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A METHODOLOGY FOR SEISMIC RETROFIT USING OPTIMAL CONTROL

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Abstract

This paper proposes a new seismic modification methodology for optimal design of multistory frame structures to avoid damage and attain a robust response to extreme hazards. The methodology determines added damping devices of optimal size at strategic locations and modifies story stiffnesses, by solving a newly formulated constrained optimization problem. The objective is to minimize a cost function representing total energy while satisfying the equations of motion and allowable interstory drifts for given severe ground motions. The proposed methodology combines the classical Linear Quadratic Regulator (LQR), which searches for an optimal gain matrix, with the analysis-redesign procedure of Levy and Lavan [1] to produce a Robust Analysis Redesign (RAR) iterative approach. A new algorithm approximates the equivalent stiffness to fit the displacement gains. At convergence, the RAR procedure will yield optimal sizing and new topology for the designed system.
The RAR procedure is developed here for shear structures. However, RAR can be utilized for complex structures by modeling them first as equivalent shear-type structures having stiffness properties that yield the same maximum interstory drifts for the given records. Two numerical examples of shear-type structures are examined. These examples use structures already having an "optimal" configuration which are further optimized using the methodology developed herein. In addition, a numerical example comprised of a 10-story MRF structure is presented to illustrate the use of the equivalent shear structure approach. The results of the exemplified retrofitted MRF structure show great improvements in its dynamic response.

Keywords: Earthquake engineering; optimal control; added stiffness and damping; robustness

1. Introduction

During the lifetime of a structure, extreme hazards such as earthquakes may impact the structural system and result in damage that endangers human life. Newer approaches which totally avoid damage, or which produce robust structures with minimum losses that can be quickly repaired to continue their function were suggested after the major earthquakes in New Zealand in 2013. An efficient design and retrofit solution for multistory buildings which must withstand seismic loads should mitigate structural vibrations and associated damage. Passive energy dissipation devices (dampers) have been well received as effective means of control and mitigation of the effects of dynamic loadings caused by strong earthquakes (Constantinou et al. [2]) because of their attractive properties, (stable performance during energy dissipation, not requiring external power supply and long-term reliability).

Viscous dampers, in particular, are found to be very efficient and attractive for linear retrofit. They are velocity dependent and the forces in the dampers are thus, out of phase with the columns axial loading and proven to be reliable. Various procedures for the design of added viscous damping for structures were proposed by many researchers (for example, Inaudi et al. [3]; Shen and Soong [4]; Lopez-Garcia [5]; and Levy and Lavan [1]). Viscous dampers can control the deformations within desirable limits (Levy and Lavan [1]).

Where mitigating non-structural components damage is concerned, the accelerations become important Weakening of the structure and adding damping devices as suggested by Reinhorn et al. [6] can lower the accelerations and limit deformations. Weakening can be achieved by disconnecting some of the moment resisting frames, or walls or by adding devices like the adaptive negative stiffness system (ANSS) of Pasala et al., [7]). The negative stiffness device (NSD), which is basically a pre-compressed spring placed vertically between the two chevron braces (Sarlis et al. [8]), reduces the stiffness of the structure to almost nil, simulating an “apparent” yielding system. When coupled with passive viscous dampers it can reduce both accelerations and deformations.

Various procedures can be found in the literature for seismic design and retrofit of multistory structures by changing their damping and stiffness coefficients. Gluck et al [9] developed a comprehensive procedure based on the well-known Linear Quadratic Regulator (LQR) which handles changes in stiffness and damping to obtain an optimal retrofit solution. Nakamura and Tsuji [10], Takewaki [11] and others followed suit and more recent work is that of Cimellaro et al. [12]. Most of these methodologies will usually require mathematics of stochastic processes, optimization methods, and/or variational mathematics—tools, not that familiar to the practicing engineer.

Analysis-redesign procedures which are based on limiting the desired maximum response may be used with time history analyses or design spectra to provide more suitable solutions. An analysis-redesign methodology that was proposed by Levy and Lavan [1] can distribute and size viscous dampers optimally. This paper proposes a methodology that combines the classical LQR which searches for an optimal gain matrix with the analysis-redesign procedure of Levy and Lavan [1] into a Robust Analysis Redesign (RAR) iterative approach. A new algorithm approximates the equivalent stiffness to fit the displacement gain. At convergence RAR procedure will yield an optimal sizing and topology of the system.

The RAR procedure is developed here for shear structures. However, RAR can be utilized for complex structures by modeling them first as equivalent shear-type structures having stiffness properties that yield the same maximum interstory drifts (for a given ensemble of ground motion records) and then applying the RAR procedure.

2. Problem Formulation

The problem at hand is formulated as an optimization problem where the objective is to mitigate story displacements and velocities, while minimizing the added transmitted force so as to attain maximum structural robustness. In addition, interstory drifts should be limited to allowable values as prescribed by seismic design standards.

The optimization problem is comprised of the following cost function under the constraints of dynamic equilibrium equations and allowable interstory drifts as:

$$\left.\begin{aligned}
&\min_{\Delta\mathbf{f}_{\mathrm{T}}[0,t_f]}\left\{\int_0^{t_f}\mathbf{x}(t)^{\mathrm{T}}\mathbf{Q}_1\mathbf{x}(t)+\dot{\mathbf{x}}(t)^{\mathrm{T}}\mathbf{Q}_2\dot{\mathbf{x}}(t)+\Delta\mathbf{f}_{\mathrm{T}}(t)^{\mathrm{T}}\Delta\mathbf{f}_{\mathrm{T}}(t)dt\right\}\\
&\text{s.t.}\\
&\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{f}_{\mathrm{T}}{}^{\text{total}}(t)=-\mathbf{M}\boldsymbol{\iota}a_g(t)\\
&\mathbf{f}_{\mathrm{T}}{}^{\text{total}}(t)=\mathbf{f}_{\mathrm{T}}(t)+\ \Delta\mathbf{f}_{\mathrm{T}}(t)\\
&\mathbf{f}_{\mathrm{T}}(t)=\mathbf{K}\,\mathbf{x}(t)+\mathbf{C}\,\dot{\mathbf{x}}(t)\\
&\Delta\mathbf{f}_{\mathrm{T}}(t)=\Delta\mathbf{K}\,\mathbf{x}(t)+\Delta\mathbf{C}\,\dot{\mathbf{x}}(t)\\
&\max_i(\max_t(|d_i(t)|))\le d_{\mathrm{all}_i}
\end{aligned}\right\}\forall\mathbf{a}_g(t) \tag{1}$$

Here $\mathbf{f}_{\mathrm{T}}{}^{\text{total}}(t)$ is the total transmitted force vector where $\mathbf{f}_{\mathrm{T}}(t)$ is the *initial transmitted force* vector which can be written as $\mathbf{f}_{\mathrm{T}}(t)=\mathbf{K}\,\mathbf{x}(t)+\mathbf{C}\,\dot{\mathbf{x}}(t)$ and $\Delta\mathbf{f}_{\mathrm{T}}(t)$ is the *added transmitted force* vector and is presented as $\Delta\mathbf{f}_{\mathrm{T}}(t)=\Delta\mathbf{K}\,\mathbf{x}(t)+\Delta\mathbf{C}\,\dot{\mathbf{x}}(t)$. The matrices $\mathbf{K}$ and $\mathbf{C}$ are the initial stiffness and inherent damping matrices respectively, $\Delta\mathbf{K}$ and $\Delta\mathbf{C}$ are the added stiffness and supplemental damping matrices respectively, $\mathbf{x}(t)$ is the displacements vector and $\dot{\mathbf{x}}(t)$ is the velocities vector. t_f is the duration time of seismic vibrations, t is the time variable, $\mathbf{M}$ is the mass matrix of the structure, $\ddot{\mathbf{x}}(t)$ is the story accelerations vector, $\mathbf{a}_g(t)$ is the ground acceleration, $\boldsymbol{\iota}$ is the influence vector of ground acceleration, $d_i(t)$ is the drift of the i_{th} story in time and d_{all_i} is the allowable drift for the i_{th} story. The matrices $\mathbf{Q}_1$ and $\mathbf{Q}_2$ are diagonal and are assigned to set the relative importance between displacements, velocities and added transmitted force (referred to as weighting matrices). Assigning coefficients of the weighting matrices should lead to least changes (adding or reducing) of stiffness and total damping in the retrofitted structure while minimizing the maximum interstory drifts.

In the proposed formulation the *changes* in stiffness and damping are emphasized so as to minimize the newly proposed cost function, which includes the state variables and the transmitted force. This control formulation differs from the usual LQR formulation in that two weighting matrices are used for the state variables (one for displacements and another for velocities) and with the weighting matrix for the transmitted forces taken as an identity matrix. Both formulations lead to Riccati's equation. However, the newly proposed cost function is advantageous since it enables the flexibility of assigning importance factors to stiffness and damping and thus caters to the structural robustness needs.

Solution of the optimal problem of Eq. (1) is obtained using a robust analysis-redesign methodology that is described herein. The methodology features the continuous time Linear Quadratic Regulator (LQR) formulation of optimal control for the assessment of the structural system variables in the form of displacement and velocity related gain matrices. A novel approach for assessing changes in stiffness uses the displacement related gain matrix and the iterative analysis-redesign scheme of Levy and Lavan [2006] that was originally applied to optimal sizing and location of viscous dampers.

3. Methodology

3.1 Continuous Time LQR Solution Algorithm

The LQR cost function may be regarded as a special case of an optimization problem where the story displacements and velocities are optimized in time, by calculating the added\reduced stiffness and damping of a shear structure (i.e. optimal control gain), subject to the physical equations of motion.

To solve the optimization problem in Eq. (1), the continuous time LQR optimization problem is introduced for the case of stochastic noise with full state information feedback:

$$\begin{gathered}
\min_{\mathbf{u}[0,t_f]}E\{J\}=\min_{\mathbf{u}[0,t_f]}E\left\{\int_0^{t_f}[\mathbf{z}^{\mathrm{T}}(t)\mathbf{Q}\mathbf{z}(t)+\mathbf{u}^{\mathrm{T}}(t)\mathbf{R}\mathbf{u}(t)]dt\right\}\\
\text{s.t. }\dot{\mathbf{z}}(t)=\mathbf{A}\mathbf{z}(t)+\mathbf{B}\mathbf{u}(t)+\mathbf{w}(t),\ \ \mathbf{w}(t)\sim\mathcal{N}(\mathbf{0},\mathbf{W})
\end{gathered} \tag{2}$$

The LQR optimization problem minimizes the expected value of the cost function "J" subject to the differential state equation of the closed-loop system. Here $\mathbf{z}(t)$ is the state vector, $\mathbf{A}$ is the closed-loop system matrix, $\mathbf{u}(t)$ is the control signals vector, $\mathbf{B}$ is the control-to-state matrix and $\mathbf{w}(t)$ is a Gaussian zero-mean white-noise process with intensity matrix (covariance) $\mathbf{W}$, $\mathbf{Q}$ and $\mathbf{R}$ are positive semi-definite and positive definite matrices respectively, referred to as weighting matrices, whose magnitudes are assigned according to the relative importance between $\mathbf{z}(t)$ and $\mathbf{u}(t)$.

As shown by Anderson and Moore [13] the closed-loop control policy which minimizes the value of the cost can be expressed as:

$$\mathbf{u}(t) = \mathbf{G}\,\mathbf{z}(t);\ \mathbf{G} = [\mathbf{G_x} \quad \mathbf{G_{\dot{x}}}];\text{and } \mathbf{G} = -\mathbf{R}^{-1}\mathbf{B}^{T}\mathbf{P} \tag{3}$$

Here $\mathbf{G}$ is a constant gain matrix and the matrix $\mathbf{P}$ is called the time-invariant Riccati Matrix, and is used to define an optimal control policy (control gain). It is determined by solving the continuous algebraic Riccati equation:

$$\mathbf{A}^{T}\mathbf{P} + \mathbf{PA} - \mathbf{PBR}^{-1}\mathbf{B}^{T}\mathbf{P} + \mathbf{Q} = \mathbf{0} \tag{4}$$

The optimization problem presented in Eq. 2 is a LQR optimization problem with ground accelerations of zero-mean in the time interval $[0, t_f]$.

The control gain sub-matrices that are presented here, $\mathbf{G_x}$ and $\mathbf{G_{\dot{x}}}$, lead to the changes $\Delta\mathbf{K}$ and $\Delta\mathbf{C}$ respectively. However, these sub-matrices are not necessarily symmetric or tri-diagonal and their practical implementation within the structure may not be possible. Gluck et al. [9] cater to this difficulty for shear structures by using Least Square Approximation (LSA) formulation on the story forces, in drift coordinates to attain interstory changes in stiffness and damping. This concept may not lead to equivalence and optimal solution in maximum response since none of their solutions associates with dynamic response and maximum response limit simultaneously.

3.2 Approximation for Stiffness Changes

Developing a new methodology for determining the changes in floor stiffness, Δk_i, is one important goal of this paper. The added stiffness matrix $\Delta\mathbf{K}$ should have the same optimal attributes as the displacement related gain matrix $\mathbf{G_x}$. Since stiffness is the extent to which a structure resists deformation to applied forces, interstory drifts are taken as the measure for equivalence. The structure with the stiffness of $\mathbf{K} + \mathbf{G_x}$ (controlled structure) is required to have the same maximum interstory drifts as the shear structure with the stiffness of $\mathbf{K} + \Delta\mathbf{K}$ (equivalent structure) for the specified seismic records.

The interstory drifts of the controlled structure, $\mathbf{d}_{con}(t)$, are determined by transforming the floor displacements, $\mathbf{x}_{con}(t)$, using $\mathbf{d}_{con}(t) = \mathbf{T}^{-1}\,\mathbf{x}_{con}(t)$. The floor displacements are obtained by solving the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}_{con}(t) + \mathbf{C}\,\dot{\mathbf{x}}_{con}(t) + (\mathbf{K} + \mathbf{G_x})\,\mathbf{x}_{con}(t) = -\mathbf{M}\boldsymbol{\iota}a_g(t) \text{ for all } a_g(t) \text{ in the active set} \tag{5}$$

The interstory drifts of the equivalent structure, $\mathbf{d}(t)$, are similarly determined from the floor displacements, $\mathbf{x}(t)$, using $\mathbf{d}(t) = \mathbf{T}^{-1}\,\mathbf{x}(t)$. The floor displacements are obtained by solving the following equation of motion:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\,\dot{\mathbf{x}}(t) + (\mathbf{K} + \Delta\mathbf{K})\,\mathbf{x}(t) = -\mathbf{M}\boldsymbol{\iota}a_g(t) \text{ for all } a_g(t) \text{ in the active set} \tag{6}$$

The controlled structure in Eq. (5) has the mass and inherent damping matrices of the initial shear structure $\mathbf{M}$ and $\mathbf{C}$ respectively and a stiffness matrix $\mathbf{K} + \mathbf{G_x}$, whereas the equivalent shear structure in Eq. (6) has the same mass and inherent damping matrices as that of the shear structure but a stiffness matrix $\mathbf{K} + \Delta\mathbf{K}$, that is yet to be determined.

When transformed to drift coordinates, the stiffness matrix $\mathbf{K}$ becomes a diagonal matrix, for shear structures and each coefficient, $(k_d)_i$, of that diagonal matrix, $\mathbf{K}_d$, defines its corresponding story stiffness. Decreasing the story stiffness, will increase the interstory drifts and vice versa. The new story stiffness coefficients are now evaluated according to the ratio of the maximum interstory drifts of the retrofitted shear structure and the maximum interstory drifts of the controlled structure:

$$(k_d + \Delta k_d)_i^{k+1} = (k_d + \Delta k_d)_i^k \left(\frac{(\max_t(|d_i(t)|))}{(\max_t(|d_{con_i}(t)|))}\right)^q \tag{7}$$

Here q is a convergence parameter.

3.3 Analysis-Redesign for Supplemental Damping

Using the LQR solution of Eq. (2) to mitigate story displacements, velocities and the added transmitted force in time, does not assure optimal limitations of interstory drifts as dictated by Eq. (1). In addition, the solution is obtained for a base excitation of a zero-mean Gaussian white-noise process, which may not be the case for real seismic excitations. Combining LQR with an approach for controlling maximum drift response, using ground motion records, would derive a retrofit solution for the optimization problem of Eq. (2) which is the aim of this work's methodology.

Levy and Lavan [2006] suggested an iterative analysis-redesign procedure for controlling maximum dynamic response. Their procedure is composed of three main stages. Stage 1 identifies the "active" ground motion from a given ensemble of ground motion records. Stage 2 uses an iterative analysis-redesign approach with the following recurrence relationship for the redesign of supplemental damping:

$$\Delta c_{d_i}{}^{k+1} = \Delta c_{d_i}{}^{k} \left(pi_i{}^k\right)^q \tag{8}$$

Here $\Delta c_{d_i}{}^k$ is the i_{th} story damper at the k_{th} iteration, $pi_i{}^k$ is the i_{th} story performance index that is defined as the ratio of the current maximum interstory drift divided by the allowable interstory drift and q is the convergence parameter. Time-history analysis is utilized to calculate the current interstory drift (analysis). In stage 3, the maximum response of the damped structure for each of the remaining ground motions in the ensemble is separately evaluated using time-history analysis. If the design achieved in stage 2 violates constraints of other records in the ensemble, i.e., $\max_i\left(pi_i{}^k\right) > 1.0$, the ground motion for which $\max_i\left(pi_i{}^k\right)$ achieves the largest value is added to the active set. Limiting interstory drifts will satisfy the constraints of Eq. (1) which the LQR solution formulation may not be able to achieve. It might be worth noting that this procedure is valid for all types of frame structures for linear as well as nonlinear analysis.

3.4 Retrofit Algorithm for Shear Structures

In this section the LQR solution algorithm of section 3.1 is combined with the iterative procedures of sections 3.2 and 3.3 for attaining the added stiffness matrix (ΔK) and the supplemental damping matrix (ΔC) into a robust analysis redesign (RAR) iterative retrofit method for shear structures. The procedure solves, in essence, the optimization problem of Eq.(1). Following is a step by step description of the RAR procedure.

Step 1. Define the physical parameters of the shear structure (**K**, **C**, **M**) and select the "active" ground motion ($a_g(t)^0$) from the given ensemble of ground motion records.

Step 2. Calculate the optimal control gain matrix $\mathbf{G}$ by solving Ricatti algebraic Eq. (4) and hence the displacements-related gain matrix $\mathbf{G_x}$.

Step 3. Attain the updated story stiffnesses using Eq. (7) that stems from the procedure of section "3.2 Approximation for Stiffness Changes" for the records within the active ground motion set.

Step 4. Derive the added damping for the records within the active ground motion set using the analysis-redesign approach using Eq. (8) as described in section 3.3

Step 5. Check for convergence for two consecutive updated story stiffnesses obtained from step 3. If convergence is not met, go to Step 2 with the initial stiffness matrix, **K,** and updated damping matrix $\mathbf{C} + \Delta\mathbf{C}$. If met, continue to step 6.

Step 6. Perform time-history analysis for all the ground motion records. If constraints on drifts are violated, then add the record with largest violation to the active set of ground motions records and go to Step 2.

Step 7. Stop.

The above RAR algorithm calculates the optimal changes in stiffness, ΔK and damping, ΔC for shear structures. The change in stiffness can be implemented by adding common braces or special negative devices (Sarlis et al. [8]). Changes in damping are easily implemented by simply adding fluid viscous dampers.

4. Equivalent Shear Structures

The suggested RAR algorithm applies to shear structure. This section describes a new method for forming an equivalent shear structure to multistory frame buildings, and thus, enabling the application of the RAR algorithm. The added damping and change in stiffness matrices are then added to the original structure as equivalent added matrices.

The new method proposes to use the procedure of section 3.2 to generate that equivalent shear structure that models multistory frame structures.

The following recurrence relationship is used to obtain the individual shear equivalent story stiffnesses:

$$(k_d)_i^{k+1} = (k_d)_i^k \left(\frac{(\max_t(|d_i(t)|))}{(\max_t(|d_{o_i}(t)|))}\right)^q \tag{9}$$

Where $\max_t(|d_{o_i}(t)|)$ is the maximum interstory drift at i^{th} story of the original structure. When convergence is reached the equivalent shear structure will have identical maximum interstory drifts as the original structure for a given ensemble of ground motions records. We note that at every iteration, the inherent damping matrix of the equivalent structure is recalculated in order to maintain its classic damping proportionality properties and its initial damping ratios.

5. Numerical Examples

Three structures, taken from the literature, were re-examined using the RAR method: A 6-DOF damped mass-spring model from Takewaki [11], a 9-story shear-type building from Cimellaro et al. [12] and a 10-story MRF structure from Levy et al. [14].

All the examples are initially processed according to the analysis-redesign procedure of Levy and Lavan [2006] (in section 3.3) for the redistribution of damping coefficients, so as to highlight the efficiency of changing stiffness coefficients using RAR.

5.1 Example 1: 6-DOF damped mass-spring model

Takewaki [11] applied his optimization method to a 6-DOF damped mass-spring model which is presented in Fig 1.

The Maximum response for his optimal model is attained here using time history analysis for the "LA 10% in 50 years" earthquake records ("http://nisee.berkeley.edu/data/strong_motion/sacsteel/ground_motions.html"). Results are shown in Table 1(A).

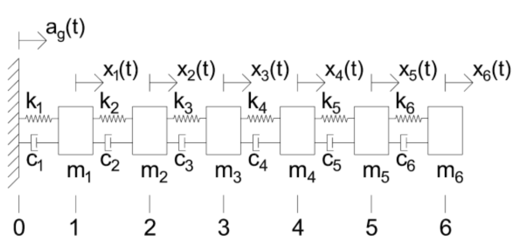

Fig. 1 - Scheme of the model in example 1 (Takewaki [11])

The analysis-redesign procedure of section 3.3 is now applied to his optimal structure for the same ground motion records and for an allowable s drift of $0.0237m$ and $\mathrm{q} = 2.0$ for convergence. Results in Table 1(B) show a minor reduction in the sum of damping coefficients (about 2.7%) a small increase in maximum story acceleration (about 4.7%), practically no change in maximum force. The analysis-redesign procedure had a negligible effect due to initially well distributed damping coefficients. Finally, the RAR algorithm is applied to the original optimal structure of Table 1(A) as a starting point.

It took a total of 14 iteration (10 for one active record; 2 more for the two active records and another 2 for the three active records) to reach convergence and the results presented in Table 1, C.

Comparing the response of the optimal structure in Table1(B), with the results of the RAR methodology that are described in Table1(C), a 13% decrease in maximum story acceleration is observed along with a 25% decrease in maximum base shear by adding 50% more damping while reducing the story stiffness by 20%. The optimal solution in terms of performance requires increase of damping and reduced stiffness, emphasizing the tradeoff between stiffness and damping. Another important aspect here is the successful application of the methodology.

5.2. Example 2: 9-story shear structure

Cimellaro et al. [12] presented a two-step algorithm for retrofitting multistory structures. The first step, determines the control gain matrix by applying the LQR algorithm. The second step utilizes the approach of Smith et al. [15] for linear-elastic structures where the control gain is considered as the sum of the active and passive control gain matrices. The passive gain matrix (which is composed of the added stiffness, damping and mass matrices) is determined by minimizing the active control power needed.

In one of their examples, Cimellaro et al. [12] simplified the 9-story MRF structure of Ohtori et al. [16] into a shear-type structure while keeping the story heights of the original MRF and retrofitted it. . Their optimal retrofit includes changes in story mass, stiffness and damping.

The optimal retrofit of Cimellaro et al. [12] is analyzed here for "BO 10% in 50 years" ground motion records from PEER database: (http://nisee.berkeley.edu/data/strong_motion/sacsteel/motions/bo10in50yr.html) with a 2% inherent damping ratio assigned to the first and second modes according to Rayleigh classical damping. Table 2(A) shows selected results, for the 1st story the 9th story and relevant intermediate stories in terms of maximum interstory drifts, accelerations and shear forces.

The analysis-redesign procedure by Levy and Lavan [1] (described in section 3.3) for assigning added damping coefficients is applied to the optimal structure of Table 2(B) for an allowable interstory drift of 1.04% (which is the maximum value attained by Cimellaro et al. [12] optimal design) and convergence parameter $\mathrm{q} = 2$. The results which are presented in Table 2(C) show that the total damping was reduced by 90% for attaining the same maximum interstory drift. The maximum story accelerations and shear forces, however, increased by about 43% and 23% respectively.

The RAR method is now applied to the optimal structure of Cimellaro et al. [12] using the same steps as in previous example with weighting matrices of $\mathrm{Q}_1 = \mathrm{diag}\{15.0 \quad 15.0 \quad 15.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0\} \cdot 10^{13}$ and $\mathrm{Q}_2 = \mathrm{diag}\{15.0 \quad 15.0 \quad 15.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0\} \cdot 10^{13}$, an allowable drift of 1.04% and a convergence parameter of $\mathrm{q} = 2$. Selected results for this configuration are presented in Table 2(C). It is seen that the total sum of stiffness coefficients and damping coefficients decreased by 24% and 91%, respectively. In addition, the RAR results show roughly the same maximum acceleration and a decrease in maximum base shear (by 22%). The RAR algorithm produces a retrofit solution with improved maximum response and less added damping.

Table 1 –Maximum response of the optimal 6-DOF damped mass-spring model {Example 1}

Degree of Freedom	**Added Damping (kN s/m)**	**Stiffness (kN/m)**	**Maximum Drift (m)**	**Maximum Acceleration (g)**	**Maximum Force (kN)**
A: Optimal 6-DOF model by Takewaki [2000]					
0					**2,080**
1	1,800	87,500	**0.0237**	0.9009	1,927
2	1,500	82,500	0.0234	0.9341	1,686
3	1,300	72,500	0.0233	1.1131	1,379
4	1,220	60,000	0.0230	1.3038	964
5	1,220	42,500	0.0227	1.5379	422
6	1,300	18,000	0.0234	**1.7475**	0
SUM	**8,340**	**363,000**			
B: Retrofitted 6-DOF model using analysis-redesign procedure by Levy and Lavan [1]					
0					**2,074**
1	2,420	87,500	**0.0237**	0.9183	1,954
2	1,648	82,500	**0.0237**	0.9349	1,718
3	1,437	72,500	**0.0237**	1.1240	1,423
4	401	60,000	**0.0237**	1.3342	1,008
5	832	42,500	**0.0237**	1.6060	427
6	1,375	18,000	**0.0237**	**1.8299**	0
SUM	**8,114**	**363,000**			
C: Retrofitted 6-DOF damped mass-spring model using RAR procedure					
0					**1,667**
1	3,699	70,360	**0.0237**	0.8970	1,479
2	3,471	62,420	**0.0237**	0.7937	1,262
3	2,941	53,260	**0.0237**	0.9220	1,104
4	1,490	46,520	**0.0237**	1.0526	879
5	0	41,920	0.0210	1.2944	412
6	901	17,400	**0.0237**	**1.5222**	0
SUM	**12,501**	**291,880**			

Table 2 –Maximum response of optimal and retrofitted 9-story shear structure {Example 2}

Floor Number	Mass (kN s^2/m)	Added Damping (kN s/m)	Stiffness (kN/m)	Maximum Drift (%)	Maximum Acceleration (g)	Maximum Shear (kN)
A: Optimal 9-story shear structure by Cimellaro et al. [12]						
Ground						**920**
1	441	724	36,210	0.46%	**0.1063**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	361	290	16,890	**1.04%**	0.1037	694
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	350	1,353	18,210	0.30%	0.0657	219 0
SUM	**3,006**	**14,971**	**329,510**			
B: Retrofitted 9-story shear structure using analysis-redesign procedure by Levy and Lavan [1]						
Ground						**1,125**
1	441	0	36,210	0.57%	0.1178	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	361	964	16,890	**1.04%**	**0.1431**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	336	381	15,680	**1.04%**		
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	350	0	18,210	0.48%	0.0606 0.1012	348 0
SUM	**3,006**	**1,349**	**329,510**			
C: Retrofitted 9-story shear structure using RAR procedure						
Ground						**709**
1	441	0	31,650	0.41%	0.1058	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	361	1,267	7,460	**1.04%**	**0.1249**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	350	0	5,770	0.89%	0.0594	203 0
SUM	**3,006**	**1,267**	**245,930**			

5.3 Example 3: 10 story Moment Resisting Frame Structure

In this example a 10-story MRF structure previously analyzed by Levy et al. [14] is retrofitted using the proposed RAR methodology after its transformation into an equivalent shear structure. For applying the RAR method, the MRF structure is first converted to an equivalent shear structure according to the methodology that is described in section 4.

The 10-story MRF is first analyzed for the strong "SE 10% in 50 years" ground motion records (http://nisee.berkeley.edu/data/strong_motion /sacsteel/motions/se10in50yr.html). Their peak maximum response is shown in Table 3(A). The inherent damping coefficients were calculated using Rayleigh classical damping assuming 2% damping ratio for the first two modes.

Table 3 –Data and maximum response of original and retrofitted structures {Example 3}

Floor Number	**Added Damping (kN s/m)**	**Maximum Drift (%)**	**Maximum Story Acceleration (g)**	**Maximum Shear (kN)**
A: Original 10-story MRF structure by Levy et al. [14]				
Ground 1		**4.66%**	0.6330	**1,544**
⋮	⋮	⋮	⋮	⋮
9 Roof		1.55%	**1.2498**	659 0
B: Retrofitted 10-story MRF structure using analysis-redesign procedure by Levy and Lavan [1]				
Ground 1	14,803	**1.00%**	**0.6569**	491 **864**
⋮	⋮	⋮	⋮	⋮
3 4 5 6	2,533 4,659 932 2,160 682	**1.00%** **1.00%** **1.00%** **1.00%** **1.00%**	0.4008 0.3642 0.3219 0.2425	578 574 562 595
⋮	⋮	⋮	⋮	⋮
9 Roof	0	0.60%	0.4675	246 0
SUM	**25,770**			

Preliminary retrofit is first determined using the analysis-redesign procedure by Levy and Lavan [1] (described in section 3.3) applied to the MRF structure for an allowable interstory drift of 1.0% and convergence parameter of q=2. The results shown in Table 3(B) present the added damping coefficients and the resulting maximum interstory drifts, story accelerations and story shear forces.

When the RAR method is applied to the equivalent shear structure, using the weighting matrices $\mathbf{Q}_1 = \mathrm{diag}\{1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0\} \cdot 10^9$and
$\mathbf{Q}_2 = \mathrm{diag}\{1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 1.0 \quad 20.0 \quad 25.0 \quad 30.0 \quad 35.0\} \cdot 10^9$ for an allowable interstory drift of 1.0% and convergence parameter of $q = 2$, changes in added damping and in stiffness are determined (see Table 4). The changes obtained by RAR are added to the original MRF structure and the damping and stiffness matrices are updated for the suggested improved retrofit.

Table 4 -Data and maximum response of retrofitted 10-story MRF using RAR {Example 3}

Floor Number	Added Damping (kN s/m)	Added/Reduced Stiffness (kN/m)	Maximum Drift (%)	Maximum Story Acceleration (g)	Maximum Shear (kN)
Retrofitted 10-story MRF using RAR procedure					
Ground					504
1	17,582	-8,770	**1.00%**	**0.6809**	**787**
2	0	-1,630	0.87%	0.4531	690
3	1,216	-1,350	**1.00%**	0.4175	627
4	5,545	-2,420	**1.00%**	0.3756	608
5	66	-1,120	**1.00%**	0.3319	552
6	3,287	-1,960	**1.00%**	0.2677	567
7	907	-1,330	**1.00%**	0.2368	565
8	0	-150	0.93%	0.2434	438
9	0	150	0.82%	0.3848	242
10	0	290	0.59%	0.4576	0
SUM	**28,603**	**-18,290**			

Analysis results for the retrofitted MRF are also shown in Table 4. The results indicate that lowering the stiffness by 19.5% (according to the sum of stiffness coefficients in the equivalent shear structure) and increasing total added damping by 11% (comparing to the analysis-redesign of Levy and Lavan [2006]) a decrease in maximum story shear forces of 8.9% is achieved with roughly the same maximum story acceleration for the maximum interstory drift of 1%.

6. Conclusions

The proposed seismic retrofit method is aimed towards utilizing, enhancing and combining two design methodologies into one practical and robust approach for maximum structural robustness and retrofit of multistory frame structures using stiffness changes and added fluid viscous dampers.

Results of RAR showed superior retrofit results when compared to the other seismic design shear methods in terms of maximum response and sum of damping and stiffness coefficients. This is due to a central advantage of RAR which is the ability of the designer to “toy” with the weighting matrices so as to control the priorities in maximum seismic response and retrofit.

For the case of moment resisting frames (MRFs), the suggested novel approach requires first a conversion to an equivalent shear model before applying the RAR to determine the required stiffness changes and damping. Determining the equivalent shear model for the MRF is based on maximum response expressed by equal maximum interstory drifts for various ground acceleration records. The approach, proved efficient and sufficiently accurate in yielding optimal results for the retrofitted MRF structure. When this approach is combined to the RAR it produces structures with better performance by using less additional resources, such as stiffness and damping changes.

7. References

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