https://github.com/arkottke/pyrvt</u>.



Fig. 3 – Process of computing  $\kappa_0$  from a response spectrum computed from a GMM: (Left) calculate response spectra from a GMM, (Center) compute compatible FAS, (Right) visually select linear frequency range and calculate the slope and associated  $\kappa$ .

RVT uses a peak factor formulation to relate the Fourier spectral shape to the time domain peak value. The sensitivity to the calculated  $\kappa$  is evaluated using three alternative peak factor formulations: Cartwright and Longuet-Higgins (1956) [21] abbreviated herein as CLH56, Vanmarcke (1976) [20] abbreviated herein as V75, and Toro and McGuire (1987) [22] abbreviated herein as TM87. The calculated compatible FAS and selected  $\kappa$  values for the three peak factor formulations are shown in Fig. 4. The relative difference in the calculated  $\kappa$  values is relatively minor. The V75 peak factor is used for subsequent calculations.



Fig. 4 – Influence of peak factor formulation on the  $\kappa$  computed for a compatible FAS.

#### 5 Discussion

Using the ground motion models defined in Section 3, the response spectra are computed for vertical strike-slip faults with the following parameters:

- magnitudes of 5, 6, and 7;
- distances of 5, 10, and 20 km; and
- V<sub>s30</sub> values of 200, 400, 600, 800, 1000 and 1,500 m/s.



For simplicity, the effects of directivity and hanging-wall effects are ignored. If the  $V_{s30}$  is greater than the maximum  $V_{s30}$  of the model, then no result is computed. For each response spectrum, the compatible FAS is computed, and the linear frequency range is visually selected and the  $\kappa$  is computed.

The calculated  $\kappa$  values are compared with the range in published  $V_{s30}$ - $\kappa_0$  models (shaded region in Fig. 5) defined in Section 2. Most of the GMMs show similar trends with increasing  $\kappa$  with decreasing  $V_{s30}$ . At  $V_{s30}$  greater than 800 m/s, many of the GMMs predict  $\kappa$  values that are higher than the published range. The cause of this bias is uncertain. One potential source of bias is that more data from soil sites than rock sites are contained within the GMM datasets. Another potential difference is that considered  $V_{s30}$ - $\kappa_0$  models include data from active and stable tectonic regions, which have been observed to have different attenuation characteristics.



Fig. 5 – Comparison of  $V_{s30}$ - $\kappa$  scaling implied by GMMs compared to published implied  $\kappa$  models (shaded region).

The ASK14, BSSA14, CB14, CY14, and I14 models are all developed using the NGA-West2 [18] ground motion database, but show different  $V_{s30}$ - $\kappa$  trends due to the different approaches for developing the site terms. The BSSA14 model shows the flattest  $V_{s30}$ - $\kappa$  scaling with an average increase of about 20 ms from 1500 m/s to 200 m/s. The ASK14 model shows the steepest  $V_{s30}$ - $\kappa$  scaling with an average increase 60 ms over the same range. The influence of this behavior on the spectral shape can be observed in Fig. 2. Decreasing the  $V_{s30}$  in the ASK14 model results in both a shift and increase in the peak of the normalized response spectrum. While decreasing the  $V_{s30}$  of the BSSA14 model only results in a shift of the peak.

The DBC13 and HKR14 are nonparametric models without specific functional forms. The DBC13 model shows a decrease in  $\kappa$  with decreasing  $V_{s30}$ , which is the reverse of what is observed in  $V_{s30}$ - $\kappa_0$  models. This behavior may be a result of the irregular spectra shape (see Fig. 2) and the dip at moderately low periods. The HKR14 model only provides spectral acceleration at 5 distinct spectral periods. Nevertheless, the HKR14 shows good agreement with the published  $V_{s30}$ - $\kappa_0$  models, except for the dip between 600 and 800 m/s. The  $\kappa$  of the HKR14 at 1000 m/s provides the best agreement with the published range.



The Zandieh, Campbell, and Pezeshk (2016) [23] study computed the  $\kappa_0$  for four of the NGA-West2 [18] GMMs at  $V_{s30}$  of 760 m/s and magnitudes ranging from 3.5 to 8.0. The study found that computed  $\kappa_0$  values ranged from 20 to 60 ms, which agree well with the values computed by this study. One interesting finding of the study was the strong magnitude dependence. For example, the  $\kappa_0$  of the BSSA14 model increases by 20 ms as the magnitude changes from 3.5 to 8.0. One potential explanation for this increase  $\kappa_0$  with increasing magnitude is soil nonlinearity.

Soil nonlinearity may influence  $\kappa$  by an increase in damping of the soil with increasing earthquake intensity. The influence on soil nonlinearity on the implied  $\kappa$  is evaluated by comparing  $\kappa$  with predicted peak ground acceleration (PGA). The PGA- $\kappa$  scaling for the ASK14, BSSA14, CB14, and CY14 models is shown in Fig. 6. At V<sub>s30</sub> of 400 m/s and greater, the calculated  $\kappa$  show no increase with increasing PGA; indicating that the nonlinearity is not significant for  $\kappa$  above at V<sub>s30</sub>  $\geq$  400 m/s. In fact, the CB14 shows a decrease in  $\kappa$  with increasing PGA. The results for a V<sub>s30</sub> of 200 m/s show somewhat different trends with the ASK14, BSSA14, and CY14 models showing an increase in  $\kappa$  with increasing PGA. The ASK14 shows the greatest increase in  $\kappa$  with increasing PGA with an increase of over 40 ms. These results indicate potential influence of nonlinearity on  $\kappa$ , however it is not clear if these trends are a result of site terms within the GMMs or trends within the underlying data.



Fig. 6 - Comparison of PGA-ĸ scaling implied by GMMs.

### 6 Conclusion

In this study, the implied site attenuation parameter ( $\kappa$ ) was calculated for several GMMs. The V<sub>s30</sub>- $\kappa$  scaling implicitly included in these GMMs were compared with published V<sub>s30</sub>- $\kappa_0$  relationships that explicitly considered  $\kappa_0$ . The results indicate that explicit and implicit consideration of  $\kappa$  provides generally consistent V<sub>s30</sub>- $\kappa_0$  scaling. However, most models have greater than the expected  $\kappa$  at 1000 m/s, which needs to be investigated further to identify the cause. The V<sub>s30</sub>- $\kappa_0$  scaling is dependent on the modelling approach and dataset. The NGA-West2 [18] GMMs provide different V<sub>s30</sub>- $\kappa_0$  scaling for the same ground motion database.



The nonlinearity of the implied  $\kappa$  was also investigated. The results show at  $V_{s30} \ge 400$  m/s there is no increase in  $\kappa$  with increasing PGA. At  $V_{s30}$  of 200 m/s (very soft and the lowest  $V_{s30}$  considered in this study), three of the four considered models showed an increase in  $\kappa$  with increasing intensity.

### 7 Acknowledgements

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