



VALIDATION OF SIMULATED EARTHQUAKE GROUND MOTIONS BASED ON EVOLUTION OF INTENSITY AND FREQUENCY CONTENT

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Abstract

Simulated earthquake ground motions can be used in many engineering applications that require time-series as input excitations. However, they need to be validated before they can be utilized with confidence in any engineering applications. Applicability and validation of simulations are subjects of debate in the seismological and engineering communities. We propose a validation methodology at the waveform level that is directly based on characteristics that are expected to influence most structural and geotechnical response parameters. In particular, three time-dependent validation metrics are used to evaluate the evolving intensity, predominant frequency, and bandwidth of a waveform. These validation metrics capture nonstationarities in intensity and frequency content of waveforms, making them ideal to address nonlinear response of structural systems. A two-component error vector is proposed to quantify the average and shape differences between these validation metrics for a simulated and recorded ground-motion pair. Because these metrics are directly related to the waveform characteristics, they provide easily interpretable feedback to seismologists for modifying their ground-motion simulation models compared to metrics that are based on structural and geotechnical responses. To further simplify the use and interpretation of these metrics for engineers, it is shown how six scalar key parameters, including Arias intensity, duration, and predominant frequency, can be extracted from the validation metrics. The proposed validation methodology is a step forward in paving the road for utilization of simulated ground motions in engineering practice, and is demonstrated using examples of recorded and four sets of simulated ground motions from the 1994 Northridge, California earthquake.

Keywords: Ground motion simulation validation, validation metrics, ground motion parameters, waveform characteristics.

1. Introduction

Simulated ground motions have been gaining more attention in recent years and are being utilized in many engineering applications such as response-history analysis of structural or geotechnical systems. However, doubts about similarities between simulated and recorded ground motions cause hesitation among engineers to use them in such applications. Hence, simulations must first go through a rigorous verification and validation process beyond what has been shown by the model developers, because different developers tend to focus on replicating different aspects of ground motions based on their specific model type and parameters. Consequently, the challenging problem of ground-motion simulation validation has gained much interest over the past few years.

Similarities between real and simulated ground motions have been recognized at three different levels: response of Single Degree of Freedom (SDoF) systems, response of Multi Degree of Freedom (MDoF) structural systems, and ground motion waveform characteristics. As an example of the first two categories, Galasso et al. [1] and [2] used broadband simulations of historical events (1979 Imperial Valley earthquake, 1989 Loma Prieta earthquake, 1992 Landers earthquake, and 1994 Northridge earthquake) in which signals from physics-based models are used for low frequencies (below about 1 Hz) combined with stochastic simulations for the high frequency range (above about 1 Hz). Galasso et al. [2] show that the median responses of select linear and nonlinear MDoF systems to simulated motions match well with the median seismic responses produced by recorded ground motions, although, some differences were observed in the short period range. They observed different record-to-record variability of seismic responses produced by simulated and recorded ground motions, especially in the short period range. Similar results were obtained for linear and nonlinear SDoF systems [1]; however, a systematic difference between intra-event dispersion of the structural response was observed. Galasso et al. [1] show that the dispersion of SDoF response for simulations is generally lower than that for recorded ground motions at short periods and higher at long periods, mainly due to existence of strong coherent velocity pulses in simulations.

Given the results of previous validations in terms of structural responses, we decided to take a step back and look into the fundamental differences between waveforms from simulations and recordings of historical events. Studying waveforms, in contrast with scrutinizing SDoF and MDoF responses, would provide us with the opportunity to understand the “true” differences between ground motion simulations and recordings, and not the differences between the “effects” of such motions on structural systems. In Rezaeian et al. [3], we proposed a validation methodology that addresses the cause of differences between response of SDoF and MDoF systems to simulated and recorded motions of historical events by looking into the ground motion waveform characteristics. This methodology is summarized in the present paper. A few other studies have considered goodness-of-fit (GOF) measures that are based on characteristics of the waveform such as [4] and [5]. These GOFs are usually based on commonly used intensity measures such the peak ground response or Fourier amplitudes, and the duration of motion. Our validation methodology introduces three validation metrics that characterize the evolution of intensity (and by extension duration of motion) and frequency content (consisted of predominant frequency and bandwidth) of the waveform over time. Each metric is a function of time. These time-varying properties of earthquake ground motions are important in engineering applications because they influence linear and nonlinear structural responses.

In the following, the three proposed validation metrics characterizing the goodness of fit of simulated to recorded motions are summarized. The error between simulated and recorded motions is quantified using two scalars that represent the average error and shape differences of validation metrics over the entire duration of motion. A few scalar key parameters that control the intensity and shape of the first two metrics (recall that each metric is a function of time) are then suggested for simplification of the proposed methodology and comparison to other validation approaches. An example application of the proposed validation methodology is presented in this paper to examine the relative closeness of simulations to observations from the Northridge earthquake using four different simulation methods. The results of this study can be used in two ways. First, to provide quantifiable metrics through which ground motion validation can be accomplished for historical events and

simulations can be ranked for use in engineering applications. Second, validation against historical events demonstrates the advantages and shortcomings of simulation models and provides feedback to seismologists.

2. Goodness-of-Fit Criteria

As previously mentioned, in order to be useful in engineering applications, simulated motions must first be statistically validated against available strong ground motion data. Selection of an ideal validation metric is difficult because different engineering applications are interested in different characteristics of ground motions. For example, while a match to the elastic response spectrum might be satisfactory for developing simulation-based ground motion prediction equations, it is not an indication of the suitability for use in inelastic response-history analysis. Various intensity measures have been used in practice to assess the fit of simulations to recorded motions; these include peak ground responses (i.e., acceleration, PGA; velocity, PGV; and displacement, PGD), Fourier or response spectral amplitudes, measures of total energy (i.e., integrals of the squared acceleration or velocity), or various measures of shaking duration. A synthetic ground motion may accurately represent a certain intensity measure, while misrepresenting other characteristics of the waveform. In this paper, instead of intensity measures at single points in time or frequency, we consider the entire evolution of intensity and frequency content of ground motion over time. These two criteria are important in engineering applications because they control the response of the structure; many intensity measures that are known to have strong effects on structural responses such as total energy, predominant frequency, or duration can be extracted from these criteria. Furthermore, development and implementation of validation methodologies require collaboration between ground motion modelers and engineering users. The criteria under consideration provide feedback to seismologists on where their simulations deviate from reality even if they match recorded motions in terms of select few intensity measures.

The evolution of intensity and frequency content of a ground motion can be represented by quantifiable statistical characteristics of the time-series that were used by Rezaeian and Der Kiureghian [6] and [7]. Rezaeian and Der Kiureghian represented earthquake ground motions as stochastic processes that are nonstationary in both time and frequency domains. Nonstationarity in the time domain refers to the variation of intensity with time, while nonstationarity in the frequency domain refers to the variation of frequency content with time. Both are fundamental characteristics of earthquake ground motions and are important factors in controlling linear and nonlinear structural responses. The statistical characteristics of a stochastic process that represent the intensity, frequency content, and their time-variation can be numerically estimated for any given time-series; they can then be compared for any given pair of recorded and simulated motions. The three statistical characteristics under consideration are the cumulative standard deviation of the acceleration time-series, the cumulative number of zero-level up-crossings, and the cumulative number of negative maxima and positive minima. While the first metric controls the evolving intensity of the process, the second and third together control the frequency content of the process. Each metric is briefly described in the following; for more details, refer to Rezaeian et al. [3].

2.1 Validation Metric 1: Evolution of Intensity

In the time domain, a ground motion can be characterized by its evolving intensity. The evolution of intensity over time also defines the duration of motion. The intensity of a zero-mean Gaussian stochastic process can be completely characterized by its time-varying standard deviation. Taking advantage of the same concept, for an acceleration time-series $a(t)$, we represent the evolution of intensity in time t by $E_a(t) = \int_0^t a^2(\tau) d\tau$, where $0 \leq \tau \leq t$, $0 \leq t \leq t_n$, and t_n represents the total duration of motion. The Gaussian assumption is not a disadvantage here because nonstationarity in the frequency domain is separable from the nonstationarity in the time domain (see Rezaeian and Der Kiureghian [6]) and is accounted for later in metrics 2 and 3. Fig 1a shows the evolution of intensity, i.e., *validation metric 1*, for a target recorded motion (component 090 of the 1994 Northridge earthquake recorded at the LA-116th Street station) and the same quantity for a simulated motion (generated according to the model of Rezaeian and Der Kiureghian [7]).

This metric, which has previously been used in literature for different purposes (e.g., Yeh and Wen [8]), has the advantage of being a relatively smooth function (compared to others such as acceleration time-series, Fourier amplitude spectra or response spectra), making it suitable for easy visual comparison between two or

more ground motion time-series without the need for any artificial smoothing. Preliminary studies of many recorded motions show that the amplitude and shape of metric 1 depends on the type of ground motion. For example, near-fault ground motions in general show a shorter “rise-time” (5 to 20 s range in Fig 1a) than far field motions. Ground motions recorded at subduction zones such as the 2011 Tohoku earthquake show a much larger total energy (proportional to the amplitude at around 40 s in Fig 1a) compared to ground motions recorded in shallow crustal earthquakes. Furthermore, the shape of the example metric 1 in Fig 1a is typical of earthquake ground motions: intensity starts at zero, slowly builds up as low intensity but high frequency P-waves arrive, quickly rises in a short duration of time as the high intensity S-waves arrive during the strong shaking phase, and finally levels out to a constant value that is proportional to the total energy of the ground motion.

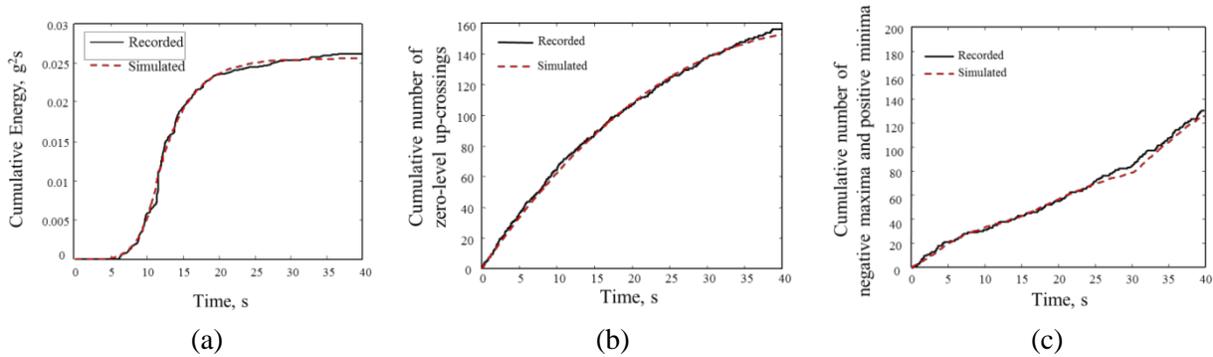


Figure 1. Validation metrics for an example simulated and recorded earthquake ground motion

2.2 Validation Metrics 2 and 3: Evolution of Frequency Content

In the frequency domain, a ground motion process can be characterized by its evolving frequency content. In particular, the frequency content may be characterized in terms of a predominant frequency and a measure of the bandwidth as they change in time. As a surrogate for the predominant frequency of the process, we employ the mean zero-level up-crossing rate, i.e., the mean number of times per unit time that the process crosses the level zero from below. For a zero-mean Gaussian process, analytical solutions are available [9], but unlike a Gaussian process earthquake ground motions are nonstationary in frequency. For a given time-series, this measure can be estimated numerically by simply counting the zero-crossings in time. Fig 1b shows the cumulative number of zero-level up-crossings, i.e., *validation metric 2*, for the same recorded and simulated motions as in Fig 1a. In this plot, the rate of up-crossings (i.e., the slope of the curve) decays with time, indicating that the predominant frequency of the ground acceleration decreases with time. Examining many recorded ground motions reveals that this decay is linear in time, suggesting a linear change in the predominant frequency of recorded ground motions [3]. For earthquake ground motions, validation metric 2 typically follows a parabolic function.

To characterize the time-varying bandwidth of the process, we use the mean rate of negative maxima and positive minima as a surrogate. In a zero-mean narrowband process, almost all maxima are positive and almost all minima are negative (e.g., a harmonic excitation). With increasing bandwidth, the rate of occurrence of negative maxima and positive minima increases. Thus, by determining the rate of negative maxima and positive minima, a time-varying measure of bandwidth can be developed. Similar to validation metric 2, an analytical expression of this rate can be derived for a zero-mean Gaussian process in terms of the well-known distribution of local peaks [9]. For a nonstationary earthquake time-series, this measure can be numerically calculated by simply counting the rate of negative maxima and positive minima in time. Fig 1c shows an example of the cumulative count of negative maxima and positive minima, i.e., *validation metric 3*, for the recorded and simulated motions of Fig 1a.

2.3 Quantification of Error

The three proposed validation metrics can be plotted as shown in Fig 1 for any given simulation, and by examining their shapes one can indicate if the evolution of intensity and frequency content in the simulation is

close to reality. Unlike most other validation techniques, these metrics are not limited to ground motion intensities at single points in time or in frequency. Evolution of intensity and frequency are important to structures, especially if nonlinear analyses are undertaken or if intensity at multiple points in time and/or frequency is of interest, as is the case in more realistic analysis techniques. As previously mentioned, a significant advantage of the proposed metrics is their smoothness and ease of visual comparison between recorded and simulated motions. For validation of a few simulations or for the benefit of model developers, the visual comparison approach is recommended. However, for mass validation of many simulations for many earthquakes, a quantifiable error measure is convenient as proposed in Rezaeian et al. [3] and shown below.

$$\epsilon_{ij} = \frac{\int_0^{t_n} |m_{ij,rec}(t) - m_{ij,siml}(t)| dt}{\int_0^{t_n} m_{ij,rec}(t) dt} \quad i = 1,2,3 \quad (1)$$

$$v_{ij} = \frac{\int_0^{t_n} (m_{ij,rec}(t) - m_{ij,siml}(t)) dt}{\int_0^{t_n} |m_{ij,rec}(t) - m_{ij,siml}(t)| dt} \quad i = 1,2,3 \quad (2)$$

A two-component error vector, denoted as $E_{ij}(\epsilon_{ij}, v_{ij})$ and defined by Eq. (1) and Eq. (2), where $i=1,2,3$ represents the number associated with each validation metric, is defined here to summarize the difference between the j^{th} pair of simulated and recorded motions. ϵ_{ij} is quantified as the normalized absolute area between validation metric i for the j^{th} simulated and recorded ground motions—see Eq. (1). v_{ij} is quantified as shown in Eq. (2), which is the ratio of (i) the algebraic summation of areas between the i^{th} validation metric for the j^{th} simulated and recorded ground motions and (ii) the absolute value summation of the same areas. Within these equations, $m_{ij,rec(t)}$ and $m_{ij,siml(t)}$, respectively represent validation metric i for the j^{th} recorded and simulated ground motions.

In essence, the proposed error vector is a tool for quantifying the differences between a validation metric for a simulated and recorded ground motion pair. Description of a few principal cases can help further understanding of the proposed error vector (see Fig 2): (1) $E_{ij}(0,a) \wedge a \in \mathbb{R}$ shows a case in which the j^{th} simulated and recorded ground motions have no difference in the i^{th} metric; (2) $E_{ij}(b,\pm 1) \wedge b \in \mathbb{R}$ corresponds to a case that the i^{th} metric follows the same trend in the j^{th} ground motion recording and simulation, but with a close to vertical shift; and (3) $E_{ij}(c,0) \wedge c \in \mathbb{R}$ represents a case in which the recording and simulation have different trends with no shift in the considered metric.

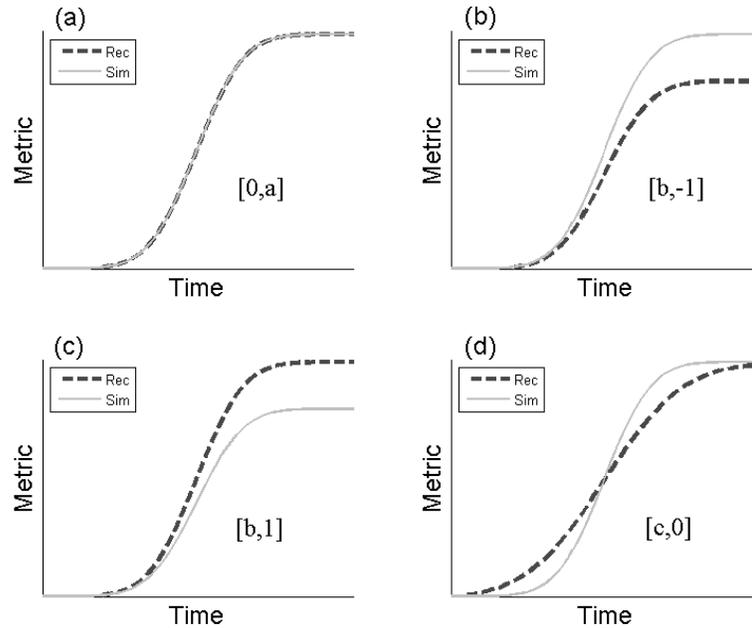


Figure 2. Principal cases of the proposed error vector: a) simulated and recorded ground motions have no difference in a metric, b and c) similar shapes between the metrics but with a close to vertical shift where the amplitude of the metric for the simulated motion always overestimates or always underestimates that of the recorded motion, and d) recording and simulation have different shapes and cross each other.

2.4 Key Scalar Parameters: Validation Proxies

In addition to the three validation metrics introduced in the previous section, we also consider a few scalar parameters, which can act as validation proxies for the response of more complicated engineered systems. Arias intensity, I_a , duration of motion, D_{5-95} , rate of energy accumulation I_a / D_{5-95} , and time at the middle of strong shaking, t_{mid} can be extracted from validation metric 1 and can be used to represent the evolution of intensity in time. Arias intensity is estimated by the value of metric 1 at t_n , I_a , and is representative of the total energy. Duration, D_{5-95} , is measured as the time between instances when the 5% and 95% levels of I_a are reached. t_{mid} , as suggested in Rezaeian and Der Kiureghian [7], is estimated as the time when 45% of I_a is reached.

Mid-Frequency, ω_{mid} , and rate of change of frequency, $\dot{\omega}$, are parameters extracted from validation metric 2 and control the predominant frequency of the motion. Predominant frequency can be estimated from the instantaneous slope of metric 2. We fit a parabola to this metric, and then differentiate it to obtain the predominant frequency as a linear function of time. The line is represented by two parameters: ω_{mid} , which is calculated at t_{mid} , and $\dot{\omega}$, which is the slope of the line. In summary, we have six scalar parameters that can be used for simplification as proxies to evaluate the fit of validation metrics.

2.5 Simulation Adjustment Technique

Given a pair of simulated and recorded ground motion time-series, one can generate the three proposed time-dependent validation metrics and examine their fit by visual inspection as well as by calculating the errors presented in Eqs. (1) and (2). Furthermore, the scalar parameters proposed in the previous section can be extracted from validation metrics 1 and 2 and compared for the simulated and recorded ground motions. However, it is common for the time discretization, total duration and starting time of a simulation to be different from the corresponding recorded motion. In such cases, the simulated motions should be adjusted (e.g., down-sampled and shortened) to be comparable to the natural recordings. The details of this procedure can be found in Rezaeian et al. [3].

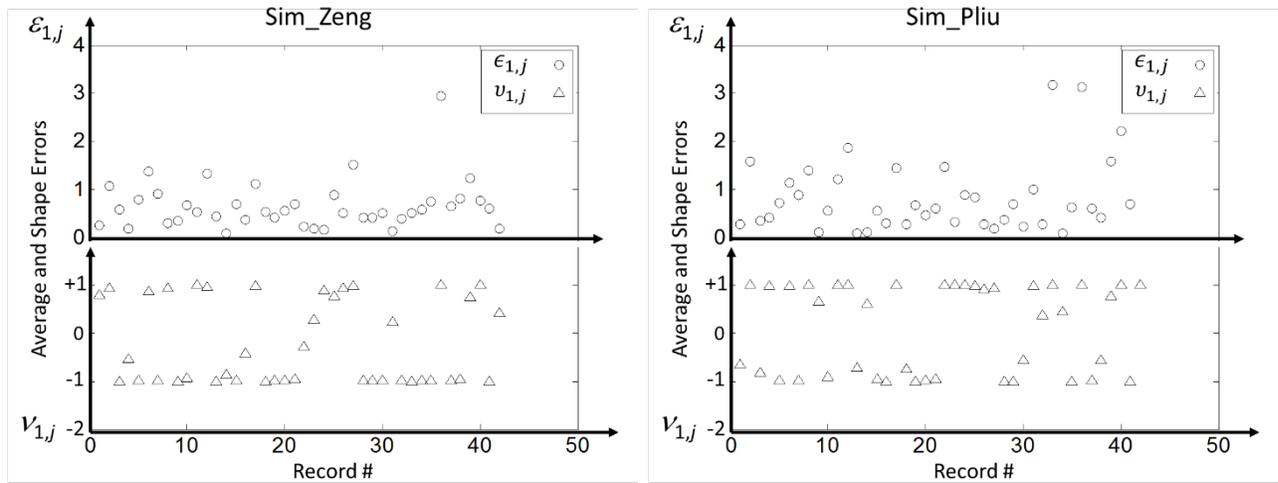
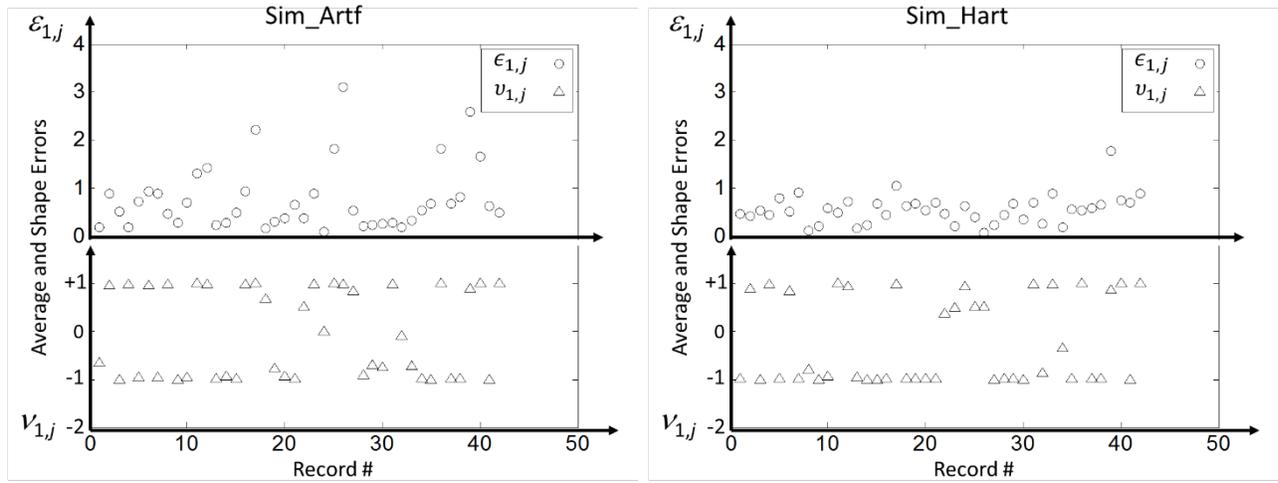
3. Example Application for Northridge Earthquake

As an example of assessing the fit of simulations to recorded motions from a historical earthquake, we calculate the proposed validation metrics and scalar parameters for four sets of 1D source-based kinematic simulation methodologies. The four simulation methods are used to simulate the same recordings from the Northridge earthquake event at 42 stations (1-32: soil sites; 33-42: rock sites). By applying the proposed validation methodology on these four sets of simulations, we are able to rank and compare them relative to each other in terms of various ground motion characteristics. The simulation methodologies are as follows:

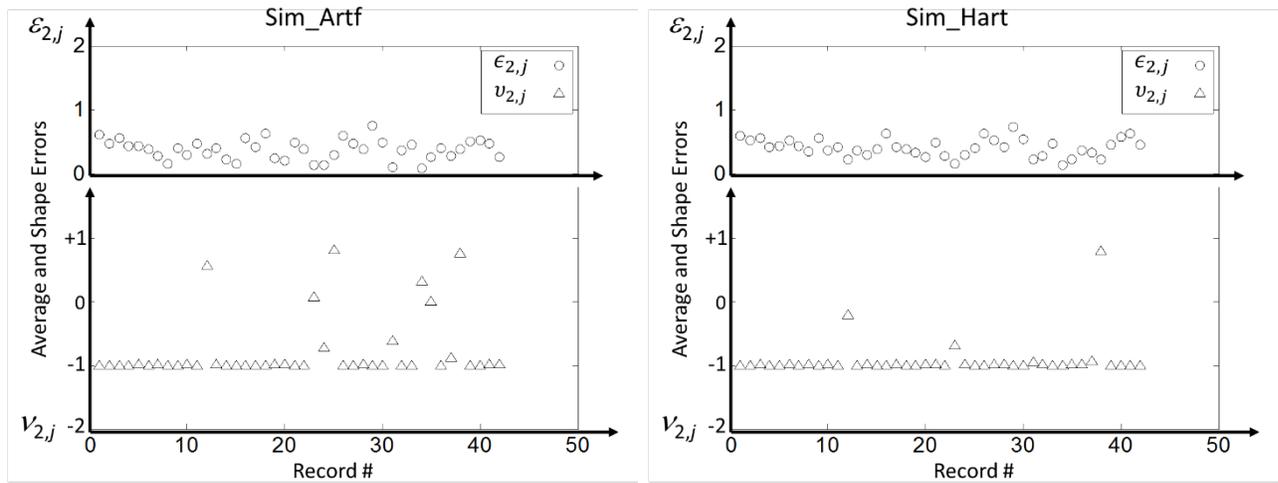
- 1) Hartzell et al. [10] and [11]. This simulation method is based on a fractal distribution of sources, where the smallest subevent size is determined using the constant stress-drop scaling model. The response of each subevent is calculated by summing theoretical Green's functions for a given velocity model for all frequencies of interest.
- 2) Liu et al. [12]. This simulation method utilizes the concept of correlated random source parameters and theoretical wave-propagation Green's functions at all frequencies. In this model, slip, rupture velocity, and rise time are represented as random, correlated variables with adjustable correlation coefficients.
- 3) Frankel [13]. This simulation method uses a source with constant stress-drop scaling with magnitude. The procedure combines long-period synthetics made using Green's functions for a prescribed velocity model with short-period synthetics made using point-source stochastic seismograms.
- 4) Zeng et al. [14]. This simulation method uses a fractal distribution of source sizes with overlapping source areas and wave propagation Green's functions for a specified velocity model. Subevents are distributed randomly on the fault plane with a size distribution based on the self-similar model.

3.1 Validation Metrics

Abbreviation of each author's name is used to present results of the corresponding simulation method: Hart, Pliu, Artf and Zeng. Quantitative error measures for the three validation metrics for the four simulation methods are shown in Fig 3. The vertical axis shows ϵ_{ij} , represented by circles, and ν_{ij} , represented by triangles. In general, the four simulations seem to behave in a similar manner regarding the average evolution of intensity, predominant frequency and bandwidth. For metric 1, Hartzell shows an overall smaller average error for the evolution of intensity. Given that a large number of triangles are on ± 1 , we can argue that the overall shape of metric 1, representing the evolution of intensity in time, is similar for simulated and recorded motions. The shape errors for all four simulations tend to be evenly distributed between overestimation (-1) and underestimation ($+1$). In metric 2, given that the majority of triangles, ν_{2j} , are -1 , we can argue that these simulated motions' cumulative zero-level up-crossings, the slope of which represents the predominant frequency, have similar shapes to recorded motions but overestimate their amplitudes. The shape errors for Zeng simulations are more evenly distributed, indicating less of an overestimation compared to the other three simulations. Metric 3 is assessing the evolution of bandwidth, and an overall similar overestimation is observed, with Hartzell simulations showing less bias compared to the other three methods. More studies have to be conducted to make further conclusions for validating and ranking these four simulation methodologies, such as using different earthquake events.



(a)



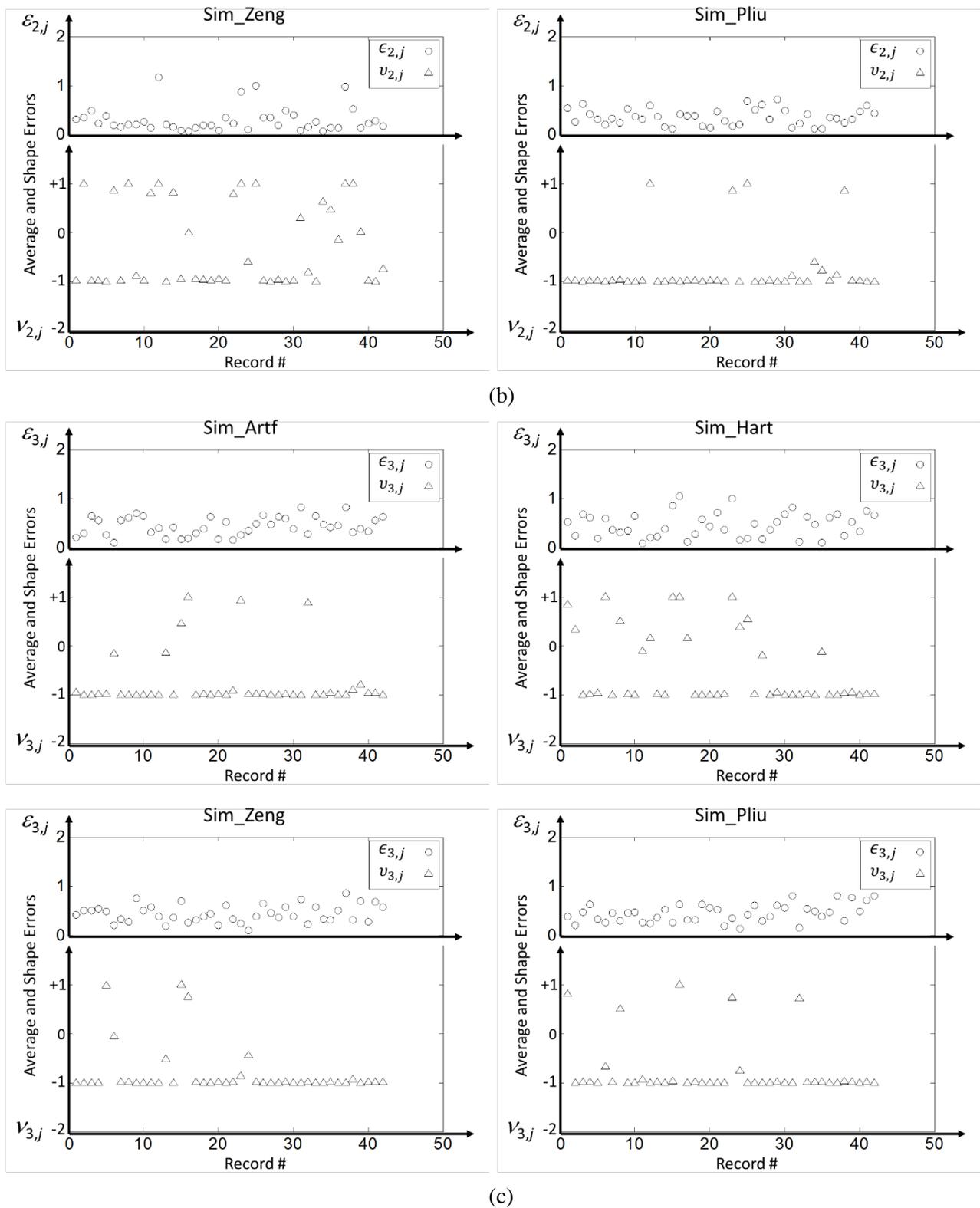


Figure 3. Results of validation-metric errors for four simulation methodologies.

3.2 Scalar Parameters: Validation Proxies

The six scalar key parameters that were introduced previously and capture the main characteristics of the validation metrics 1 and 2 are investigated. Summary of these scalar parameters for the four sets of simulations can be seen in Fig 4. These figures show box plots, where the central line is the median value of the dataset, the box indicates the 25th and 75th percentile limits, the whiskers extend to the 10th and 90th percentiles (the most extreme data points not considered outliers), and the crosses outside of the whiskers are the outliers. From these figures, we can see which simulation method produces closer medians and distributions relative to the recorded ground motions. In general, we can see that the medians of the four simulation methods are close to the medians of the recorded ground motions for Arias intensity, with Zeng simulations having a relatively lower median. Zeng and Frankel simulations have a wider distribution of Arias intensity compared to the recorded motions. These distributions are skewed, with tails controlled by a few individual and very large intensities, which lead to overestimation of building responses consistent with the findings in Zhong [15]. Variability in the significant duration of these four sets of simulated motions is relatively higher than the duration variability seen for recorded motions. Liu simulations underestimate the median duration compared to recorded motions. The simulated values for the parameter I_a/D_{5-95} , representing the rate of input energy, are overall in good agreement with the recordings, but Zeng simulations are skewed towards larger values as was the case with Arias intensity. In such a case, more energy is inputted into the structure in a shorter duration, and the ground motion is more likely to be pulse-like and create overestimation in structural response when this simulation is used in engineering applications. t_{mid} , representing the time at the middle of strong shaking, is mismatched for all the simulations in median and variability. And finally, we can see that the Zeng and Frankel simulations slightly overestimate the frequency at the middle of strong shaking, and the other methods show underestimation with narrow distribution. The rate of frequency decay is slower (less negative numbers for ω') for all four sets of simulations. In addition to estimating the median value for each parameter, these box plots give the model developers an idea of the spread of each parameter and whether the variability is well represented in the models compared to recorded ground motions.

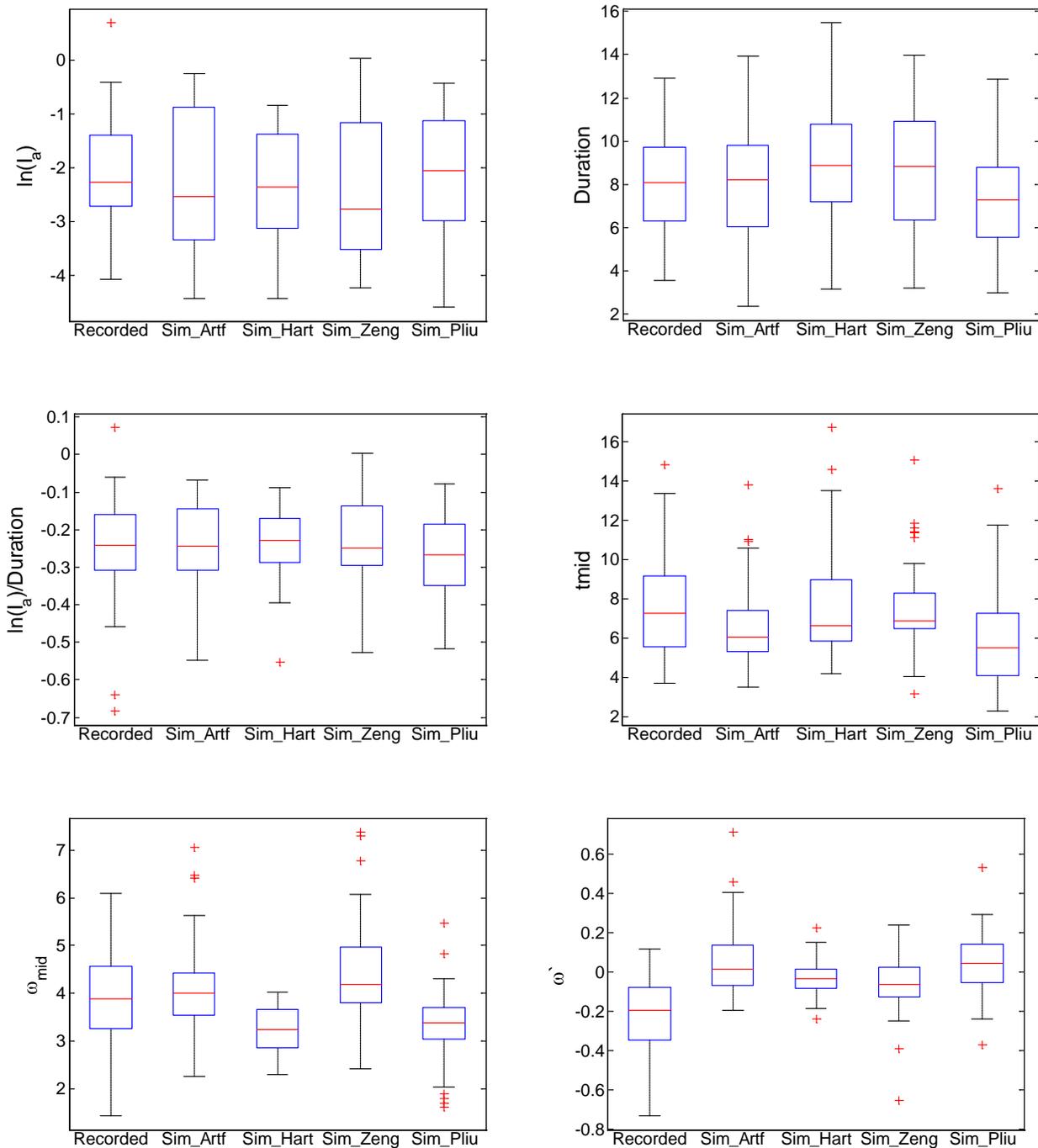


Figure 4. Comparison of key scalar parameters

4. Conclusion

We assess the validity of simulated ground motions against recordings of historical earthquakes, using three time-dependent validation metrics that characterize the evolution of intensity and frequency content of the ground motion waveform. These characteristics of earthquake ground motions are important in engineering applications because they influence linear and nonlinear structural responses. The difference between each metric for simulated and recorded motions is quantified using an error vector that represents the average and shape errors

over the entire duration of motion. We also introduce six scalar parameters that are extracted from the first two validation metrics. These parameters represent the Arias intensity, effective duration of motion, rate of input energy, time at the middle of strong shaking, frequency at this time, and rate of change of frequency in time.

The proposed methodology can benefit both engineers and seismologists. In this paper, we presented an application of this validation methodology to assess the fit of four simulation methods to recordings from the Northridge earthquake. Validating simulations for multiple earthquake events is the next step and a subject of future research. Another subject of future study is to utilize the proposed validation methodology to study the fundamental characteristics of spectrum compatible ground motions at various hazard levels in order to assess existing practical, but controversial, scaling and spectrum matching methods.

5. Acknowledgements

This research was supported by the NSF and the U.S. Geological Survey sponsored Southern California Earthquake Center (SCEC) program as part of the Technical Activity Group (TAG) on Ground Motion Simulation Validation (GMSV). Their support is gratefully acknowledged. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the NSF. We would like to thank Dr. Yuehua Zeng, Dr. Pengcheng Liu, Dr. Arthur Frankel and Dr. Stephen Hartzell for providing the simulations used in this study.

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