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SECOND-ORDER EFFECTS OF SLENDERNESS ON INELASTIC RESPONSE OF RC BRIDGE COLUMNS: A PARAMETRIC STUDY

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Abstract

Inelastic response of columns is governed by geometrical second-order effects in proportion to their slenderness. Experiments on slender reinforced concrete (RC) columns have demonstrated that second-order effects, namely, nonlinear moment gradient and P-δ moments increase the inelastic demands along the column beyond the limits predicted by current expressions for L_{pr}. In this paper, geometrical nonlinear beam theory in conjunction with a bilinear inelastic moment-curvature section response was utilized to determine: (1) the magnitude by which the inelastic response of RC columns is affected due to their slenderness, (2) a slenderness limit for RC bridge columns defined in terms of design variables, and (3) the influence of different structural, geometrical and material parameters on the vulnerability of slender columns to second-order actions. To investigate the slenderness effects, a parametric study was conducted on all the possible design configurations that are permitted by the latest seismic design guidelines for bridge columns. It was found that the susceptibility of RC bridge columns to second-order effects were mainly dominated by three parameters, namely, the longitudinal steel reinforcement, axial load, and aspect ratios. This study led to the identification of accurate slenderness limits for the noted parameters beyond which the second-order effects on the inelastic response of RC bridge columns are significant and can no longer be ignored. Recommendations are made based on the presented findings to update the current design guidelines for consideration of second-order effects. Finally, simple expressions are proposed to predict the effects of second-order P-δ moments on the length of the plastic region (L_{pr}).

Keywords: reinforced concrete; slender bridge columns; second-order effects; P-δ moments; parametric study

1. Introduction

Reinforced concrete (RC) bridge columns are generally designed to withstand earthquake events by dissipating seismic energy through inelastic deformations [1]. To ensure a ductile inelastic response without a significant loss of strength, special detailing is required along the sections of RC columns that are expected to experience significant inelastic deformations [2]. These critical regions of the column are commonly known as the plastic region. Recommendations for the length of the critical plastic region (L_{pr}), which is different from the length of the plastic hinge (L_p), are provided by most seismic design specifications [3-5]. Recent research has demonstrated the effects of slenderness on the inelastic response of RC columns [6] and on the extent of the plastic region [7]. The noted slenderness effects were also observed in tests conducted by the authors of this paper on four slender RC bridge columns with aspect ratios (shear span length to width ratio, L/D) up to 12 [8]. Despite overwhelming evidence for the slenderness effects on the extent of the plastic region (L_{pr}), current design guidelines lack provisions for the second-order effects on L_{pr}. Instead, the plastic region is typically estimated regardless of the column's slenderness.

An analytical closed-form solution to quantify the effects of second-order moments on L_{pr} using nonlinear beam theory was recently proposed by the authors [9]. The solution has been proved to be capable of accurately estimating the extent of the plastic region due to nonlinear moment gradients. Details of the derivation and verification of the analytical solution are out of the scope of this paper. Despite the robustness of the proposed analytical model, it requires a precise estimate of parameters that are not typically calculated and are less known to design engineers. Thus, a full implementation of the nonlinear model in seismic design of RC columns needs expressions in terms of well-known design parameters. In addition, the effect of common structural/material parameters, such as aspect ratio, axial load ratio, steel reinforcement ratio, and concrete strength, on the susceptibility of RC columns to second-order effects remains unclear. In this paper, a parametric study of the second-order effects on the plastic region along slender RC columns is presented. This study led to the identification of design parameters that significantly affect the second-order response of RC columns. Furthermore, the parameter ranges for which second-order effects on L_{pr} can be safely ignored are identified. Finally, in lieu of more sophisticated analyses, simple design formulas are proposed to estimate the effects of second-order moments on the plastic region of RC columns.

2. Background

2.1 Extent of the plastic Region (L_{pr})

The spread of plasticity along RC columns is typically assessed by using the distribution of curvatures along their height [9, 10]. The plastic region length, L_{pr}, is obtained by determining the length of the region over which the curvature values exceed the yield curvature (ϕ_y). Alternatively, bending moment profiles can be used to obtain L_{pr} by evaluating the length of the region along which the moment demands exceed the yield moment (M_y). Thus, a necessary step is to define the RC section's yield moment and curvature.

2.1.1 Yield Moment (M_y) and Yield Curvature (ϕ_y)

The yield curvature, ϕ_y, was calculated according to the method by Priestley et al. [1] using $\phi_y = \phi'_y \cdot M_n / M'_y$ where, M'_y and ϕ'_y respectively are the moment and curvature values associated with the first yield in the extreme tensile rebar and M_n is the moment at which either the extreme concrete fiber in compression reaches $\varepsilon_c = -0.004$ or the extreme steel fiber in tension reaches $\varepsilon_s = 0.015$, whichever occurs first. The yield moment, M_y, was defined as the moment that corresponds to the point on the moment-curvature (M-ϕ) response of the RC section at the yield curvature (ϕ_y), as schematically illustrated in Fig. 1(a).

2.1.2 Current Design Guidelines for L_{pr}

Most modern design specification codes provide guidelines for estimating the length of the critical plastic region on RC columns to provide special detailing and enhanced confinement that ensure stable ductile inelastic response. Some codes, such as the ACI code [3], define L_{pr} as a constant factor (one-sixth) of the column length regardless of its slenderness and moment demands. The AASHTO seismic design guidelines [4] define L_{pr} for RC columns as the length of the region where the moment demand exceeds 75% of the maximum plastic moment. Therefore, it is essential to have an accurate account of the yield to ultimate moment ratio (M_y/M_u) to correctly estimate the extent of the plastic region using profiles of moment demand along the columns' height.

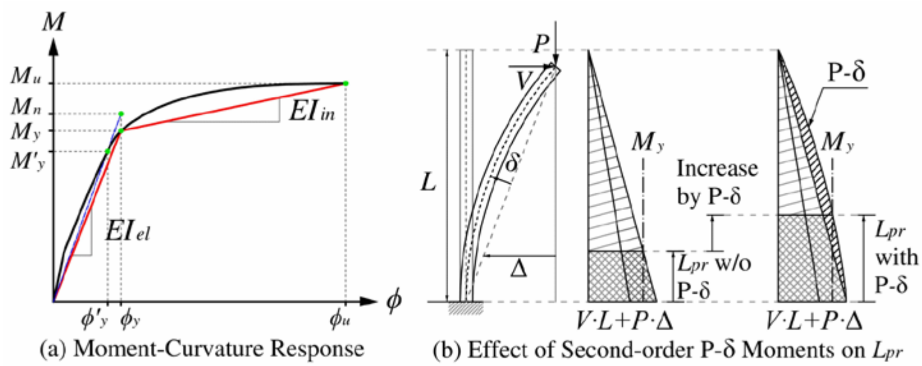

Fig. 1 – Schematic illustration of: (a) moment-curvature response for a typical RC section; (b) effect of nonlinear second-order P-δ moments on the extent of the plastic region (L_{pr})

2.1.3 Ultimate Moment (M_u) and Ultimate Curvature (ϕ_u)

The ultimate moment and curvature (M_u, ϕ_u) were defined as the point at which the strain in the extreme concrete fiber or steel rebar exceeded the ultimate crushing or fracture limits, respectively, whichever happens first. The ultimate crushing limit for confined concrete was defined according to the energy balance method [12]. The ultimate strain limit for the reinforcing steel was obtained considering low-cycle fatigue as proposed in the literature [13].

2.2 Second-order P-δ effects on L_{pr}

Slender ductile RC columns exhibit significant flexural flexibility and substantial bending deformations under extreme loading conditions. These excessive member deformations away from the column chord-line (δ) in conjunction with the applied axial load (P) generate additional second-order moments, commonly referred to as P-δ effects, which contribute to the extent of the plastic region as schematically depicted in Fig. 1(b). Therefore, nonlinear moment gradient with P-δ moments included leads to a larger plastic region along the column height, than that from a linear gradient, in proportion to the column's slenderness.

2.3 Closed-form Nonlinear Solution to L_{pr}

Previous research by the authors [9] has offered a closed-form solution to the extent of the plastic region (L_{pr}) due to nonlinear moment gradients. Nonlinear beam theory was applied to the inelastic response of RC columns to derive an expression for L_{pr} that includes second-order effects. The exact solution is provided in Eq. (1) for the ultimate state in which the end moment reached the ultimate moment (M_u). The expression in Eq. (1) is valid for predicting the spread of plasticity due to moment gradient. Therefore, it only applies to RC columns with positive post-yield flexural stiffness (EI_{in}), as defined by $EI_{in} = (M_u - M_y)/(\phi_u - \phi_y)$. Details of the closed-form solution are out of the scope of this paper and are provided in a separate communication [9].

$$L_{pr} = L - \frac{L}{\kappa_{el}} \sin^{-1}\left[\frac{M_y \sin(\kappa_{el})}{M_u}\right] \tag{1}$$

In Eq. (1) κ_{el} is the slenderness parameter, as defined by Eq. (2), which represents the susceptibility of RC columns to second-order effects. In Eq. (2) EI_{el} is the flexural stiffness of the cracked RC section before yielding of the longitudinal reinforcement as defined by $EI_{el} = M_y/\phi_y$.

$$\kappa_{el} = L\sqrt{\frac{P}{EI_{el}}} \tag{2}$$

By simplifying Eq. (1) the effects of second-order P-δ moments on the spread of the plastic region beyond that resulting from a linear gradient is approximated by Eq. (3) in which $L_{pr,NL}$ and $L_{pr,L}$ are the length of the plastic region obtained from nonlinear and linear moment gradients, respectively.

$$\frac{L_{pr,NL} - L_{pr,L}}{L_{pr,L}} = \frac{\kappa_{el}^2}{3} \tag{3}$$

2.4 Key Variables of the Nonlinear Solution

Expressions for the extent of the plastic region due to nonlinear moment gradient and the effect of second-order P-δ moments on L_{pr}, as respectively given by Eq. (1) and Eq. (3), are in terms of two key variables: (a) moment overstrength ratio (M_u/M_y) and (b) columns slenderness factor (κ_{el}). The accuracy of the analytical results from the nonlinear model presented above directly depends on the precision to which these key variables are estimated. It is thus essential to evaluate the relationship between typical structural design parameters and the key variables of the nonlinear analytical solution, i.e. M_u/M_y and κ_{el}.

2.4.1 Slope of Post-yield M-ϕ Branch (EI_{in})

The closed-form solution presented in this paper predicts the extent of the plastic region due to nonlinear moment gradient. Therefore, it is only applicable to RC sections with positive post-yield flexural stiffness (EI_{in} > 0). Moreover, depending on the post-yield response of RC sections, the extent of the plastic region is governed by different phenomena. For instance, moment gradient significantly contributes to the spread of plasticity in slender RC columns with positive post-yield flexural stiffness ($M_u > M_y$). Whereas, the effect of moment gradient on L_{pr} in columns with a softening branch after yield is negligible. Therefore, knowledge about the sign of the post-yield flexural stiffness (positive for hardening and negative for softening response) is essential for proper implementation of the nonlinear solution to the second-order effects on L_{pr}, as expressed by Eq. (3).

2.4.2 Moment Overstrength Ratio (M_u / M_y)

The ratio of ultimate moment (M_u) to the yield moment (M_y), also known as moment overstrength factor, is another key variable in determining the extent of the plastic region in RC columns according to the nonlinear solution presented in Eq. (3). Thus, it is crucial to have an accurate estimate of the moment overstrength factor to implement second-order effects on the inelastic response of slender RC columns, in lieu of sophisticated section analyses. This becomes even more important considering the fact that most specification codes define L_{pr} in terms of M_y/M_u ratio, which is the inverse of the moment overstrength factor. If the M_u/M_y ratio is underestimated, the actual L_{pr} can exceed the length required by design guidelines to have special detailing for ductile inelastic response. That is, the plastic demands can potentially spread beyond the ductile zone and cause brittle failures.

2.4.3 Slenderness Parameter (κ_{el})

The susceptibility of RC columns to second-order moments on their inelastic response can be fully captured by the slenderness parameters (κ_{el}) as defined by Eq. (2). Unlike the aspect ratio (L/D), which describes the slenderness of RC columns from a geometrical point of view, the slenderness parameter (κ_{el}) is capable of addressing the structural response of slender columns. The reason is that κ_{el} includes, in addition to the aspect ratio, the effects of parameters such as axial load (P) and the compressive strength of concrete (f_c'), and the area of the longitudinal steel reinforcement (A_{sl}). The combination of the noted parameters influences the degree to which second-order moments affect the inelastic response of RC columns. Yet, the contribution of these structural parameters on the susceptibility of RC columns to second-order effects has not been evaluated.

3. Methods

3.1 Structural Parameters

In this paper, 9 normalized dimensionless parameters that are sufficient to describe critical structural and material properties of circular RC bridge columns were considered. Section diameter (D), steel reinforcement ratios (ρ_{sl} , ρ_{st}), diameter of rebars (d_{sl} , d_{st}), spiral hoop spacing (s), concrete compressive strength (f_c'), steel yield and ultimate stress values (f_{ysl} , f_{usl}), axial load ratio ($P/A_g\ f_c'$) and the shear span length (L) were the considered geometric and material parameters. The subscripts of (l) and (t) denote longitudinal and transverse steel reinforcement layouts. To normalize the geometrical parameters with length dimensions they were divided by the section diameter (D), which ranged from 305 mm to 1219 mm. Similarly, material properties with units of stress were normalized by the yield stress for the longitudinal steel reinforcement (f_{ysl}), which was set to 455 MPa. The normalized parameters used in this study are listed in Table 1. Also presented in Table 1 is the range of values for the noted parameters. Design specifications for reinforced concrete sections according to ACI-318 [3] along with seismic design guidelines from AASHTO [4] and Caltrans [5] were utilized to establish the limiting bounds for the structural parameters. Typical values of material properties for current practice were used to define the limits for material parameters. The average values for the parameters of this study were extracted from the PEER structural performance database [14] for 52 circular RC columns that met the conditions for ductile flexural inelastic response. These average values, which are presented in Table 1, are generally consistent with the mean value of the ranges except for the volumetric transverse reinforcement ratio (ρ_{st}) and concrete compressive to steel yield strength (f_c' / f_{ysl}). The reason is that current seismic design guidelines generally require higher ratios of transverse reinforcement to provide confinement than those in effect decades ago when most of the columns reported in the PEER database were designed and tested. Moreover, recent advancements in concrete mixture design and technology have led to higher strength values than the mixes used in most columns reported in PEER database.

Table 1 – Dimensionless parameters of the study

Dimensionless Parameter	ρ_{sl}	d_{sl} / D	ρ_{st}	d_{st} / D	s / D	f_c' / f_{ysl}	f_{ysl} / f_{usl}	$P/A_g\ f_c'$	L / D
Lower bound	1%	0.029	1.2%	0.013	0.042	0.1	1.3	5%	2
Higher bound	4%	0.083	3.9%	0.042	0.23	0.2	1.7	20%	12
PEER Average	2.4%	0.040	1.1%	0.018	0.12	0.1	1.5	15%	5.5

3.2 Parametric Study Method

The effect of the structural and material parameters presented in Table 1 on the second-order response of RC bridge columns was studied. A total of 11,088 cases of possible design configurations that meet the requirements of the aforementioned code specification were generated by taking at least three equally-spaced values from the ranges presented in Table 1 and omitting the combinations that were not allowed. Section analyses were conducted on all of the allowed cases to obtain the key parameters of the nonlinear closed-form solution to the second-order effects of P-δ on the extent of the plastic region (L_{pr}).

3.3 Section Analyses

Moment-curvature (M-ϕ) responses of the study cases were obtained by conducting analyses on fiber-discretized RC sections to capture the combined effect of axial load and bending moment. Different inelastic constitutive models for confined core concrete, unconfined cover concrete, and steel reinforcement were used. A force-based formulation, as per Spacone et al. [15], was utilized to obtain a stable M-ϕ response. The effect of confinement by the transverse reinforcement on the compressive response of the core concrete was considered following the methods by Mander et al. [12]. The uniaxial constitutive relation presented by Priestley et al. [1] for steel rebars was implemented to define the post-yield hardening response. Key parameters of the closed-form solution, namely, the slenderness parameter (κ_{el}) and the moment overstrength ratio (M_u/M_y), were extracted from the M-ϕ response of the RC sections. Pre-yield flexural stiffness for the cracked RC sections was obtained by the ratio of the yield moment (M_y) to the yield curvature (ϕ_y) per $EI_{el} = M_y / \phi_y$ and the slenderness parameter was calculated according to Eq. (2). The moment overstrength ratio was also computed by the ratio of the moment at ultimate state (M_u) to the yield moment (M_y).

3.4 Regression Models

Results of the parametric study were used to identify the most effective parameters in defining the key variables of the nonlinear closed-form solution. Linear regression models were derived to serve as simple design formulas with the least number of parameters required to provide reasonably accurate estimates for the second-order effect on the inelastic response of RC columns. To accomplish this, the ordinary least square estimator method [16] for multi-variable regression problem was utilized. Regression models that provided the highest accuracy with the least number of variables were selected.

3.5. Factor of Safety

Regression models were adjusted using factors of safety to ensure conservative estimates of the key variables. Factors of safety were calculated based on 95% confidence intervals. That is, the regression models were adjusted to provide conservative predictions for at least 95% of the cases. For both key variables, i.e., M_u/M_y and κ_{el}, larger values and overestimates led to more conservative designs. Therefore, the regression models were magnified by a factor of safety to meet the aforementioned condition.

4. Parametric Study Results

4.1 Sign of Post-yield Flexural Stiffness (EI_{in})

A parametric study was conducted to evaluate the slope of post-yield moment-curvature response (EI_{in}). Eight out of nine parameters from Table 1 were used since the aspect ratio (L/D) has no effect on the sign of EI_{in}, which is a property of the RC section. The results are illustrated in Fig. 2, in which the correlation between 8 parameters and the sign of the post-yield flexural stiffness (EI_{in}) is provided. Also provided in Fig. 2 is the equation for a linear regression line obtained by the least square estimator method. The slope of the regression line multiplied by the average value of the parameter simply indicates the significance of the parameter in determining the sign of EI_{in}. Using the adjusted partial correlation coefficients it was found that the sign EI_{in} is mostly dominated by two parameters: (a) axial load ratio ($P/A_g f_c'$), and (b) concrete compressive strength (f_c').

The probability of positive EI_{in} is depicted in Fig. 3(a) as a probabilistic distribution based on different values of the effective parameters. It is evident from Fig. 3(a) that RC sections with lower axial load and lower concrete strength have higher probability of a positive post-yield flexural stiffness. That is, the moment-curvature response of the section has a positive slope after yield and the flexural stiffness of the section has a positive value, i.e., $M_u > M_y$.

Statistical analysis of the results from the parametric study revealed that positive EI_{in} occurred in 69% of all design configurations studied here. This percentage increased to 91% for sections with normal-strength concrete (f_c' = 45 MPa) and reduced to 52% for high-strength concrete (f_c' = 90 MPa). Therefore, the closed-form solution presented in this paper for the second-order effects is applicable to almost all RC columns with normal-strength concrete (NSC). It is also appropriate for RC sections with high-strength concrete (HSC) with axial load ratios of less than 10%. The valid ranges for the noted parameters are better highlighted in Fig. 3(b), in which the shaded area represents the ranges that are most likely to cause negative post-yield stiffness. Also shown in Fig. 3(b) is the distribution of test columns from the PEER database that have axial load ratios less than or equal to 20%. It can be seen that all test columns are located in a region with higher probability for positive EI_{in}. This shows that the closed-form solution in Eq. (1) is applicable to most plausible design configurations.

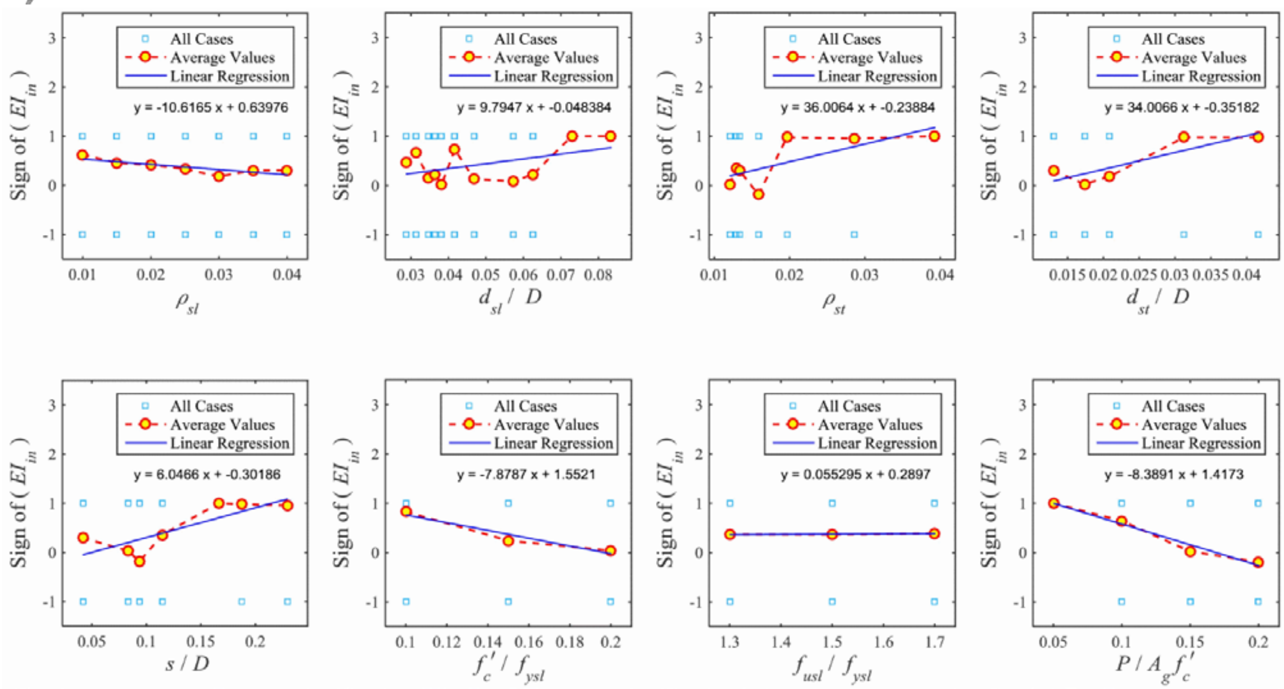

Fig. 2 – Parametric study of the sign of post-yield flexural stiffness (EI_{in})

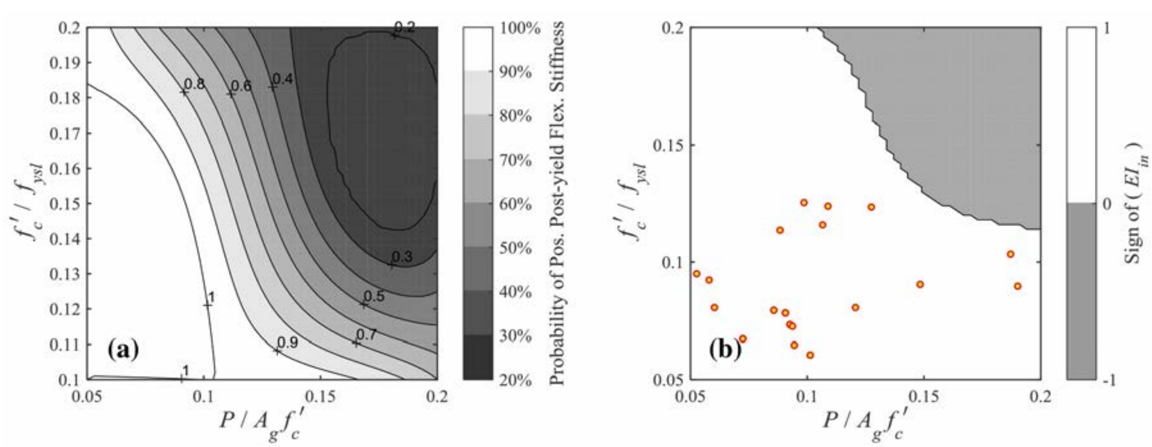

Fig. 3 – (a) Probabilistic distribution of sign of post-yield flexural stiffness (EI_{in}); (b) Distribution of test columns reported in PEER database

4.2 Moment Overstrength Ratio (M_u / M_y)

A parametric study was carried out to identify parameters that significantly affect the ultimate to yield moment ratio (M_u/M_y) of RC sections that meet the precondition of Eq. (1), which is a positive post-yield flexural stiffness ($M_u/M_y > 1$). Similar to the sign of EI_{in}, only 8 parameters were used for this study since aspect ratio does not affect the ultimate to yield moment ratio (M_u/M_y), which is governed only by the properties of RC sections. The correlation between the 8 parameters used and M_u/M_y is depicted in Fig. 4. Equations for the best fitted lines are also provided. The products of the line slopes and the average values of the parameters can be used to identify the most significant parameters. By computing the adjusted partial correlation coefficients it was found that three parameters dominate the moment overstrength ratio (M_u/M_y) of RC sections. These parameters, in order of significance, are: (a) axial load ratio ($P/A_g\ f_c'$), (b) concrete compressive strength (f_c'), and (c) the ultimate to yield stress for the longitudinal steel reinforcement (f_{usl}/f_{ysl}).

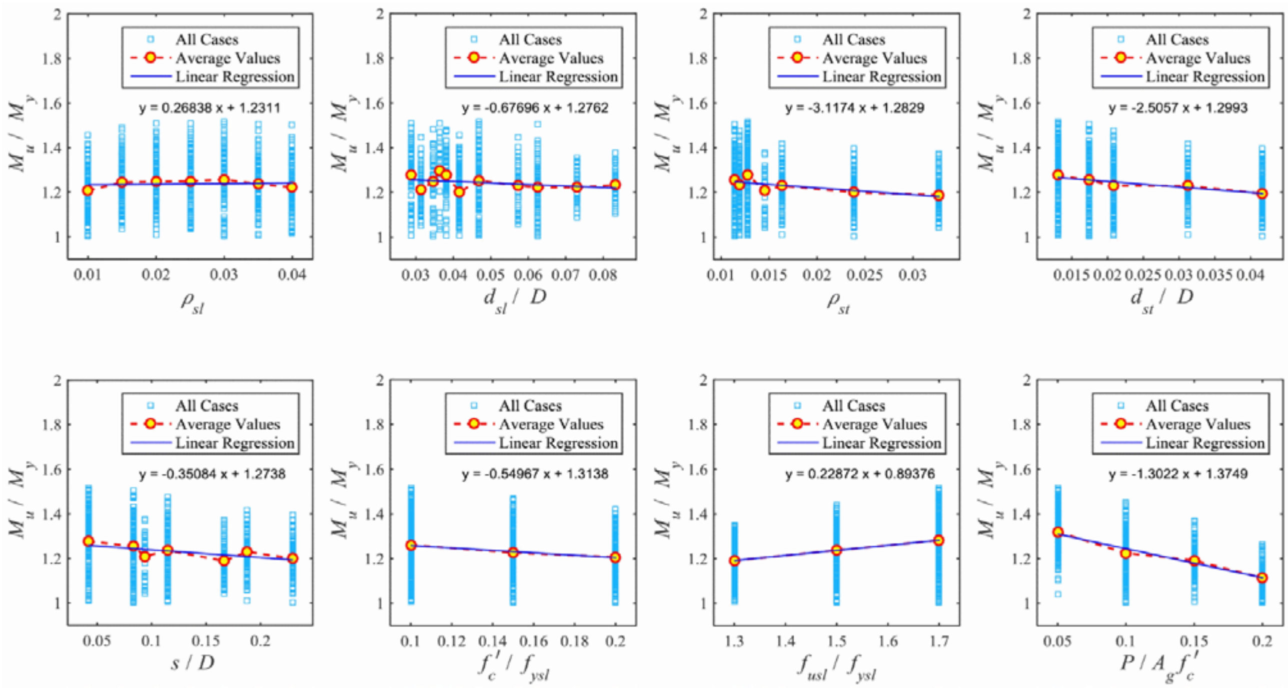

Fig. 4 – Parametric study of the moment overstrength ratio (M_u/M_y)

The distribution of M_y/M_u ratio with respect to two main parameters, i.e., axial load ratio ($P/A_g\ f_c'$) and ultimate to yield stress for longitudinal reinforcement steel (f_{usl}/f_{ysl}), is plotted in Fig. 5(a) and Fig. 5(b) for NSC and HSC sections, respectively. It is evident from the plots that the M_y/M_u ratio is generally higher for greater values of the f_{usl}/f_{ysl} ratio. The direct relationship between M_y/M_u and f_{usl}/f_{ysl} is explained by the fact that the strain-hardening of the steel reinforcement is the main factor that contributes to the moment overstrength in RC sections. However, the axial load ratio has an adverse effect on the M_y/M_u ratio. That is, the M_y/M_u ratio generally decreases with increasing $P/A_g\ f_c'$ up to 20%. The ratio of M_y/M_u is also of importance since most design specification codes describe the extent of the plastic region as the region over which the moment demands exceeds a factor of the maximum end moment. The M_y/M_u ratios recommended by design guidelines, which depend on the axial load ratio, vary from 75% to 80% for columns with axial load ratios less than 25%. The results of the parametric study presented in this paper were used to reevaluate the accuracy and reliability of the M_y/M_u ratios suggested by design guidelines. It was found that the average M_y/M_u ratio for RC section with NSC is 77% and 89% for axial load ratios 10% and 20%, respectively. However, the average ratios may not be appropriate to use in design guidelines since the average values provide a non-conservative overestimate of the M_y/M_u ratio for about half of the design configurations. Therefore, the 95% confidence interval values were calculated as 71% and 81% for axial load ratios 10% and 20%, respectively. Although current design guidelines for the extent of the plastic region successfully estimate the M_y/M_u ratio for columns with axial load ratio of 20%, it is evident from the results of the parametric study that current design specifications lead to non-conservative overestimates for columns with axial load ratio of 10%. The ratio of M_y/M_u was generally higher in RC sections with HSC. The 95% confidence interval for HSC sections was 78% and 86% for axial load ratios 10% and 20%, respectively. Increase in the M_y/M_u ratio due to higher strength of concrete has been confirmed by Pam et al. [11] through experiments conducted on the extent of the plastic region along HSC columns.

4.3 Slenderness Parameter (κ_{el})

Effects of the 9 parameters listed in Table 1 on the slenderness factor (κ_{el}) were also evaluated through a parametric study. The relationship between κ_{el} and 8 parameters from Table 1, excluding the ultimate to yield stress ratio for reinforcement steel (f_{usl}/f_{ysl}) is illustrated in Fig. 6. The relationship between κ_{el} and f_{usl}/f_{ysl} was omitted from the results since no significant correlation was observed. This can be explained by the fact that the slenderness parameter (κ_{el}) is defined using properties of RC section prior to yield, whereas the f_{usl}/f_{ysl} ratio comes into play in the hardening response of RC sections after yield. A strong linear correlation between κ_{el} and aspect ratio (L/D) was observed as depicted in Fig. 6. Therefore, it is more convenient to conduct the parametric study on κ_{el} /(L/D), which is the slope of the best fitted line relating κ_{el} to L/D. The most significant parameters for predicting κ_{el} /(L/D) were identified by using adjusted partial correlation coefficients,. These parameters, in order of significance, are: (a) axial load ratio ($P/A_g\ f_c'$), (b) longitudinal steel reinforcement ratio (ρ_{sl}), and (c) concrete compressive strength (f_c').

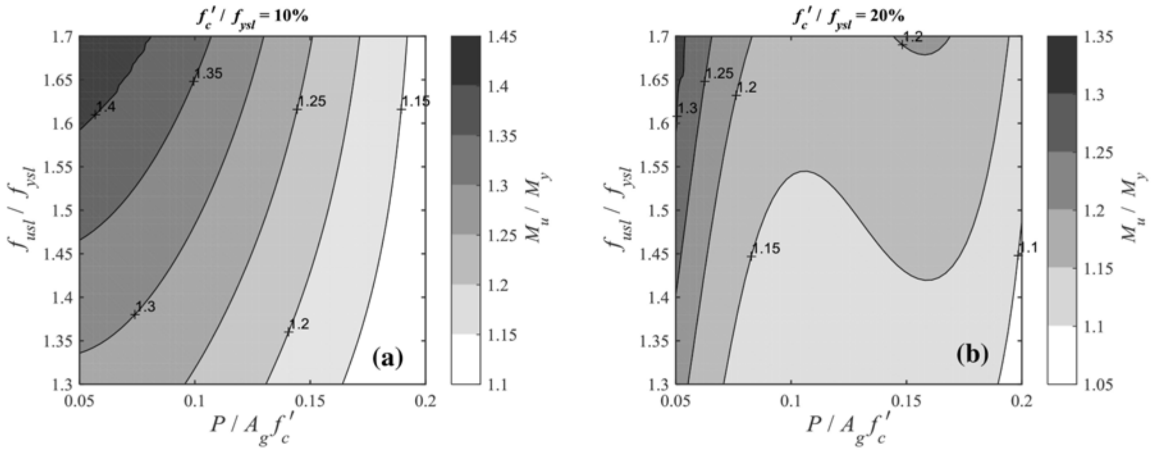

Fig. 5 – Distribution of M_u/M_y ratio with respect to $P/A_g\ f_c'$ and f_{usl}/f_{ysl} for: (a) normal strength concrete (f_c' = 45 MPa); and (b) high-strength concrete (f_c' = 90 MPa)

Distribution of κ_{el} /(L/D) with respect to its two most significant parameters, i.e., $P/A_g\ f_c'$ and ρ_{sl}, is shown in Fig. 7(a), which can be used to estimate the slenderness parameter by multiplying the slope from the graph by the aspect ratio (L/D). It is evident from the plots axial load ratio contributes to the slenderness of RC columns as defined by their susceptibility to second-order effects. By contrast, the longitudinal reinforcement ratio has an adverse effect on κ_{el}, which can be explained by the fact that RC sections with higher longitudinal reinforcement exhibit greater flexural rigidity to bending and P-δ effects. The third significant parameter, concrete strength (f_c'), is not shown in Fig. 7(a) but also contributes to the slenderness of RC columns. That is, columns with higher concrete strength are more vulnerable to second-order effects.

Results from Fig. 7(a) were also used to find limits for the aspect ratio of RC columns beyond which second-order effects of P-δ moments on the extent of the plastic region (L_{pr}) cannot be ignored. This was accomplished by assuming a limit for the second-order effects and combining Eq. (3) with the distribution of the slenderness parameter (κ_{el}) with respect to $P/A_g\ f_c'$ and ρ_{sl}. The aspect ratios beyond which second-order moments increase the extent of the plastic region by more than 10% are shown in Fig. 7(b). Therefore, second-order effects cannot be ignored in RC columns with aspect ratios greater than the values depicted in Fig. 7(b).

4.4 Effect of Second-order P-δ Moments on L_{pr}

The effect of second-order P-δ moment on the extent of the plastic region (L_{pr}) was evaluated using Eq. (3) for combinations of the parameters listed in Table 1 that provided positive post-yield flexural stiffness. Statistical analyses were conducted to identify the most significant parameters. It was found that the aspect ratio and the axial load ratio play a significant role in the second-order effects on L_{pr}. In addition, longitudinal reinforcement ratio and concrete strength were found to have a slight effect on the susceptibility of RC columns to second-order moments. Results from this parametric study are illustrated in Figs. 8(a) and 8(b) for ρ_{sl} equal to 1% and 4%, respectively. The percentages shown in Fig. 8 represent the increase in L_{pr} due to second-order P-δ moments. It can be seen that the length of the critical region can increase by more than 50% in columns with an aspect ratio of 12, axial load ratio of 20% and longitudinal reinforcement ratio of 1%.

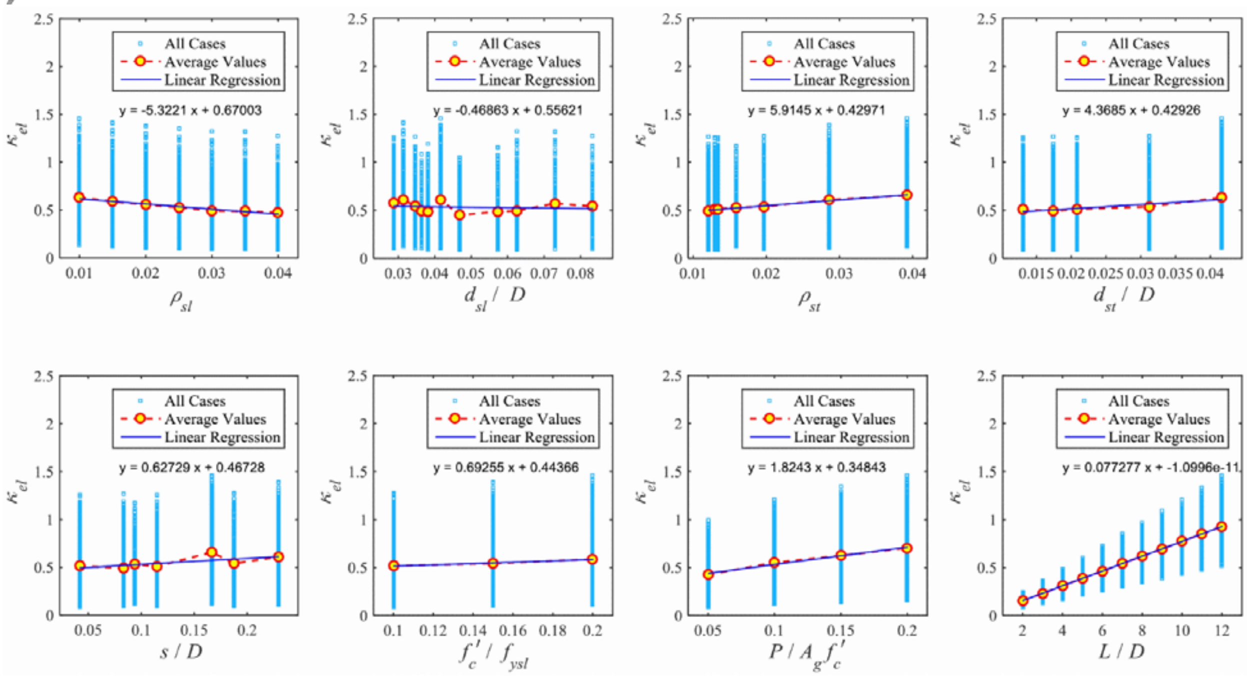

Fig. 6 – Parametric study of the slenderness factor (κ_{el})

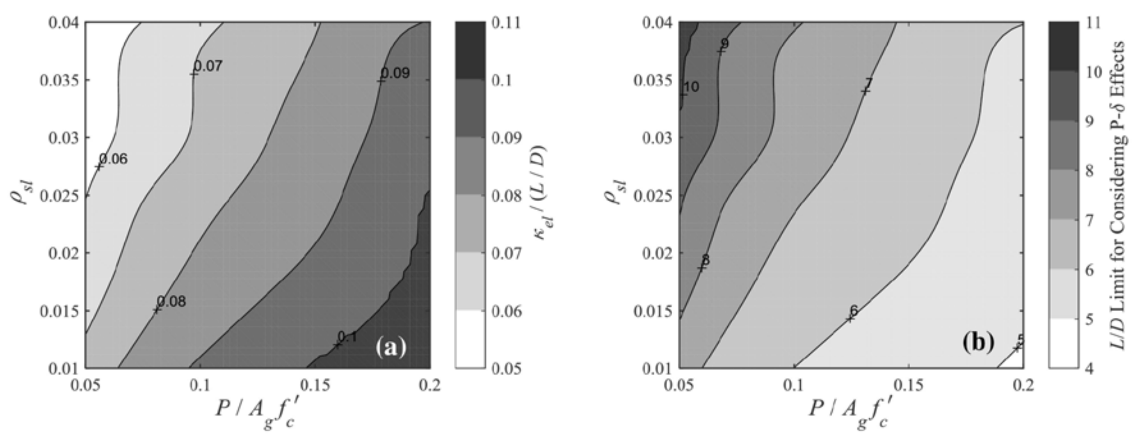

Fig. 7 – Distribution of (a) $\kappa_{el}/(L/D)$ and (b) L/D limit for considering P-δ effects with respect to $P/A_g f_c'$ and ρ_{sl}

5. Regression Models and Design Formulas

A regression model was developed for moment overstrength ratio (M_u/M_y) using the least square estimator method. The correlation between exact values and predictions from the regression model is shown in Fig. 9(a). It can be seen that the regression model underestimates the M_u/M_y ratio for almost half of the design configurations. Thus, the regression model was adjusted by a factor of safety to ensure that the approximate values were greater than the exact ones for more than 95% of the cases. The adjusted regression model for M_u/M_y ratio, which was obtained by adjusting the best fitted linear model by a factor of 1.087, is given in Eq. (4). Similarly, the regression model for the slenderness parameter (κ_{el}) was obtained and adjusted by a factor of 1.074. The result is provided in Eq. (5). The correlation between the exact values and predictions from the adjusted models for M_u/M_y and $\kappa_{el}/(L/D)$ is depicted in Fig. 9(b) and (c), respectively. It can be seen that the linear model for the slenderness parameter, Eq. (5), correlates very well with exact values. Whereas, the linear model for the moment overstrength ratio, Eq. (4), provides a conservative overestimate of the M_u/M_y ratio with reasonable accuracy.

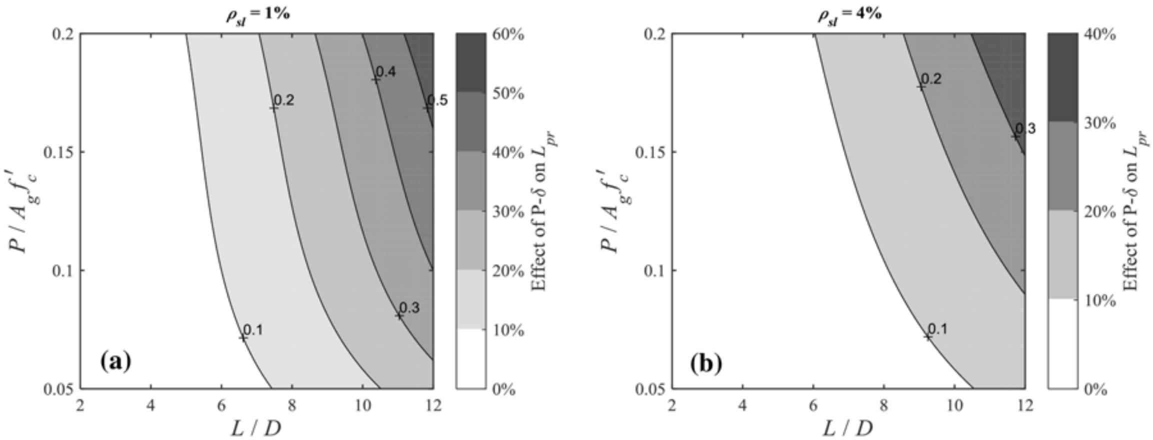

Fig. 8 – Effect of P-δ moments on L_{pr} in terms of L/D and $P/A_g\,f_c'$ for sections with (a) ρ_{sl} = 1% and (b) ρ_{sl} = 4%

$$M_u/M_y = 1.28 - 1.03\frac{f_c'}{f_{ysl}} + 0.254\frac{f_{usl}}{f_{ysl}} - 1.62\frac{P}{A_g f_c'} \quad > 1 \tag{4}$$

$$\kappa_{el} = \frac{L}{D}\left(0.043 - 0.742\rho_{sl} + 0.182\frac{f_c'}{f_{ysl}} + 0.312\frac{P}{A_g f_c'}\right) \quad > 0 \tag{5}$$

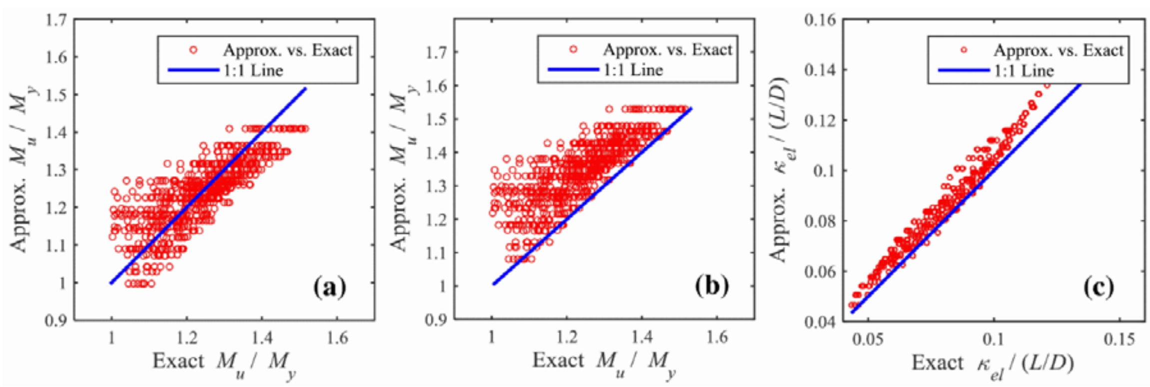

Fig. 9 – Approximate versus exact values for (a) M_u/M_y, (b) adjusted M_u/M_y, and (c) adjusted $\kappa_{el}/(L/D)$

6. Conclusions

A parametric study was conducted to evaluate second-order effects on the extent of the plastic region on slender RC columns. Accordingly, the following conclusions were reached:

- A majority of RC sections (91%) with normal-strength concrete ($f_c' \leq 45$ MPa) that were designed according to the current seismic design guidelines exhibited positive post-yield flexural stiffness. Therefore, analytical models based on a nonlinear moment gradient were applicable to almost all NSC sections. For cases of sections with high-strength concrete (f_c' = 90 MPa), only half of the feasible design configurations led to positive post-yield flexural stiffness. As a result, the use of L_{pr} models based on moment gradient is limited in RC columns with HSC.
- The ratio of yield moment to ultimate moment (M_y/M_u), which is used as a measure for the length of the critical plastic region (L_{pr}), was found to be non-conservatively overestimated by current design guidelines for

NSC columns with axial load ratios less than 10%. Through parametric study on M_y/M_u ratio, it was found that the limit needs to be modified to 70% to ensure a conservative estimate of L_{pr} along such columns.

- The findings of this study helped to determine limits for the aspect ratio (L/D) of RC columns beyond which the effects of the second-order moments on L_{pr} can no longer be ignored. The limiting values for L/D were found to vary from 5 (for RC sections with axial load ratio of 20% and longitudinal reinforcement ratio of 1%) to 10 (for RC sections with axial load ratio of 5% and longitudinal reinforcement ratio of 4%).
- The length of the critical region (L_{pr}) on slender columns can increase by up to 50% due to second-order P-δ moments. This can potentially extend inelastic deformation demands beyond the critical region over which special detailing is provided.

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