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# SIZE EFFECT ON THE ROTATIONAL CAPACITY OF REINFORCED CONCRETE BEAMS: NUMERICAL INVESTIGATION VS. EUROCODE 8

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## *Abstract*

The ductility is a very important parameter in the seismic design process of reinforced concrete structures. Under seismic loading, ductility is required to allow a significant deformation and energy dissipation before the collapse. The plastic design of beams under bending-loads is based on the formation of plastic hinges. The dissipation is characterized by the rotational-capacity of these zones. The nonlinear seismic analysis of reinforced concrete structures could be performed using a global approach based on the direct use of moment-curvature laws in zones of plastic hinges.

Concrete is known to be a quasi-brittle material. Its cracking behavior depends on the specimen's size. At the scale of engineering structures, the concrete has a brittle behavior due to the phenomenon of the size-effect. The design codes produce general empirical formulas fitted by test data. The reinforced concrete structures in daily practice are generally much larger than the testing laboratory samples. Do not take into account of the size-scale effect leads to an unsafe design. Numerical models are required to reproduce the effect of scale-change on the structural design.

In the context of the seismic design, the experimental observations confirm the dependence of the ductility on the structural dimension. Hence, the size-scale effect should be considered in the seismic design process. To establish the moment-rotation curves needed in dynamic non-linear analysis of RC structures, Eurocode 8 gives empirical formulas to provide the moment-rotation curves. The main disadvantage of using this practical design code is that the size effect is not taken into account in evaluating the rotational capacity which produces a bad estimation of the energy dissipation.

In this present paper, a global model is proposed in order to into account the phenomena of size effects and its influence on the ductility assessment. Both compression and tensile behavior are described by stress-strain curves where the localization process is taken into account. A comparison with the experimental results is firstly performed in order to validate the proposed model. Furthermore, a comparison between the EC8 and the proposed model regarding the diagram providing the rotational capacity of RC beams as a function of the relative neutral axis positions x/d is proposed.

*Keywords: Size effect, Rotation capacity, Ductility, EC8.*

## 1. Introduction

The ductility is a fundamental property required for safe design of reinforced concrete structures. It contributes to achieve different objectives: redistribution of internal solicitations, preventing unstable crack propagation... For seismic design, the ductility is required to enable major energy dissipation accompanied with large deformations in order to avoid a brittle failure of structures. From a practical point of view, in the context of a seismic design, such requirement is satisfied by complying with some empirical prescriptions provided by existing design codes (such as Eurocode8). Most of these empirical prescriptions are based on simplified theoretical approaches and/or experimental results.

For RC elements under bending-loads, the ductility is characterized by the rotation capacity of plastic hinges. Due to the complexity of the phenomenon, the assessment of the ductility is still difficult to achieve. Different non-linear mechanisms affect the global mechanical behavior: concrete-cracking in tension, steel behavior (yielding, bond…), concrete crushing. The rotation capacity of reinforced concrete elements has been the subject of numerous experimental investigations to study the influence of the aforementioned phenomena [3], [4]. The experimental observations confirm the dependence of the ductility on the structural dimension. Hence, the size-scale should be considered in the seismic design process.

The size-scale effect is mostly related to the localization process both in compressive and tension zones. Generally speaking, during the localized cracking process, the local behavior is governed by a dissipation of energy (per unit area) in response to a stress applied. The relationship between these two parameters (Energy / stress) leads to a size-dependency of the global behavior of the structures. Experimental results confirm the influence of the size effects on the rotational capacity of RC elements under bending. The "Overlapping Crack Model" developed by Carpinteri [5] is a pioneer model that has been developed to take into account the size effect in the ductility assessment.

In the present work, we propose a numerical global-model to take into account the influence of the size effect on the rotational capacity. In addition to the strain-localization phenomenon produced during the crushing of the concrete in compression, another phenomenon related to the strain-localization in the tension-cracked region is considered. In the context of a non-linear seismic design, the model is able to produce the curves Moment-Rotation necessary for seismic non-linear analysis. Based on the experimental results from the literature [6]. We also offer a practical comparison of the numerical results produced by the model with those provided by the design codes EC8.

In the next section, the formulas of EC8 used to evaluate the rotation capacity of RC element are presented. The fundamental and mathematical bases of the developed model are then discussed (concrete behavior, steel behavior). The work ends with a practical comparison and conclusions.

## 2. Rotational capacity in EC 8

The rotational capacity $\theta_{pl}$ is calculated as the difference between the ultimate rotation $\theta_u$ and the yielding rotation $\theta_y$:

$$\theta_{pl} = \theta_u - \theta_y \tag{1}$$

The rotation $\theta$ is defined as the integral of the curvature $\phi$ along the beam $l$:

$$\theta = \int_0^l \phi \, dx \tag{2}$$

The curvature is the deformation diagram-angle:

$$\phi = \frac{\varepsilon_c}{c} = \frac{\varepsilon_s}{d-c} = \frac{\varepsilon_c + \varepsilon_s}{d} \tag{3}$$

Where $\varepsilon_c$ and $\varepsilon_s$ are the strain of concrete and steel respectively, $d$ is the effective depth of the beam cross section, and $c$ is depth of the neutral axis.

When the demand of ductility in a structure is not uniform (for example in complex structures), advanced analysis methods should be considered. The simplified approached (static or spectral analysis) are no longer applicable. For nonlinear dynamic-analysis of structural elements under bending, the non-linearity of the structural elements is taken into account through the development of plastic hinges. The material nonlinearity in the hinges zones is introduced via the moment-curvature laws which are determined from the curvatures and moment-resistance formulas [1]

$$\phi_u = \frac{\varepsilon_s - \varepsilon_c}{d} \tag{4}$$

$$\phi_y = \frac{2.1\ \varepsilon_{sy}}{d} \tag{5}$$

From the moment-curvature laws, we obtain the moment-rotation curves by multiplying the curvature by the length of the plastic hinge. The EC8 adopts the formulation proposed by Panagiotakos and Fardis [2]. The rotational capacity of a reinforced concrete beam of length $L$ is given by the following formula:

$$\theta_{pl} = \left(\phi_u - \phi_y\right) L_p \left(1 - \frac{L_p}{L}\right) \tag{6}$$

In this formula, the influence of size effects for estimating the moment-rotation law is not considered.

Where $\phi_u$ and $\phi_y$ are the curvature ultimate and yielding respectively, $\varepsilon_{sy}$ is steel yielding strain, and $L_p$ is length of the plastic hinge.

## 3. The model basis

The proposed model is developed at the scale of a RC section. For a reinforced concrete beam, an element of this beam subjected to a bending moment is considered. This element has a length-to-height ratio equal to the unity and is assumed to be representative of the central zone of the beam where the plastic hinge is supposed to appear.

In this model, the main nonlinearities observed in a RC element are considered in order to obtain moment-rotation curves. The failure of a reinforced concrete beam could be caused either by the steel rupture (or slippage) or by the crushing of the concrete under compression. In what follows, each phenomenon is studied separately. The behavior laws of both steel and concrete materials, which introduce the influence of the size-scale effect, are developed. strain localization process is taken into account both in tension and compression.

3.1 Behavior of concrete under compression

Strain localization is a phenomenon that is associated with softening. It is well known that this phenomenon is more pronounced in tension than in compression. However, the softening character of the behavior of concrete under compression leads to a strain localization that should be considered in order to control the energy dissipation during the localization process. Experimentally, this phenomenon has been already observed [7]–[9]. Before the peak, the stress-strain is size-independent. However, the post-peak stress-strain regime is strongly dependent on the specimen-size. A RC failure caused by concrete-crushing could be reached before the steel rupture. This process is characterized by an energy-dissipation that should be taken into account to ensure an energy regulation. We propose to describe the behavior of concrete under compression by a stress-strain curve with three phases (Fig.1). The crushing energy $G_c$, is defined as the area under the post-peak stress-displacement curve. This energy is supposed to be a material parameter, ie, size-independent. The concrete behavior law is described by three phases (Fig. 1)

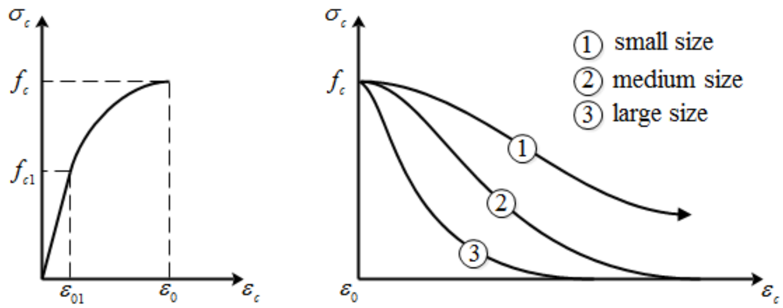



Fig. 1 – Stress-strain relationship for concerte compression

Phase 1 ($0 \rightarrow \varepsilon_{01}$): undamaged concrete, the behavior is elastic-linear

$$\sigma_c = E_c \varepsilon_c \tag{7}$$

Where $\sigma_c$ is the stress in compressive concrete and $E_c$ is concrete Young's modulus.

Phase 2 ($\varepsilon_{01} \rightarrow \varepsilon_0$): A ductile-damaged zone characterized by inelastic deformation. The behavior law is an equation described by a second-degree parabola.

$$\sigma_c = f_c \left[ \frac{2\varepsilon_c}{\varepsilon_0} - \left( \frac{\varepsilon_c}{\varepsilon_0} \right)^2 \right] + A_1 \left( \varepsilon_c - \varepsilon_{01} \right) - A_2 \tag{8}$$

Where $\varepsilon_{01}$ is the strain at the elastic limit of the concrete, $\varepsilon_0$ is the compressive strain of concrete at peak stress, $A_1$, and $A_2$ are constant to ensure continuity between the first and the second phase.

Phase 3 ($\varepsilon_0 \rightarrow \infty$): when exceeding the maximum stress, the crushing of concrete is associated with an energy dissipation $G_c$. The stress-strain relationship is described by an exponential equation.

$$\sigma_c = f_c Exp \left[ B \left( \varepsilon_0 - \varepsilon_c \right) \right] \tag{9}$$

$B$ is a parameter controlling softening. It is related to the size of the element via the following equations:

$$G_c = \int_{\varepsilon_0}^{\infty} \sigma_c dw \tag{10}$$

$$G_c = h \int_{\varepsilon_0}^{\infty} \sigma_c d\,\varepsilon_c \Rightarrow \quad G_c = h \int_{\varepsilon_0}^{\infty} f_c Exp \left[ B \left( \varepsilon_0 - \varepsilon_c \right) \right] d\,\varepsilon_c \Rightarrow \quad G_c = h\, f_c \left[ \frac{-Exp \left[ B \left( \varepsilon_0 - \varepsilon_c \right) \right]}{B} \right]_{\varepsilon_0}^{\infty} \tag{11}$$

It should be noted that the parameter $G_c$ represents only the crushing energy (we adopted this definition to identify dissipation under the softening part). We could also rewrite the equations in terms of the total dissipated energy (area under the total stress-strain curve).

3.2 Steel-concrete behavior in the tensile zone

Concerning the tensile part, we adopt a simplified behavior law for steel (elastic-perfectly plastic). The possibility of strain localization in the steel near the cracked regions is taken into account.

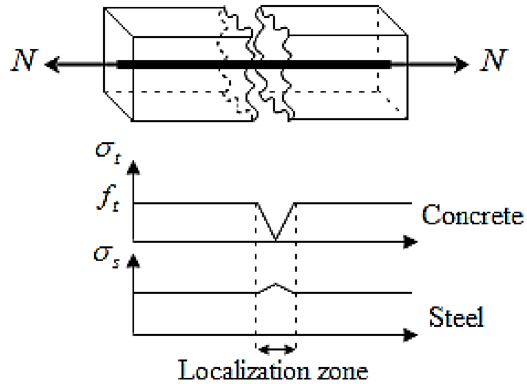



Fig. 2 – Stress distribution in steel and concrete after the first crack

In this zone (Fig. 2), the increase in stresses implies an increase of strains in the steel, the strain in these regions exceeds the average strain in the steel bar, this phenomenon is most remarkable when the behavior of the armatures is elastic-perfectly plastic [11]. In this case, when the maximum stress $f_y$ is achieved in the steel, the strain in the cracked zones instantly rises and reaches its ultimate value. The failure of the bar is reached.

The phenomenon of strain localization in the steel is related to the slip which occurs between the concrete and steel in the cracked regions. In order to highlight this phenomenon, an analysis of a Tie-beam element subjected to tensile loading (see Fig. 3) is presented.

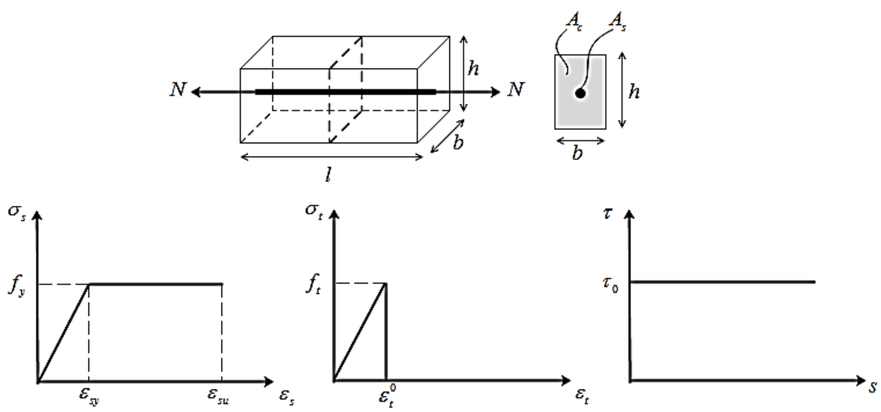



Fig. 3 – Concrete-Steel behavior

After the elastic behavior, cracks in concrete and slippage occur with a redistribution of stresses and strains. By increasing the applied strain, the steel yielding is reached in the cracked section in the middle-section. The evolution of stresses and strains in the beam element is given by the resolution of the two following equilibrium conditions: equilibrium of the section at mid-span (Eq. (12)), and equilibrium of the steel bar (Eq. (13)). A simplified bond law is used.

$$A_s\sigma_s + A_c\sigma_t = N \tag{12}$$

$$A_s d\sigma_s = -\tau\,\pi\,d_s dx \tag{13}$$

Where $A_s$ and $A_c$ is the area of the steel and concrete respectively, $d_s$ is the rebar diameter, $\sigma_t$ is the stress in the tensile concrete, $\sigma_s$ is the stress in the tensile steel, and $\tau$ is the bond stress.

$$\frac{d\sigma_s}{dx} = \frac{-\tau\,\pi\,d_s}{A_s} = \frac{-4\tau}{d_s} \tag{14}$$

The average ultimate strain $\varepsilon_m^u$ in the steel over a length $l/2$ corresponding to the ultimate strain $\varepsilon_{su}$ along the length of the strain localization $l_{\max}$ , it is defined by the following relationship:

$$\varepsilon_m^u \frac{l}{2} = \varepsilon_{su} l_{max} \tag{15}$$

$$\varepsilon_m^u = \frac{2l_{max}}{l} \varepsilon_{su} \tag{16}$$

$l_{\max}$ is determined from Eq. (14) where: $\sigma_s = f_y$ ( $f_y$ is the steel yielding stress)

$$\frac{f_y}{l_{max}} = \frac{4\tau}{d_s} \Rightarrow l_{max} = \frac{f_y d_s}{4\tau} \tag{17}$$

Replacing $l_{\max}$ in Eq. (16):

$$\varepsilon_m^u = \frac{d_s}{2l\tau} f_y \varepsilon_{su} \tag{18}$$

The parameter $l\tau$ is determined by the equilibrium condition of the bar, when $\varepsilon = \varepsilon_t^0$ (formation of the first crack) in the mid-span section $(x = l/2)$:

$$l\tau = \frac{A_c f_t d_s}{2A_s} \tag{19}$$

So, the average ultimate strain $\varepsilon_m^u$ is related to the ultimate strain $\varepsilon_{su}$ by the following relationship:

$$\varepsilon_m^u = \frac{A_s f_y}{A_c f_t} \varepsilon_{su} \tag{20}$$

Eq. (20) takes into account the phenomenon of strain localization in the steel bar.

3.3 Global behavior of the beam

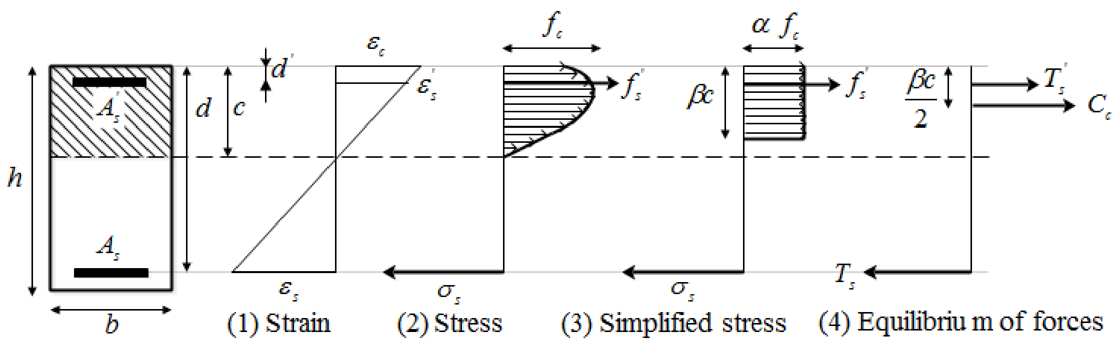



Fig. 4 – The distribution of strains and stresses in the mid-span section

To calculate the rotational capacity of the RC element, the rotation is calculated from the curvature. The loading in the mid-span section is driven by increasing the strain of the concrete. At each stage, the values of the moment and curvature are obtained by solving the equations of equilibrium (Fig. 4-4). This process is iterative. For each value of strain, we determine the position of the neutral axis and calculating the strain in the steel ($\varepsilon_s$ and $\varepsilon_s'$). Thus, plastic deformations are propagated over the section until reached the failure either by steels rupture or by concrete crushing. The structure of the proposed model is given in Fig 5.

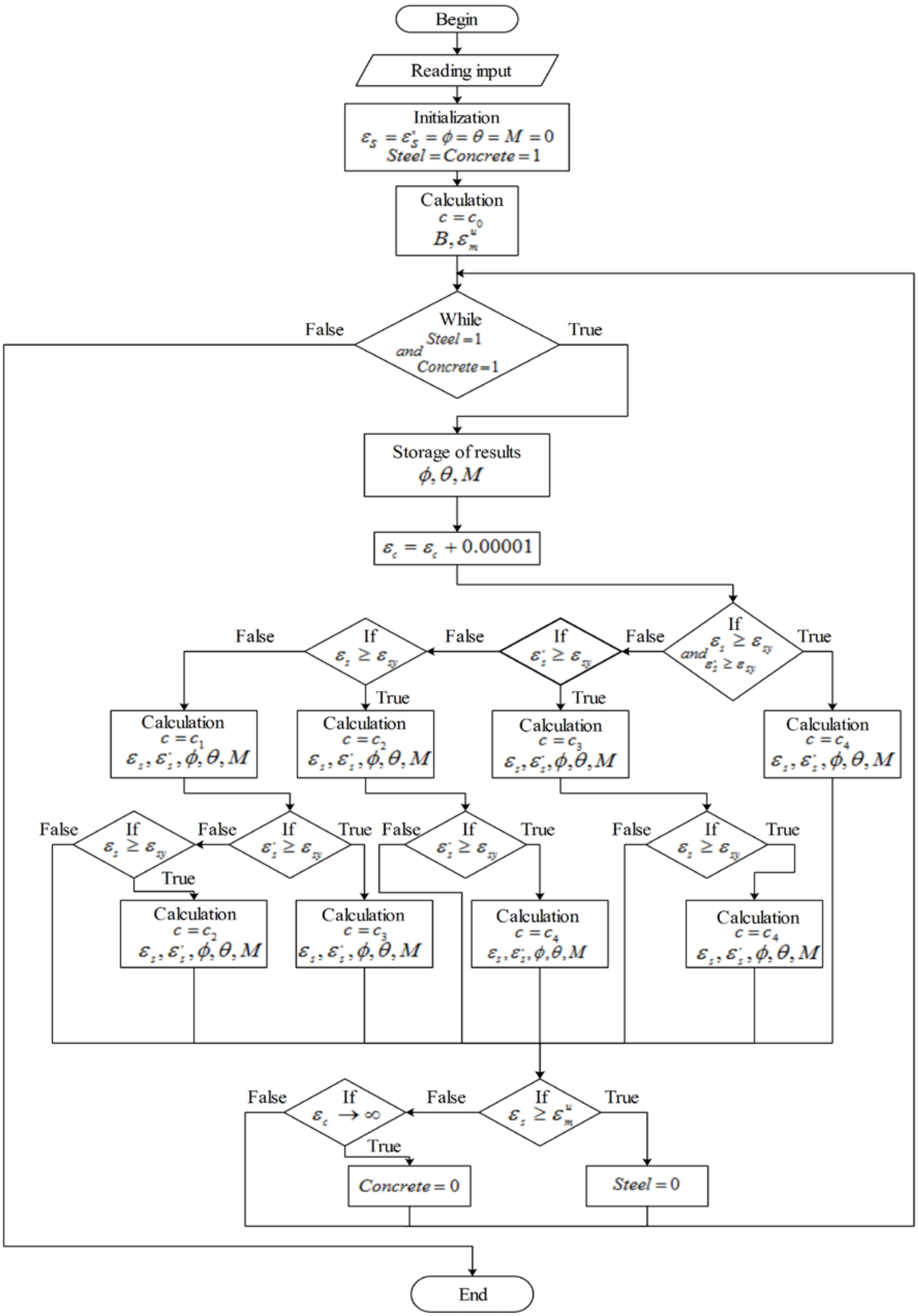



Fig. 5 – Diagram of the Model and Algorithm

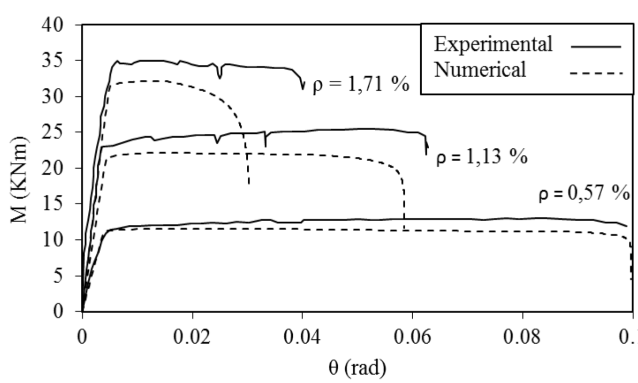
## 4. Validation

A set of experimental results has been reported by Bosco and Debernardi [6]. Eleven RC beams were tested under bending loading and the rotational-capacity curves (Moment-rotation curves) were given. In order to validate the numerical model proposed in the present paper a comparison between the numerical results and the experimental ones is proposed. Table 1. gives the data related to the eleven beams tested experimentally.

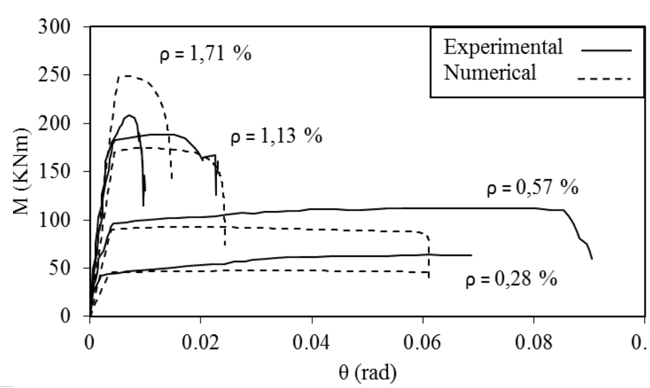
Table 1 – Data relating to the beams tested by Bosco and Debernardi [6]

| Beams | L(m) | h (m) | b (m) | ρ (%) | ρ' (%) |
|---|---|---|---|---|---|
| T1 | | | | 0.57 | 0.2 |
| T2 | 2 | 0.2 | 0.1 | 1.13 | 0.5 |
| T3 | | | | 1.71 | 0.5 |
| T4 | | | | 0.28 | 0.2 |
| T5 | 4 | 0.4 | 0.2 | 0.57 | 0.2 |
| T6 | | | | 1.13 | 0.2 |
| T7 | | | | 1.71 | 0.2 |
| T8 | | | | 0.13 | 0.12 |
| T9 | 6 | 0.6 | 0.3 | 0.25 | 0.12 |
| T10 | | | | 0.57 | 0.12 |
| T11 | | | | 1.13 | 0.12 |

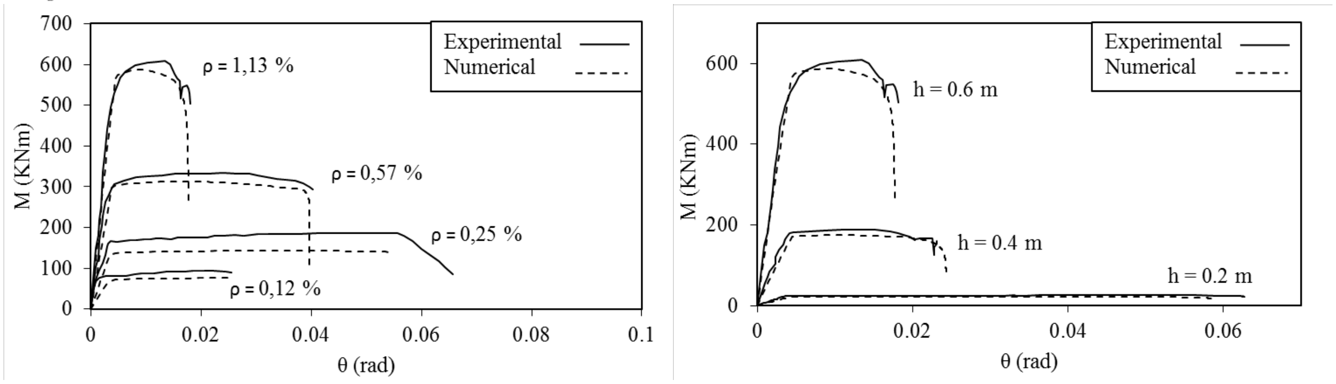
The steel used is a B500H (high ductility). For the concrete characteristics, the maximum tensile stress is about 2.97 MPa and the compressive one is 30.9 MPa.




**a-** h = 0.2m




**b-** h = 0.4m




**c-** h = 0.6m **d-** ρ = 1.13%

Fig. 6 – Comparison between experimental and numerical results.

The numerical moment-rotation curves obtained from the proposed model are compared with the experimental results in Figure 6 for different beam heights and different reinforcement ratio. For a small ratio, the final collapse of beams is caused by steel rupture. For the beams of 0.4m and 0.6m height, the maximum ductility is achieved for 0.57% reinforcement ratio and 0.25% respectively. These values correspond to optimum percentages (minimum ratio). A lower percentage results in a decrease of the rotational capacity because of the strain localization phenomenon near the cracked zones (unstable crack propagation). The model is able to correctly reproduce this phenomenon. For all the beams of different heights, exceeding the optimum percentage leads to over-reinforced beams where the final collapse at ultimate limit is caused inevitably by the crushing of concrete. The compression-failure which characterizes over-reinforced beams occurs without warning. The higher the beam, the brittle the behavior.

Fig. 6-d clearly shows the influence of the size-effect on the rotational capacity. The moment-rotation curves of beams with different height (h varies from 0.2 to 0.6 m) and a unique reinforcement ratio (1.13%) are compared. Comparing with the experimental data, the model correctly reproduces the brittle behavior of large beams. To illustrate these conclusion, the diagram providing the rotational capacity of RC beams as a function of the relative neutral axis positions x/d is proposed in Fig. 7. This diagram-type is often used by Codes provisions for assessing the admissible rotational capacity.

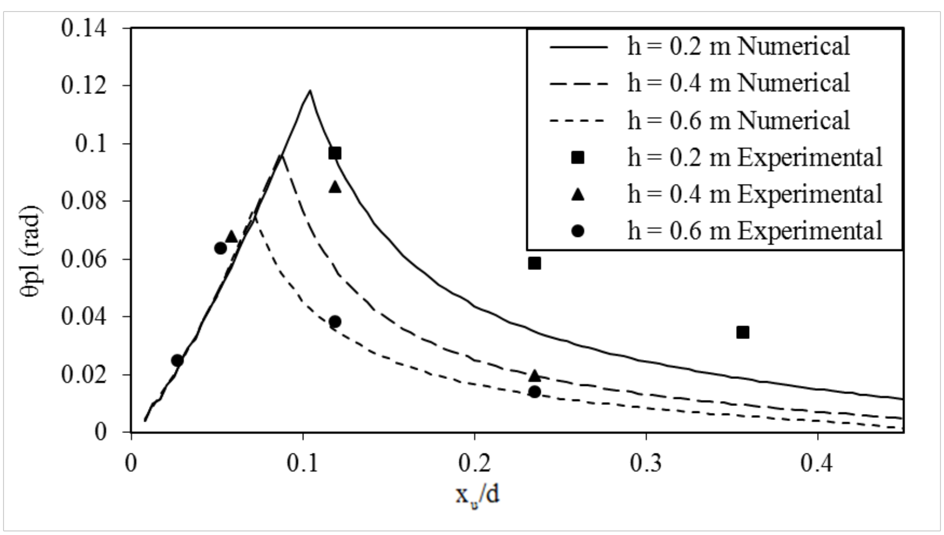



Fig. 7 – Plastic rotation Vs relative neutral axis position : Experimental Vs Numerical results

## 5. Model Vs. EC 8

The numerical results of the proposed model have been compared with the experimental ones in the previous section. Another comparison is proposed with the EC8. Based on the diagram given in Figure 7, the evolution of the plastic rotation as a function of the relative neutral axis predicted by the EC8 is compared with the numerical result. Figure 8 shows that in the zone where the behavior is driven by the steel rupture, the EC8 overestimates the rotational capacity. In fact, the strain localization near cracked region causes a decrease in the average strain of the bar. EC8 fixes the steel failure strain at 7.5% which leads to this overestimation. On the other hand, the transition zone is not well simulated which generates a bad estimate of the rotational capacity in the region characterized by a compressive failure due to concrete crushing.

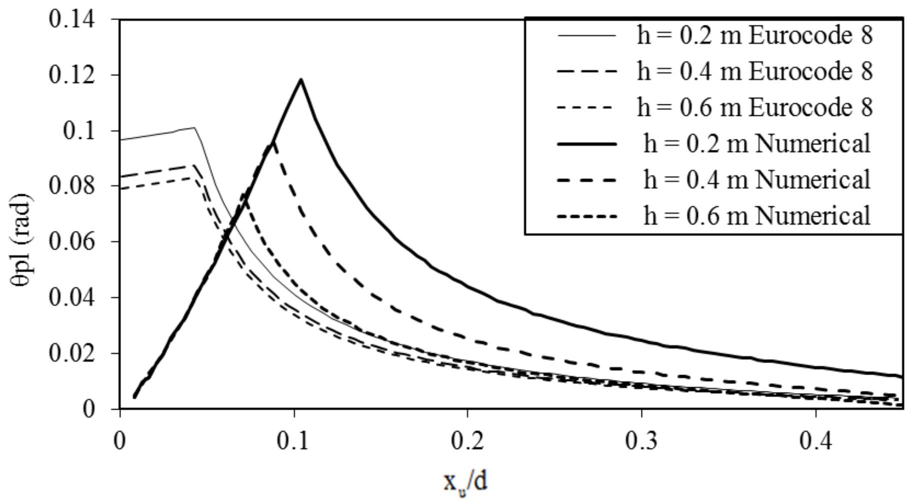



Fig. 8 – Eurocode 8-Numerical comparison

## 6. Conclusion

In this present paper, a “global” model has been proposed to describe the rotation capacity of reinforced concrete beams. The localizations process in the compressive and the tensile regions have been taken into account. The influence of the size effects on the evolution of the rotational capacity (ductility) is reproduced correctly. For reinforced concrete beams under bending loads, a lower reinforcement leads to unstable crack propagation, in the other hand; a high ratio leads to over-reinforced beams where the final collapse is caused inevitably by the concrete crushing in the compressive zone which generates a size dependency on the beam height. The model proposed is able to reproduce this phenomenon. However, the formula proposed in EC8 to assess the rotational capacity must evolve to take into account the size effect and ensure a good prediction of the Moment-rotation curves needed for seismic analysis of reinforced concrete structures.

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