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OPTIMAL GAINS IN MASS, STIFFNESS AND ADDED DAMPING FOR SEISMIC UPGRADE OF FRAME STRUCTURES

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Abstract

A novel approach for the seismic retrofit of linear frame structures is presented and described. Use is made of optimal control theory to attain structural changes in stiffness coefficients, changes in mass and supplemental damping. An optimization problem that is required to satisfy constraints on maximum total story accelerations and maximum interstory is formulated. The general system interconnections paradigm is introduced as a closed-loop control system with a passive controller that consists of the structural changes and has simulated feedback on the structural response and seismic excitation. The cost function minimizes the H_∞ norm of the maximum closed-loop response, single-input single-output (SISO) transfer function case. The first-order steepest descent of Apkarian and Noll [1] is introduced, utilized and enhanced to solve the optimization problem. A numerical example of a 9-story shear-type building is studied. Optimal changes in mass, stiffness and damping that are obtained, show significant improvement in the peak dynamic response. The results clearly indicate the efficiency of the proposed methodology that possesses the capability of attaining optimal changes in all of the structure’s physical characteristics (mass, stiffness, damping) while adhering to preassigned maximum response levels.

Keywords: Earthquake engineering; upgrade of structures; changed mass damping and stiffness; H-infinity norm

1. Introduction

The field of optimal control offers solution procedures for optimizing the performance of a control system (typically by minimizing a cost function), subjected to equilibrium and stability of the system. Optimizing changes in structural properties, that are considered as the controller gain matrix, and satisfy the equation of motion and allowable response constraints; fall within the realm of such problems. Soong [2] addressed the minimization of the Linear Quadratic Regulator (LQR) and showed the resemblance between the passive control gain to changing the stiffness and damping of a structure. Gluck et al [3] used his solution procedure to develop a method which handles simultaneous changes in story stiffness and damping by applying the least squares approximation approach to the displacement and velocity related passive gain matrices individually. Agrawal and Yang [4] followed suit and introduced an iterative procedure for minimizing the LQR cost function's gradient so as to calculate the optimal passive gain matrix. Their procedure can also apply to changes in story stiffness and linear damping. The work of Cimellaro et al. [5] considered mass changes as well. They applied the technique of Smith et al. [6] to the LQR gain matrix.

Other optimization problems deal with the minimization of system norms. The solution procedures for these types of problems have also been implemented for upgrading of structures. Baratta et al. [7] showed that the squared maximum displacement response of a single-degree-of-freedom (SDOF) structure is bounded by the multiplication of the second-norm of the excitation force and the second-norm of the Dirac-impulse response. They formulated the parametric response solution for the structure and use "random walk" in order to find the optimal solution for free-vibration radial frequency and damping ratio, which are assigned to the structure as the passive control gain. In case of multi-degree-of-freedom (MDOF) structures, they use modal analysis and apply their procedure to each mode individually. Bai et al. [8] proposed an approach for calculating and assigning linear viscous dampers to frame structures that are subjected to harmonic vibrations. They address the H_∞ norm of the transfer function for the structure (from applied force into velocities) and prove that it has smaller norm than the inverse of the symmetric added damping matrix, an inequality that allows them to achieve a required H_∞ norm from the damped structure and thus, reduce maximum velocity response due to earthquakes. Lin et al. [9] suggest imposing a H_∞ norm value so as to minimize the entropy of the closed-loop transfer function matrix, which bounds the H_2 norm of a system. It is explained that minimizing the entropy term would also push down the eigenvalues of the transfer function and thus, result in a lower H_∞ norm as well. The Langrage multipliers are utilized and applied to stability constraints in order to minimize the entropy term and calculate the optimal gain matrix which consists of damping and stiffness changes. The above methodologies provide numerical examples of shear-type buildings that show in their results improved peak dynamic response (to some extent). While this may be the case, they do not impose a constraint on the peak response quantities in their problem formulation and solution procedure.

In this paper, we introduce an approach for upgrading linear frame structures by assigning structural changes in mass, damping and stiffness while limiting peak response of total story accelerations and interstory drifts. In our formal optimization problem, the proposed objective function is subjected to constraints on the peak response quantities. For solving our optimization problems, we utilize and enhance the first-order steepest descent algorithm of Apkarian and Noll [1].

2. METHODOLOGY

The following section describes the developed methodology for upgrading linear frame structures by containing their peak response to given allowable values through attaining optimal changes in structural properties using optimal control theory. A unique control system is proposed and addressed within a formal optimization problem

2.1 Constraint Requirements

The concern of this paper lies in redistributing the story mass, changing story stiffness and adding linear viscous dampers, i.e. the structural changes. The structure is required to satisfy the controlled equation of motion:

$$\begin{aligned}&\mathbf{M}\ddot{\mathbf{x}}(t)+\mathbf{C}\dot{\mathbf{x}}(t)+\mathbf{K}\,\mathbf{x}(t)=-\mathbf{M}\boldsymbol{\iota}a_g(t)+\mathbf{u}(t)\\&\mathbf{x}(0)=0,\ \dot{\mathbf{x}}(0)=0\\&\text{where:}\\&\mathbf{u}(t)=-\Delta\mathbf{M}\boldsymbol{\iota}a_g(t)+\mathbf{u}_\Delta(t)\\&\mathbf{u}_\Delta(t)=-\Delta\mathbf{M}\,\ddot{\mathbf{x}}(t)-\Delta\mathbf{C}\,\dot{\mathbf{x}}(t)-\Delta\mathbf{K}\,\mathbf{x}(t)\end{aligned}\tag{1}$$

where $\mathbf{M}$, $\mathbf{C}$ and $\mathbf{K}$ are the mass, inherent damping and stiffness matrices of the structure respectively. The vectors $\mathbf{x}(t)$, $\dot{\mathbf{x}}(t)$ and $\ddot{\mathbf{x}}(t)$ are the relative to ground displacements, velocities and accelerations of the story floors (respectively), $\boldsymbol{\iota}$ is the influence vector and $a_g(t)$ is the ground motion record. The vector $\mathbf{u}(t)$ contains the control forces induced by the structural changes. These structural changes are the supplemental damping, $\Delta\mathbf{C}$, changes in mass and stiffness matrices, $\Delta\mathbf{M}$ and $\Delta\mathbf{K}$, having the following form:

$$\begin{aligned}&\Delta\mathbf{M}=\mathrm{diag}\{\Delta m_1,\dots,\Delta m_N\}\\&\Delta\mathbf{C}=\mathbf{T}^T\Delta\mathbf{C}_d\mathbf{T},\ \Delta\mathbf{C}_d=\mathrm{diag}\{\Delta c_1,\dots,\Delta c_N\}\\&\Delta\mathbf{K}=\mathbf{T}^T\Delta\mathbf{K}_d\mathbf{T},\ \Delta\mathbf{K}_d=\mathrm{diag}\{\Delta k_1,\dots,\Delta k_N\}\\&\mathbf{T}=\begin{bmatrix}1&&&\\-1&1&&\\&\ddots&\ddots&\\&&-1&1\end{bmatrix}\end{aligned}\tag{2}$$

Where $\mathbf{T}$ is a transformation matrix, from displacements into drifts coordinates, and $[\cdot]^T$ denotes the transpose of a matrix. The parameters Δm_n, Δc_n and Δk_n are the assigned changes of mass, damping and stiffness to the n^{th} story respectively. The total story accelerations $\mathbf{a}^{tot}(t)$ and interstory drifts $\boldsymbol{\delta}(t)$ become:

$$\mathbf{a}^{tot}(t)=\ddot{\mathbf{x}}(t)+\boldsymbol{\iota}a_g(t)\tag{3}$$

$$\boldsymbol{\delta}(t)=\mathbf{T}\,\mathbf{x}(t)\tag{4}$$

The peak response quantities of the controlled structure are to satisfy the following behavioral constraints:

$$\left.\begin{aligned}&\max_t\{|a^{tot}{}_n(t)|\}/a^{all}{}_n\le 1.0\\&\max_t\{|\delta_n(t)|\}/\delta^{all}{}_n\le 1.0\\&\forall\ n=1,\dots,N\end{aligned}\right\}\forall a_g(t)\tag{5}$$

Where $\boldsymbol{\delta}^{all}$ and $\mathbf{a}^{all}$ are the allowable interstory drifts and total story accelerations vectors.

2.2. General System Interconnections

The general system interconnection is a paradigm which defines the transfer function interconnections, within a control system of some kind, between the external input/controller input, and the regulated outputs/simulated feedback. The scheme in Fig 1 describes the proposed general system interconnection paradigm.

In Fig 1, $\mathbf{Q}_\Delta$ is the controller (gain matrix), the four partitions $\mathbf{G}_{11}(s)$, $\mathbf{G}_{12}(s)$, $\mathbf{G}_{21}(s)$ and $\mathbf{G}_{22}(s)$ are the transfer function matrices that connect the external input vector $\boldsymbol{\omega}_1(s)$ and the controller's input $\boldsymbol{\omega}_2(s)$ to the regulated outputs vector $\mathbf{y}_1(s)$ and the simulated feedback vector $\mathbf{y}_2(s)$. Consequently, their linear connectivity is given by:

$$\begin{aligned}&\mathbf{y}_1(s)=\mathbf{G}_{11}(s)\boldsymbol{\omega}_1(s)+\mathbf{G}_{12}(s)\boldsymbol{\omega}_2(s)\\&\mathbf{y}_2(s)=\mathbf{G}_{21}(s)\boldsymbol{\omega}_1(s)+\mathbf{G}_{22}(s)\boldsymbol{\omega}_2(s)\\&\text{and:}\\&\boldsymbol{\omega}_2(s)=\mathbf{Q}_\Delta\,\mathbf{y}_2(s)\end{aligned}\tag{6}$$

Where $\mathbf{G}(\mathrm{s})$ is the generalized plant transfer function matrix. The closed-loop transfer function matrix for the general system ($\mathbf{CL}_{\omega_1\rightarrow y_1}(\mathrm{s})$), which is the linear relation between $\boldsymbol{\omega}_1(\mathrm{s})$ to $\mathbf{y}_1(\mathrm{s})$, is formulated by substituting the control law $\boldsymbol{\omega}_2(\mathrm{s}) = \mathbf{Q}_\Delta\, \mathbf{y}_2(\mathrm{s})$ into $\mathbf{y}_1(\mathrm{s})$ and $\mathbf{y}_2(\mathrm{s})$ terms of Eq. (6) and then substituting the new term of $\mathbf{y}_2(\mathrm{s})$ into $\mathbf{y}_1(\mathrm{s})$ to yield:

$$\begin{aligned} &\mathbf{y}_1(\mathrm{s}) = \mathbf{CL}_{\omega_1\rightarrow y_1}(\mathrm{s})\, \boldsymbol{\omega}_1(\mathrm{s}) \\ &\text{where:} \\ &\mathbf{CL}_{\omega_1\rightarrow y_1}(\mathrm{s}) = [\mathbf{G}_{11}(\mathrm{s}) + \mathbf{G}_{12}(\mathrm{s})\mathbf{Q}_\Delta[\mathbf{I} - \mathbf{G}_{22}(\mathrm{s})\, \mathbf{Q}_\Delta]^{-1}\, \mathbf{G}_{21}(\mathrm{s})] \end{aligned} \tag{7}$$

The particular transfer function matrix $\mathbf{P}(\mathrm{s})$ of Fig 1 is related to the common transfer matrix $\mathbf{H}(\mathrm{s})$, used to solve the equation of motion in its state space representation using the Laplace transform, denoted by $\mathcal{L}[\cdot]$, through:

$$\begin{aligned} &\mathbf{P}(\mathrm{s}) = \boldsymbol{\Lambda}\mathbf{H}(\mathrm{s}) \\ &\text{where:} \\ &\boldsymbol{\Lambda} = [\mathbf{I} \quad \mathbf{0}] \\ &\mathbf{H}(\mathrm{s}) = (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{B} \end{aligned} \tag{8}$$

More specifically, for excitation force $\mathbf{F}(\mathrm{s}) = \mathcal{L}[\mathbf{f}(\mathrm{t})]$ the transfer function matrix is $\mathbf{H}(\mathrm{s}) = (\mathrm{s}\mathbf{I} - \mathbf{A})^{-1}\, \mathbf{B}$ so that $\mathbf{Z}(\mathrm{s}) = \mathbf{H}(\mathrm{s})\mathbf{F}(\mathrm{s})$ and $\mathbf{z}(\mathrm{t}) = \mathcal{L}^{-1}[\mathbf{Z}(\mathbf{s})]$. The state space representation of the system is defined as:

$$\begin{aligned} &\dot{\mathbf{z}}(\mathrm{t}) = \mathbf{A}\, \mathbf{z}(\mathrm{t}) + \mathbf{B}\, \mathbf{f}(\mathrm{t}) \\ &\text{where:} \\ &\mathbf{z}(\mathrm{t}) = [\mathbf{x}(\mathrm{t}) \quad \dot{\mathbf{x}}(\mathrm{t})]^{\mathrm{T}};\ \mathbf{z}(\mathbf{0}) = [\mathbf{0} \quad \mathbf{0}]^{\mathrm{T}} \\ &\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix};\ \mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}; \end{aligned} \tag{9}$$

The external input $\omega_1(\mathrm{s})$ is scalar and is the ground acceleration $\mathrm{a_g}(\mathrm{t})$ in the "S" domain. The ground acceleration is the excitation force of the system that introduces the force $\mathbf{f}(\mathrm{t})$ so that $\mathbf{f}(\mathrm{t}) = -\mathbf{M}\boldsymbol{\iota}\mathrm{a_g}(\mathrm{t})$. The controller, $\mathbf{Q}_\Delta$, receives $\mathbf{y}_2(\mathrm{s})$ as negativity feedback of $\ddot{\mathbf{x}}(\mathrm{t})$, $\dot{\mathbf{x}}(\mathrm{t})$, $\mathbf{x}(\mathrm{t})$ and $\boldsymbol{\iota}\mathrm{a_g}(\mathrm{t})$ in the "S" domain (i.e. $\mathrm{s}^2\mathbf{X}(\mathrm{s})$, $\mathrm{s}\mathbf{X}(\mathrm{s})$, $\mathbf{X}(\mathrm{s})$ and $\boldsymbol{\iota}\mathcal{L}\left[\mathrm{a_g}(\mathrm{t})\right]$ respectively). These feedbacks are multiplied by the proposed controller's partitions so that it produces the controller's input $\boldsymbol{\omega}_2(\mathrm{s})$ of "$-\Delta\mathbf{M}\boldsymbol{\iota}\mathrm{a_g}(\mathrm{t})$" and $\mathbf{u}_\Delta(\mathrm{t})$ in the "S" domain (i.e. $-\Delta\mathbf{M}\boldsymbol{\iota}\mathcal{L}\left[\mathrm{a_g}(\mathrm{t})\right]$ and $\mathbf{U}_\Delta(\mathrm{s})$ respectively). We note that the proposed passive controller $\mathbf{Q}_\Delta$ is unique in that it receives feedback on structural response as well as dynamic excitation. The regulated outputs are simply $\boldsymbol{\delta}(\mathrm{t})$ and $\mathbf{a}^{\mathrm{tot}}(\mathrm{t})$ in the "S" domain (i.e $\Delta(\mathrm{s})$ and $\mathbf{A}^{\mathrm{tot}}(\mathrm{s})$ respectively).

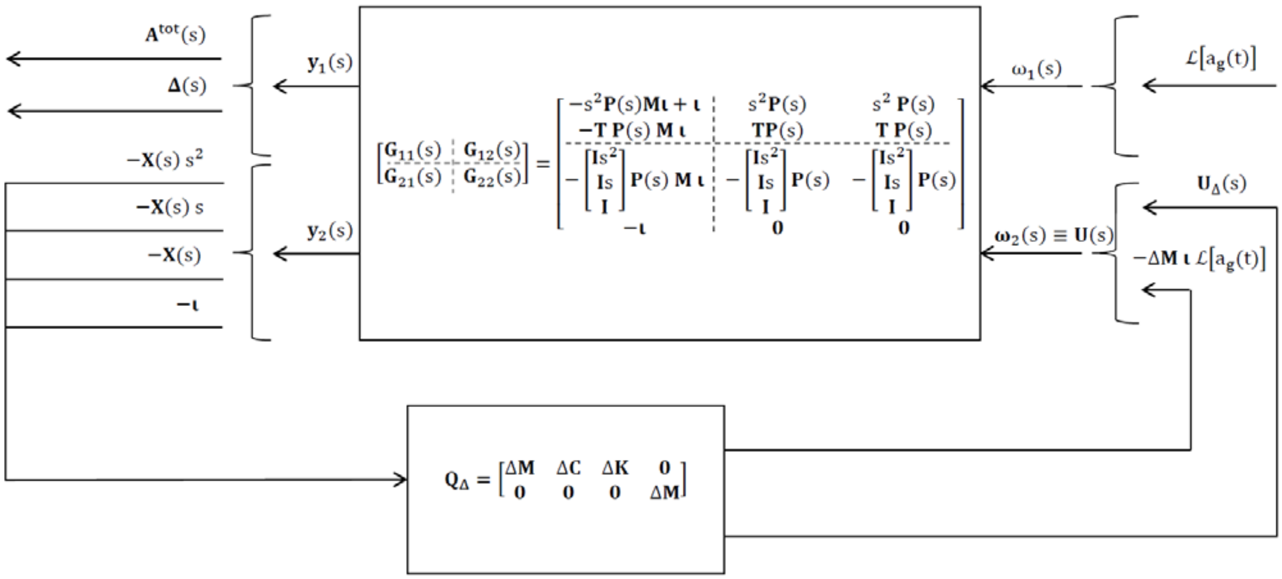

Fig 1. The proposed general system interconnections

Substituting $\mathbf{G}_{11}(s)$, $\mathbf{G}_{12}(s)$, $\mathbf{G}_{21}(s)$, $\mathbf{G}_{22}(s)$, $\mathbf{Q}_\Delta$ and $\omega_1(s)$ into Eq. (7) yields the following linear relations, in the "S" domain, between $\mathcal{L}[a_g(t)]$ and the response of $\mathbf{A}^{tot}(s)$ and $\Delta(s)$:

$$\begin{aligned}
&\mathbf{A}^{tot}(s) = s^2\mathbf{P}(s)\left(-\mathbf{M}\iota\mathbf{L}[a_g(t)] + \mathbf{U}(s)\right) + \iota\,\mathcal{L}[a_g(t)] \\
&\Delta(s) = \mathbf{T}\,\mathbf{P}(s)\left(-\mathbf{M}\iota\mathcal{L}[a_g(t)] + \mathbf{U}(s)\right) \\
&\text{where:} \\
&\mathbf{U}(s) = -\Delta\mathbf{M}\iota a_g(t) + \mathbf{U}_\Delta(s) \\
&\mathbf{U}_\Delta(s) = -\Delta\mathbf{M}\,s^2\mathbf{X}(s) - \Delta\mathbf{C}\,s\mathbf{X}(s) - \Delta\mathbf{K}\,\mathbf{X}(s) \\
&\mathbf{X}(s) = \mathbf{P}(s)\left(-(\mathbf{M}+\Delta\mathbf{M})\iota\mathcal{L}[a_g(t)]\right)
\end{aligned} \tag{10}$$

We note that the terms of Eq. (10) is in essence the Laplace transform of Eq. (1), while considering the terms of interstory drifts and total story acceleration (given in Eq. (3) and (4) respectively). Consequently, the closed-loop transfer function matrix $\mathbf{CL}_{\omega_1\to y_1}(s) = \mathbf{CL}_{L[a_g(t)]\to[\mathbf{A}^{tot}(s)\,;\,\Delta(s)]^T}(s)$, which will be henceforth denoted as $\mathbf{CL}(s)$, takes the explicit form of:

$$\mathbf{CL}(s) = \begin{bmatrix} -s^2\,\mathbf{P}(s)\left((\mathbf{M}+\Delta\mathbf{M})\,\iota + (\Delta\mathbf{M}\,s^2 + \Delta\mathbf{C}\,s + \Delta\mathbf{K})\mathbf{P}(s)\mathbf{M}\iota\right) + \iota \\ -\mathbf{T}\,\mathbf{P}(s)\left((\mathbf{M}+\Delta\mathbf{M})\,\iota + (\Delta\mathbf{M}\,s^2 + \Delta\mathbf{C}\,s + \Delta\mathbf{K})\mathbf{P}(s)\mathbf{M}\iota\right) \end{bmatrix} \tag{11}$$

In the formulation of the formal optimization problem below, the H_∞ norm of $\mathbf{CL}(s)$ will be taken as the objective function and the solution of that problem will yield the optimal design variables Δm_n, Δc_n and Δk_nthat construct $\Delta\mathbf{M}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$ according to Eq. (2).

2.3 Formal Optimization Problem

The seismic upgrade in this paper involves in calculating the components of $\Delta\mathbf{M}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$ in order to attain peak response lower than allowable quantities of total story accelerations and interstory drifts. This is regarded by the constraints of our proposed optimization problem:

$$\left.\begin{array}{l}\underset{\Delta m_n,\Delta c_n,\Delta k_n}{\text{minimize}}\left\{J = \max_{i\in[1,\dots,2N]}\{\gamma_i\|CL_i(s)\|_\infty\}\right\}\\ s.t\\ \max_t\{|a^{tot}{}_n(t)|\}/a^{all}{}_n \le 1.0\\ \max_t\{|\delta_n(t)|\}/\delta^{all}{}_n \le 1.0\\ \forall\, n = 1,\dots,N\end{array}\right\}\forall a_g(t) \quad (12)$$

The first “N” responses ($CL_i(s){:}\, i = 1,\dots,N$) are the total story accelerations and the next “N” responses ($CL_i(s){:}\, i = N+1,\dots,2N$) are interstory drifts. The value for $\|CL_i(s)\|_\infty$, is the “peak response” factor for the i^{th} dynamic response and is calculated by substituting $s = j\omega$ (“j” denotes imaginary number) into the i^{th} term of $\mathbf{CL}(s)$, from Eq. (11), and searching for the largest absolute value (of complex number):

$$\|CL_i(s)\|_\infty = \sup_{\omega\in[0,\infty].}\{|CL_i(j\omega)|\}\ \forall\ i = 1,\dots,2N \quad (13)$$

The objective function is, actually, minimizing the maximal peak harmonic response for $s = j\omega$ in the “S” domain (resonant response). Now, since $CL_i(s)$ is a single-input single-output (SISO) scalar transfer function, the peak response is equal to its H_∞ norm. The value for $\|CL_i(s)\|_\infty$ indicates the maximum response possible due to $a_g(t)$, that is:

$$\left.\begin{array}{l}\max\limits_t\{|a^{tot}{}_n(t)|\}/\max\limits_t\{|a_g(t)|\} \le \|CL_n(s)\|_\infty\\ \max\limits_t\{|\delta_n(t)|\}/\max\limits_t\{|a_g(t)|\} \le \|CL_{N+n}(s)\|_\infty\\ \forall\, n = 1,\dots,N\end{array}\right\}\forall a_g(t) \quad (14)$$

it is worth noting that the value of the objective function J will always satisfy the following ratios:

$$\left.\begin{array}{l}\max\limits_t\{|a^{tot}{}_n(t)|\}/\max\limits_t\{|a_g(t)|\} \le J/\gamma_n\\ \max\limits_t\{|\delta_n(t)|\}/\max\limits_t\{|a_g(t)|\} \le J/\gamma_{N+n}\\ \forall\, n = 1,\dots,N\end{array}\right\}\forall a_g(t) \quad (15)$$

where the parameter γ_i is chosen by the designer to gear to a practical and acceptable engineering solution.

2.4 Solution of H_∞ Control Problems

Apkarian and Noll [1] developed a first-order steepest descent algorithm for minimizing the H_∞ norm of a general closed-loop transfer function matrix. The algorithm applies to systems that can be modified into the first-order state-space presentation and adopts the iterative approach of the steepest descent method. Consequently, the solution converges into a local minimum.

This paper adopts their algorithm for solving a problem of minimizing the H_∞ norm by using a “structured controller”. This type of controller has passive gain with defined “free parameters” that are the variables in the minimization procedure, where all other variables are kept “fixed”. MATLAB©’s “hinfstruct” function handles this type of algorithm and is used herein.

The algorithm is applied to the objective function of Eq. (12) with $\Delta m_1,\dots,\Delta m_N,\Delta c_n,\dots,\Delta c_N,\Delta k_1,\dots,\Delta k_N$ as the free parameters. The algorithm is applied, repeatedly, until the total story acceleration and interstory drifts constraints are satisfied. Let the handled response for minimizing the objective function have the index “i”, then the generalized plant transfer function for the hinfstruct function is:

$$\begin{array}{l}\bar{\mathbf{G}}(s)^k = \begin{bmatrix}\bar{G}_{11}(s)^k & \bar{\mathbf{G}}_{12}(s)^k\\ \mathbf{G}_{21}(s) & \mathbf{G}_{22}(s)\end{bmatrix}\\ \text{where:}\\ \bar{G}_{11}(s)^k = \mathbf{e}_i\mathbf{G}_{11}(s)\\ \bar{\mathbf{G}}_{12}(s)^k = \mathbf{e}_i\mathbf{G}_{12}(s)\end{array} \quad (16)$$

Where k is the iteration index and $\mathbf{e}_i$ denotes a vector where all components are zero besides the i^{th} component, which equals one.

The output for the hinfstruct function is the optimal controller $\mathbf{Q}_\Delta$ which consists of the structural changes $\Delta\mathbf{M}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$. The implementation of the hinfstruct function and our retrofit procedure, for solving Eq. (12), are described in the next section.

2.5 Iterative Procedure for Seismic Upgrade

The proposed procedure for seismic upgrading of frame structures is summarized herein as a 5-steps iterative algorithm:

Step 1. Determine the matrices $\mathbf{M}$, $\mathbf{C}$, $\mathbf{K}$.
Assign initial structural changes $\Delta m_1{}^0, \dots, \Delta m_N{}^0, \Delta c_n{}^0, \dots, \Delta c_N{}^0, \Delta k_1{}^0, \dots, \Delta k_N{}^0$.
Choose the set of ground motion records.
Define $\boldsymbol{\delta}^{all}$ and $\mathbf{a}^{all}$.

Step 2. Perform time-history analysis to derive the i^{th} index for which the following ratio is maximal:

$$\max_i \left\{ \frac{\max_t\{|a^{tot}{}_i(t)|\}^k}{a^{all}{}_i} : i = 1, \dots, N \, ; \, \frac{\max_t\{|\delta_{i-N}(t)|\}^k}{\delta^{all}{}_{i-N}} : i = N+1, \dots, 2N \right\} \tag{17}$$

If the value is less than 1.0, then stop. Else, proceed to step 3.

Step 3. Determine the reduced general plant transfer function matrix $\overline{G}(s)^k$ according to Eq. (16) where the components of e_i are all zero besides the i^{th} component, which equals one.

Step 4. Perform H_∞ minimization for $CL_i(s)$ by applying MATLAB©'s hinfstruct to $\overline{\mathbf{G}}(s)^k$ to obtain the upgraded $\Delta\mathbf{M}$, $\Delta\mathbf{C}$ and $\Delta\mathbf{K}$ from $\mathbf{Q}_\Delta$.

Step 5. Perform time-history analysis for the upgraded structure, with $\Delta\mathbf{M}^k + \Delta\mathbf{M}$, $\Delta\mathbf{C}^k + \Delta\mathbf{C}$ and $\Delta\mathbf{K}^k + \Delta\mathbf{K}$, to drive the i^{th} index with maximal value. If that value is less than 1.0, then stop. Else, go to the next step 3.

3. Numerical Example

The following numerical example reexamines the simplified 9-story shear-type building from Cimellaro et al. [9], using the proposed seismic upgrade procedure. Elevation scheme of the building is shown in Fig 1. The structure is analyzed here for "BO 10% in 50 years" ground motion records from PEER database: (http://nisee.berkeley.edu/data/strong_motion/sacsteel/motions/bo10in50yr.html) with a 2% inherent damping ratio assigned to the first and second modes according to Rayleigh classical damping. Table1(A) and Table 1(B) show the selected results for their simplified and retrofitted structure respectively. The results are for the 1st story the 9th story and relevant intermediate stories in terms of maximum interstory drifts, total story accelerations and shear forces.

The proposed procedure of section 2.5 is applied to the simplified structure in Table1(A). A step by step description of its application is presented below.

Step 1. Initial structural changes were taken as $\Delta\mathbf{M}^0 = \mathbf{0}\,kg$, $\Delta\mathbf{C}^0 = \mathbf{T}^T\mathbf{T}\cdot 10^5\,Nsec/m$ and $\Delta\mathbf{K}^0 = \mathbf{0}\,N/m$. The allowable interstory drifts and total story acceleration were set as:

$$\mathbf{a}_{all} = \{0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1, 0.1\}^{\mathbf{T}}\,g$$
$$\boldsymbol{\delta}_{all} = \{3.84, 2.77, 2.77, 2.77, 2.77, 2.77, 2.77, 2.77, 2.77\}^{\mathbf{T}}\cdot 10^{-2}m$$

Step 2. Time-history analysis is performed for the initial structure by using all "BO 10% in 50 years" records. The output with the largest maximum to allowable response ratio is:

$$\max_t\{|a^{tot}{}_4(t)|\}^0 / a_{all_4} \cong 2.85$$

Step 3. The generalized plant transfer function matrix for $i = 4$, $\bar{\mathbf{G}}(s)^0$, is defined according to Eq. (16).

Step 4. H_∞ minimization is performed for $CL_4(s)$. The hinfstruct function is applied to $\bar{\mathbf{G}}(s)^0$ to derive the optimal controller $\mathbf{Q}_\Delta$. The matrices $\Delta\mathbf{M}$, $\Delta\mathbf{C}$, $\Delta\mathbf{K}$ are extracted from $\mathbf{Q}_\Delta$, to yield:

$$\Delta\mathbf{M} \cong \text{diag}\{1.00, 1.00, 1.00, 1.00, 0.36, 0.64, 0.25, 0.15, 0.34\} \cdot 10^5\ kg$$
$$\Delta\mathbf{C} \cong \mathbf{T}^T\text{diag}\{0, 0, 0, -0.0064, -0.0003, 1.7, 0, 0, 0\}\mathbf{T} \cdot 10^7\ Nsec/m$$
$$\Delta\mathbf{K} \cong \mathbf{T}^T\text{diag}\{-2.87, -5.26, -4.5, -3.4, -1.5, 0, -1.4, -2.35, -0.66\}\mathbf{T} \cdot 10^5\ N/m$$

The matrices $\Delta\mathbf{M}$, $\Delta\mathbf{C}$, $\Delta\mathbf{K}$ are then added to the structural changes: $\Delta\mathbf{M}^1 = \Delta\mathbf{M}^0 + \Delta\mathbf{M}$, $\Delta\mathbf{C}^1 = \Delta\mathbf{C}^0 + \Delta\mathbf{C}$ and $\Delta\mathbf{K}^1 = \Delta\mathbf{K}^0 + \Delta\mathbf{K}$.

Here, we changed the options of the hinfstruct function so that damping is added only to stories where allowable drifts are not satisfied and deducted otherwise.

Step 5. Time-history analysis is performed for the upgraded structure by using all “BO 10% in 50 years” records. The output with the largest maximum to allowable response ratio is:

$$\max_t\{|a^{tot}{}_4(t)|\}^0 / a_{all_4} \cong 2.5$$

The value is greater than 1.0. Therefore, the algorithm proceeds to Step 3.

The current simulation stops after 5 iterations. For comparing the upgraded design with that of the simplified structure and to highlight the redistribution of mass in the structure, we scaled the mass, stiffness and damping of the upgraded structure by:

$$\frac{\sum_{n=1}^{N} m_n}{\sum_{n=1}^{N} \Delta m_n{}^{k=4} + m_n} = 0.4634$$

This scaling ensures an upgraded mass equal to the initial mass and does not change the dynamic response. The structural changes $\Delta\mathbf{M}^*$, $\Delta\mathbf{C}^*$, $\Delta\mathbf{K}^*$, after scaling, now become:

$$\Delta\mathbf{M}^* \cong \text{diag}\{9, 9, 9, 9, -14, 9, 9, 9, -49\} \cdot 10^3\ kg$$
$$\Delta\mathbf{C}^* \cong \mathbf{T}^T\text{diag}\{0, 0, 0, 0, 5.05, 0, 2.03, 0, 0\}\mathbf{T} \cdot 10^6\ Nsec/m$$
$$\Delta\mathbf{K}^* \cong \mathbf{T}^T\text{diag}\{-120, -221, -190, -143, -63, -197, -43, -230, -77\}\mathbf{T} \cdot 10^6\ N/m$$

The structural coefficients and maximum response, for the 1st story the 9th story and relevant intermediate stories of the upgraded structure, are given in Table1(C). It is shown that all maximum responses of the upgraded structure satisfy the allowable quantities. The total stiffness coefficients reduced by approximately 80%. On the other hand, total damping remained roughly the same, attaining the allowable interstory drift between the 4th and 5th floors while all the rest having results lower than the allowable. This is mainly due to better distribution of damping coefficients. Consequently, weakening of the structure and adding dampers provided lower base-shear force (reduction of 75%). The nature of these results is in accordance with the concept of Reinhorn et al. [10], where weakening of the structure while adding damping is very effective for reducing accelerations, deformations and base-shear simultaneously. The optimal retrofit of Cimellaro et al. [5], in Table 1(B), suggests the same concept, but with more than twice the amount of added damping for about the same total stiffness. As for the total story accelerations, they are lower than allowable in all stories.

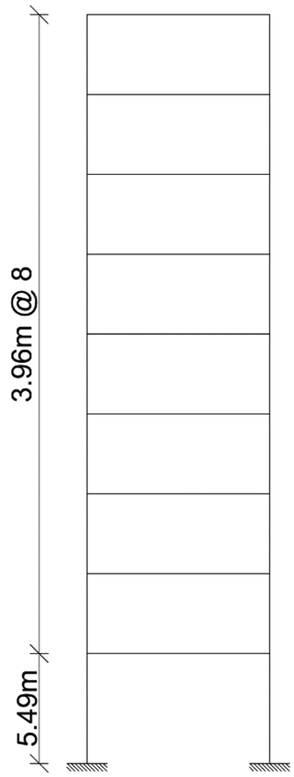

Fig. 1 – Elevation scheme of the simplified 9-story shear-type building from Cimellaro et al. [9]

Table 1 –Maximum response of optimal and retrofitted 9-story shear-type building

Floor Number	**Mass (kN s²/m)**	**Added Damping (kN s/m)**	**Stiffness (kN/m)**	**Maximum Drift (%)**	**Maximum Acceleration (g)**	**Maximum Shear (kN)**
A: Simplified 9-story shear-type building by Cimellaro et al. [9]						
Ground						**2,218**
1	503.5	906.5	143,480	0.28%	**0.1828**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
6	494.7	391	71,520	**0.70%**	0.2309	1,977
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	534.1	411	100,020	0.29%	0.2172	1,130 0
SUM	**4,501**	**7,725**	**1,586,660**			
B: Retrofitted 9-story shear-type building by Cimellaro et al. [9]						
Ground						**920**
1	441	724	36,210	0.46%	**0.1063**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	361	290	16,890	**1.04%**	0.1037	694
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	350	1,353	18,210	0.30%	0.0657	219 0
SUM	**3,006**	**14,971**	**329,510**			
C: Retrofitted 9-story shear-type building using the proposed retrofit procedure						
Ground						**556**
1	513	0	22,800	0.44%	**0.0935**	
⋮	⋮	⋮	⋮	⋮	⋮	⋮
4	480	5,050	11,990	**0.70%**	0.0642	327
⋮	⋮	⋮	⋮	⋮	⋮	⋮
8 Roof	486	0	22,710	0.28%	0.0529	252 0
SUM	**4,501**	**7,080**	**302,930**			

6. Conclusions

The current paper presents a new procedure for seismic upgrading of structures so as to satisfy allowable values of total accelerations and interstory drifts. Attained optimal changes in story mass, interstory stiffness and added damping show the validity of the approach through the results of a numerical example, which addresses a 9-story shear-type building.

A clear relation between the field optimal control and seismic design is shown. The relation is between the proposed objective function of an H_∞ norm minimization and the upper bound for the peak seismic responses. The H_∞ norm minimization is performed to reduce a single seismic response within the structure iteratively, until all peak responses are satisfied. The optimum is reached through the solution of a sequence of such SISO problems.

The hinfsturct function of MATLAB© is applied for solving the proposed objective function. In our numerical example, we intervened with the free parameters so that damping is only added where interstory drifts are not satisfied and reduced otherwise, to result in very fast convergence to the optimum. This intervention led to a full allowable drift of 0.7 between the 4th and 5th floors and a full allowable floor acceleration of 0.1g in the 1st floor to yield a "fully stressed" design (Levy [11]; Levy [12]; Levy and Lavan [13]).

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