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A FAST AND DIRECT ALGORITHM FOR STRUCTURAL DAMAGE IDENTIFICATION BASED ON MODIFIED MODAL RESIDUAL FORCE

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Abstract

Structural damage detection is the most important part of a structural health monitoring program which has been devoted to damage identification by analyzing recorded data from monitored structure. Among different methods which have been proposed for damage identification, vibration-based methods are more preferred. This is due to a rational and one-to-one relation which exists between vibrational data and damage features. Despite the availability of various vibration-based methods which use different computational strategies for damage localization and quantification, proposing fast and practical approach for identifying structural damages is more desirable in the field of structural maintenance.

In this paper, a fast damage prognosis method is proposed to detect and estimate structural damages in shear frame structures. The basic idea of the proposed algorithm relies on turning the tri-diagonal stiffness matrix of a shear frame into a diagonal matrix by means of the diagonalization method. This transform results in a modification on the modal residual force vector which causes to formulate a direct damage detection approach. Finally, the computation of damage severity is defined and established by a fast, simple, and direct concept which is able to quantify damage magnitude in a high level of accuracy. The important advantages of the presented algorithm are its simplicity and fast speed in the determination of damage severity in comparison with soft-computing- and optimization-based damage identification methods. To demonstrate the efficiency and robustness of the proposed approach, a 25-story shear frame was numerically considered with different damage scenarios. In addition, the performance of the presented approach was demonstrated when the measured modal data is contaminated with different levels of random noises. In order to validate the application of the damage diagnosis method in real structures, a five-story building structure on a shaking table was considered. Most of the obtained results from the numerical and experimental studies show the good performance and reliability of the presented method for damage identification and estimation and introduced it as a fast and powerful method for damage diagnosis in engineering structures that have a tri-diagonal stiffness matrix.

Keywords: Damage detection and localization; estimation; modal analysis; modal residual force; diagonalization method

1. Introduction

As the important part of a Structural Health Monitoring (SHM) program, damage localization and quantification has received considerable attention in the last decades. Early detection of damage not only can prevent calamitous events, but also may help engineers to prepare appropriate plans for structural rehabilitation. Although different methods have been proposed for damage identification, modal data (i.e. natural frequencies and mode shape vectors) have been considered in many methods for localizing and/or quantifying damages in plate- or frame-like structures [1-4]. To detect damages in the shear frames, Koo et al. [2] and Sung et al. [4] presented algorithms by the calculation of static displacements based on the flexibility matrix and verified with different numerical and experimental studies. Using the sensitivity analysis of the first derivative of the first mode shape vector, Zhu et al. [5] estimated structural damage features. They defined the first mode shape slope as a damage-sensitive parameter in occurring single damage scenarios and then developed this hypothesis for cases with multiple damaged stories. Sung et al. [6] identified damages in beams by proposing a novel modal flexibility-based method employing angular velocities measured from gyroscopes as input data.

In the above-mentioned approaches, the damage detection problem is solved directly by introducing a damage index. However, there are other types of damage detection methods which are based on defining the problem as an inverse problem. To solve such a problem, the soft computing or optimization strategies may be followed. In the soft computing-based methodologies (such as [7, 8]), at the first stage, the forward problem is considered for different probable damage patterns on a special structure. Then, the inverse problem is solved based on the trained system using those data extracted from studied damage cases in evaluation of the forward problem. Therefore, training a suitable system is a key for getting accurate results. In the optimization-based methods (such as [9-12]), a parametric scheme is employed for problem formulation by introducing a damage-sensitive cost function. In such strategies, the main concept is finding global extremum of a multi-variable function. Finally, the cost function is solved by means of optimization algorithms and the optimal solution is reported as damage detection results. Although all mentioned approaches are applicable for damage identification in common structures, there are concerns about their convergence to global extremums. In addition, when a large and complex structure is considered, the computational time using these approaches is very much and this can be considered as one of the main drawbacks for such methods.

In the present paper, a direct and fast speed strategy for damage localization and quantification in the shear frames is proposed. Using the diagonalization method, tri-diagonal stiffness matrix of a shear frame is turned into a diagonal matrix and a modified version of the modal residual force is introduced. Then, damage detection problem is solved by proposing a simple direct mathematical algorithm which is able to identify damage location and severity with high level of accuracy. Simplicity and fast speed are the most important advantages of the proposed method for damage identification in those structures which have tri-diagonal stiffness matrices. To demonstrate the applicability of the suggested method, different damage patterns on a 25-story shear frame are simulated and then, the proposed method is applied to damage identification when ideal or noisy modal data are fed as input data. Moreover, the described approach has been experimentally validated by studying a five-story shear frame structure on shaking table.

2. The Proposed Method

The details of the proposed method are explained in the present section. The free vibration eigenvalue problem of a damaged structure with N degrees of freedom (DOFs) is written as below:

$$\left(\mathbf{K}^d - \left(\omega_i^d\right)^2 \mathbf{M}\right)\mathbf{\Phi}_i^d = \mathbf{0} \quad i = 1, 2, ..., N \tag{1}$$

where $\mathbf{M}$ and $\mathbf{K}^d$ are the mass and damaged stiffness matrices of a shear frame, respectively. ${\omega_i}^d$ and ${\mathbf{\Phi}_i}^d$ are the ith natural frequency and corresponding mode shape vector of damaged structure, respectively.

Generally, damage causes some changes in the stiffness matrix. Therefore, a stiffness reduction matrix $\Delta\mathbf{K}$ can be defined between undamaged and damaged structure as follows:

$$\Delta\mathbf{K} = \mathbf{K}^u - \mathbf{K}^d \tag{2}$$

where $\mathbf{K}^u$ is the undamaged stiffness matrix of system. By substituting Eq. (2) into Eq. (1), Eq. (3) can be yielded:

$$\left(\mathbf{K}^u - \left(\omega_i^d\right)^2 \mathbf{M}\right)\mathbf{\Phi}_i^d = \Delta\mathbf{K}\mathbf{\Phi}_i^d \tag{3}$$

Modal residual force is defined as below:

$$\mathbf{r}_i = \left(\mathbf{K}^u - \left(\omega_i^d\right)^2 \mathbf{M}\right)\mathbf{\Phi}_i^d \tag{4}$$

By means of Eq. (4), Eq. (3) can be written more compactly as:

$$\Delta\mathbf{K}\mathbf{\Phi}_i^d = \mathbf{r}_i \tag{5}$$

Using the value of the vector $\mathbf{r}_i$ for ith mode, damage locations in shear frames can be identified [13]. In undamaged stories, the value of corresponding entries in the vector $\mathbf{r}_i$ is zero while it is nonzero for damaged stories. It is important to mention that this concept is true just for measured data without noise. On the other hand, the amount of entries of the vector $\mathbf{r}_i$ is equal or very close to zero for healthy stories. Thus, it seems that the vector $\mathbf{r}_i$ is a suitable index for damage localization. However, this index has a big drawback in some damage scenarios. Assume a case in which the jth and (j+1)th stories of a shear frame structure have been damaged. Therefore, the amount of the jth, (j+1)th, and (j+2)th entries of the vector $\mathbf{r}_i$ have a nonzero value. Using these results, we can't certainly judge on the healthy state of the (j+1)th story. This problem is one of the important issues that can be seen in the most of damage detection methods. In the present study, we employ the diagonalization method to overcome this problem.

The damaged stiffness matrix for an N-story shear building can be written as:

$$\mathbf{K}^d = \begin{bmatrix} \left(1-\beta_{1,i}\right)k_1 + \left(1-\beta_{2,i}\right)k_2 & -\left(1-\beta_{2,i}\right)k_2 & \cdots & 0 \\ -\left(1-\beta_{2,i}\right)k_2 & \left(1-\beta_{2,i}\right)k_2 + \left(1-\beta_{3,i}\right)k_3 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \left(1-\beta_{N,i}\right)k_N \end{bmatrix}_{N\times N} \tag{6}$$

where $\beta_{j,i}$ is the stiffness reduction in jth story that coming from the ith mode and k_j is the stiffness of jth story. As obvious, this matrix is a tri-diagonal matrix. It can be turned into a diagonal matrix as:

$$\left(\mathbf{K}^d\right)^* = \left(\mathbf{\Gamma}^{-1}\right)^T \mathbf{K}^d \left(\mathbf{\Gamma}^{-1}\right) \tag{7}$$

where $(\mathbf{K}^d)^*$ is the diagonal damaged stiffness matrix and the matrix $\mathbf{\Gamma}$ is defined as follows [14]:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & \ddots & 0 \\ 0 & 0 & 0 & \ddots & \ddots & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{8}$$

This diagonalization method can be also applied for the undamaged stiffness matrix. If diagonal stiffness matrix for undamaged structure is denoted with $(\mathbf{K}^u)^*$, the relation between $(\mathbf{K}^d)^*$ and $(\mathbf{K}^u)^*$ can be declared as:

$$\left(\mathbf{K}^{d}\right)^{*}=\mathbf{B}\left(\mathbf{K}^{u}\right)^{*} \tag{9}$$

in which **B** is the residual stiffness coefficient matrix as:

$$\mathbf{B}=\begin{bmatrix} 1-\beta_{1,i} & 0 & \dots & 0 \\ 0 & 1-\beta_{2,i} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & 1-\beta_{N,i} \end{bmatrix} \tag{10}$$

Also, the stiffness reduction matrix in the diagonal form can be expressed as follows:

$$\Delta\mathbf{K}^{*}=\Delta\mathbf{B}\left(\mathbf{K}^{u}\right)^{*} \tag{11}$$

where $\Delta\mathbf{B}$ is the reduced stiffness coefficient matrix:

$$\Delta\mathbf{B}=\begin{bmatrix} \beta_{1,i} & 0 & \dots & 0 \\ 0 & \beta_{2,i} & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \beta_{N,i} \end{bmatrix} \tag{12}$$

The diagonal stiffness reduction matrix can be turned into a tri-diagonal matrix using the inverse of the diagonalization method:

$$\Delta\mathbf{K}=\mathbf{\Gamma}^{T}\Delta\mathbf{K}^{*}\mathbf{\Gamma} \tag{13}$$

By substituting Eq. (11) into Eq. (13) and then in Eq. (5), Eq. (5) is rewritten as:

$$\mathbf{r}_{i}=\mathbf{\Gamma}^{T}\Delta\mathbf{B}\left(\mathbf{K}^{u}\right)^{*}\mathbf{\Gamma}\mathbf{\Phi}_{i}^{d} \tag{14}$$

Next, with pre-multiplying Eq. (14) in $(\mathbf{\Gamma}^{T})^{-1}$, Eq. (14) can be rewritten as follows:

$$\left(\mathbf{\Gamma}^{T}\right)^{-1}\mathbf{r}_{i}=\Delta\mathbf{B}\left[\left(\mathbf{K}^{u}\right)^{*}\mathbf{\Gamma}\mathbf{\Phi}_{i}^{d}\right] \tag{15}$$

The vectors $\mathbf{d}_i$ and $\mathbf{c}_i$ are defined as:

$$\mathbf{d}_{i}=\left(\mathbf{\Gamma}^{T}\right)^{-1}\mathbf{r}_{i} \tag{16}$$

$$\mathbf{c}_{i}=\left(\mathbf{K}^{u}\right)^{*}\mathbf{\Gamma}\mathbf{\Phi}_{i}^{d} \tag{17}$$

In Eq. (16), $\mathbf{r}_i$ is computed using Eq. (5) and actually, $\mathbf{d}_i$ is the modified version of the modal residual force. By means of Eq. (16) and Eq. (17), Eq. (15) can be rewritten as:

$$\Delta\mathbf{B}\mathbf{c}_{i}=\mathbf{d}_{i} \tag{18}$$

Therefore, for the *j*th story, we can compute the damage severity by using the *i*th mode's data as:

$$\beta_{j,i} = \frac{d_j}{c_j} \geq 0 \tag{19}$$

in which d_j and c_j are the jth entries of vectors $\mathbf{d}_i$ and $\mathbf{c}_i$, respectively. Finally, the damage severity β_j for the jth story through the first m modes' data can be computed using the root mean square method as:

$$\beta_j = \left(\frac{1}{m} \sum_{i=1}^{m} \left(\beta_{j,i} \right)^2 \right)^{\frac{1}{2}} \tag{20}$$

3. Numerical Study

This section is aimed at evaluating the performance of the suggested method by studying different damage patterns on a 25-story shear frame structure. Table 1 describes the material properties for this structure. The simulated three damage patterns are as follows:

- Pattern I: The stiffness of story 10 is reduced by 15%,
- Pattern II: The stiffness of story 6 is reduced by 20% and the stiffness of stories 14 and 18 are reduced by 15%,
- Pattern III: The stiffness of stories 7 and 25 are reduced by 10% and the stiffness of stories 12 and 14 are reduced by 5% and 15%, respectively.

Although the first damage pattern simulates a case with single damage, the second and third damage patterns consists of multiple damage scenarios. Since it is possible that in real SHM programs the input data are polluted with noises, investigation of the robustness of a damage detection method by feeding noisy input data is more desirable. In the present study, this issue is addressed by contaminating natural frequencies with random noises using below-mentioned strategy:

$$\omega_i^n = \omega_i (1 + n\varepsilon_i) \tag{21}$$

Table 1 – The physical properties of the 25-story shear frame

Story number	Mass (ton)	Stiffness (MN/m)
1~5	100	500
6~10	100	400
11~15	100	300
16~20	100	200
21~25	100	100

where ω_i^n is the ith natural frequency contaminated with noise, ω_i is ith natural frequency without noise, n is the noise level, and ε_i is a random value between [-1 1].

After generating input data, the proposed method is applied to damage identification and quantification. Here, not only free noise state, but also three noisy states (with 3%, 5% and 8% noises in the natural frequencies) are considered. Moreover, to evaluate the influence of the number of utilized modal data on the reliability of the proposed method, the problem is solved by considering two different sets of modal data: when the first one modes' data is available (m=1), and when the first three modes' data are fed as input data (m=3). The obtained results for the first damage pattern are shown in Fig.1. In addition, Fig.2 and Fig.3 show damage detection results for the second and third damage patterns, respectively. From these figures, it is obvious that in all cases the obtained results have an acceptable accordance with the simulated damage patterns. It should be noticed that in the noisy states, not only may some differences between actual damage severities and obtained results be seen, but also it is possible that some of the undamaged stories are reported as damaged ones. In addition, by increasing the number of utilized noisy modal data, above-mentioned differences increase. However, the amount of differences between actual damages and reported results is very small and cannot adversely influence the robustness of the method in the presence of noisy input data.

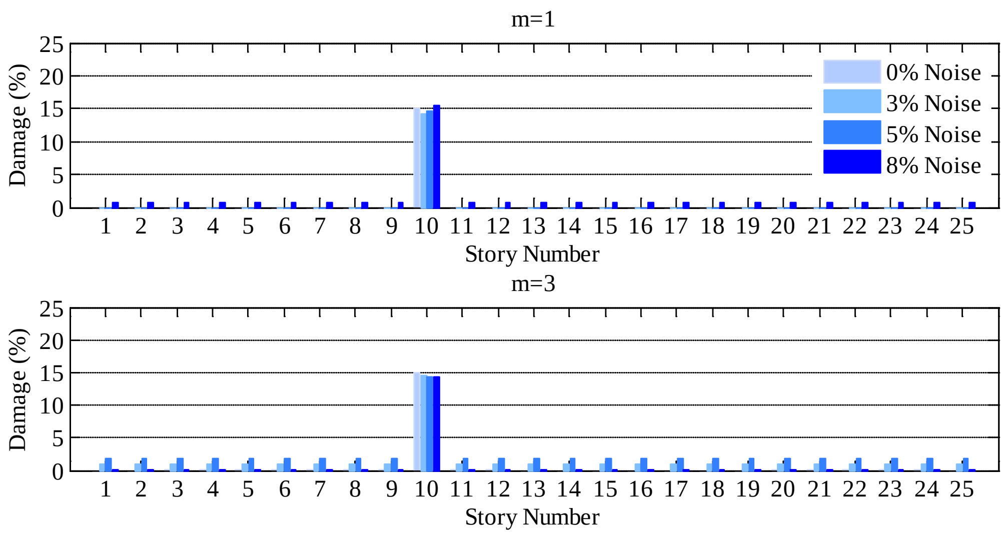

Fig. 1 – Damage detection results for the first damage pattern of the 25-story shear frame

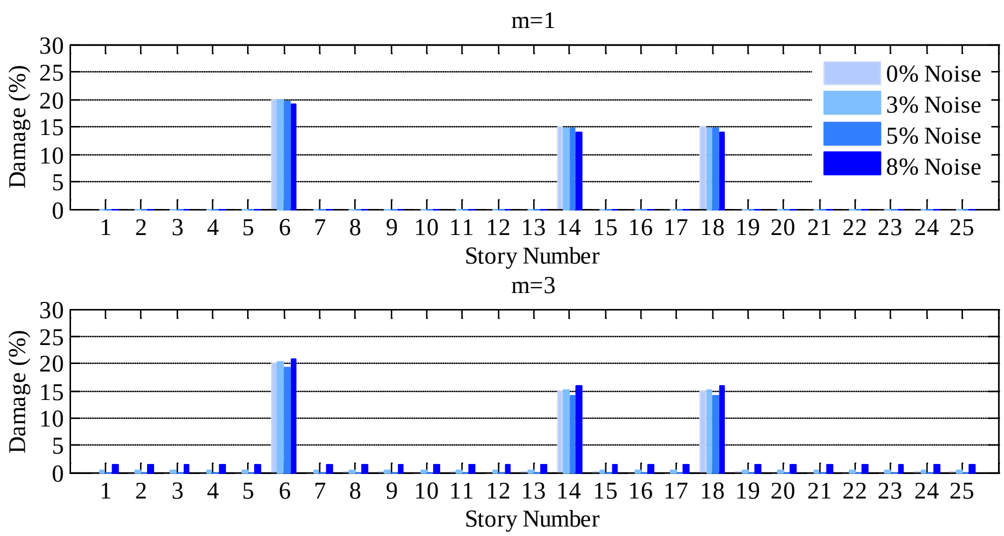

Fig. 2 – Damage detection results for the second damage pattern of the 25-story shear frame

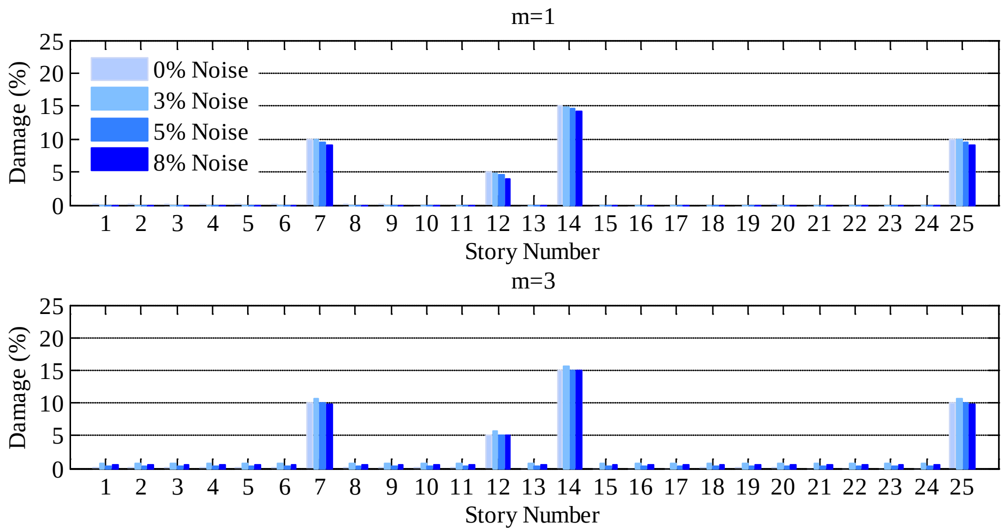

Fig. 3 – Damage detection results for the third damage pattern of the 25-story shear frame

Fig. 4 – Experimental setup of the five-story shear frame

4. Experimental Validation

In this section the applicability of the presented method in real conditions is validated by studying a five-story shear frame structure tested on a shaking table (Fig.4). For each story, the value of story mass and story stiffness are 16.09 *kg* and 11.89 *KN*/*m*, respectively. Damage was simulated by reducing the original cross section of columns in the damaged story. The selected damage pattern for damage simulation is as follows:

- A single damage at the first story with 10% reduction in the story stiffness.

Shaking table excitation was conducted by a random load for 10 *min* and five accelerometers on each floor were sampled at 20 Hz. In this experiment, measurement was repeated three times for damaged structure to investigate the uncertainty of the modal data and damage diagnosis.

Table 2 – Modal parameter changes for the simulated damage in the experimental study

Mode number	$\Delta\omega/\omega$ (%)	MAC
1	-2.42	0.9999
2	-1.89	0.9998
3	-1.18	0.9985

The modal parameters were identified employing data-drive stochastic subspace identification method described in [15]. The calculated comparative errors of the natural frequencies ($\Delta\omega/\omega$) as well as MAC values for the first three modes are declared in Table 2. For the *i*th mode, these parameters have been calculated using below-mentioned formula:

$$MAC_i = \frac{\left| \left(\boldsymbol{\Phi}_i^u\right)^T . \left(\boldsymbol{\Phi}_i^d\right) \right|^2}{\left(\left(\boldsymbol{\Phi}_i^u\right)^T . \left(\boldsymbol{\Phi}_i^u\right) \right) . \left(\left(\boldsymbol{\Phi}_i^d\right)^T . \left(\boldsymbol{\Phi}_i^d\right) \right)} \tag{23}$$

$$\frac{\Delta\omega_i}{\omega_i}(\%) = \frac{\omega_i^d - \omega_i^u}{\omega_i^u} \times 100 \tag{24}$$

where superscripts *u* and *d* denote the undamaged and damaged states, respectively. From Table 2 it can be seen that although the natural frequencies reduced by about 1.2~2.5%, the MAC values did not change much. Therefore, such conclusion may be drawn that if the changes in the modal data are directly inspected, judgment on the health of the monitored structure cannot be accurately done.

In the following, the presented method is applied to damage identification using only the first modes' data as input data. Fig.5 shows the mean and one standard deviation value of fault diagnosis results. It is obvious that the method can identify the damage location in the structure and also it can quantify the damage severity with an acceptable accuracy. It is worth noting that there are some differences between the estimated and actual damage intensity, which is because of the existence of some noises in the input data. However, the rate of differences is small and this cannot result in false judgment on the health of different stories.

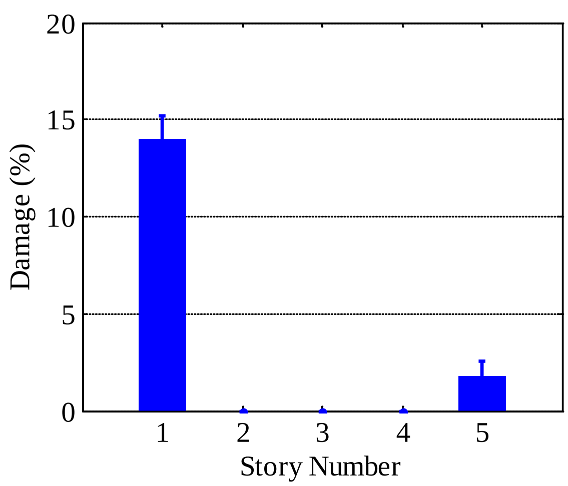

Fig. 5 – The mean and one standard deviation value of damage detection results in the experimental five-story shear frame structure

5. Conclusions

In this paper an effective method for damage prognosis in the shear frames was presented. Using diagonalization method, a modified version of the modal residual force was introduced. Then, it was employed to propose a direct mathematical algorithm for damage localization and quantification. Simplicity and fast speed are the main advantages of the proposed method which make it a viable approach in comparison with soft-computing- and optimization-based methods. To investigate the applicability of the proposed method, different damage patterns on a 25-story shear frame structure were numerically studied. Challenges such as the effects of the number of available modal data as well as noise impacts on the reliability of the method were evaluated. Moreover, a five-story shear frame structure was experimentally tested by simulating a single damage pattern and the proposed method was applied to damage identification using only the first mode's data as input data. The obtained results showed acceptable performance of the suggested method for damage identification in those engineering structures which have tri-diagonal stiffness matrix.

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