16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 3370

Registration Code: S-F1460413436

EXPERIMENTAL VALIDATION OF A MULTI-PHYSICS MODEL FOR THE SEISMIC RESPONSE OF FREESTANDING STRUCTURAL SYSTEMS

C.E. Wittich[(1)], T.C. Hutchinson[(2)]

[(1)] *Ph.D. Candidate, Department of Structural Engineering, University of California, San Diego, cwittich@ucsd.edu*
[(2)] *Professor, Department of Structural Engineering, University of California, San Diego, tara@ucsd.edu*

Abstract

Systems of unattached, or freestanding, structures are highly vulnerable to damage and overturning when subject to seismic excitation. This class of structures includes systems such as unreinforced masonry walls, multi-drum columns, statue-pedestal systems, and various mechanical or electrical equipment. Damage to these structures can result in loss of irreplaceable heritage, limited functionality of critical structures, and even loss of life. As a result, accurate prediction of their response is essential. However, traditional analytical methods cannot readily incorporate multiple bodies, multiple modes, and three-dimensional behavior – each of which has been shown to significantly affect the response in recent experiments. Therefore, in this paper, a numerical model is developed in the commercially available and widely-used multi-physics platform LS-DYNA. The fully three-dimensional numerical model employs a penalty-based contact definition lumping non-linearity at system's interfaces and allowing large displacements and rotations of the individual structures. The model is validated using the results of an extensive shake table testing campaign, which included various geometric configurations of symmetric and eccentric freestanding structures in single or stacked systems. The model is shown to sufficiently capture the salient features of the system response including three-dimensionality and the interaction of multiple modes such as sliding and rocking. Furthermore, numerical simulations indicate that the model can be used to predict the dynamic response of these structures with high fidelity.

Keywords: rocking; sliding; numerical modeling; shake table test; multi-body systems;

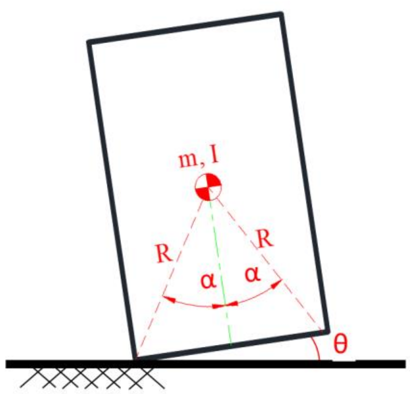
1. Introduction

Freestanding, or unanchored, structural systems encompass a wide variety of everyday, critical, or unique structures and components. This class of structures includes certain mechanical and electrical equipment, laboratory and hospital components, unreinforced masonry walls, nuclear radiation shields, classical multi-drum columns, and culturally significant statue-pedestal systems. Failure of these systems can have drastic consequences including loss of functionality for a critical facility, loss of irreplaceable cultural heritage, and even loss of life. Furthermore, earthquake reconnaissance highlights the vulnerability of these systems to even moderate seismic demands. For example, an overturned statue-pedestal system and a twisted and translated electrical transformer are shown in Fig. 1 following the M_w 6.0 2014 South Napa Earthquake [1]. These examples not only emphasize the wide variety of components effected, but also the multiple failure patterns (e.g. overturning, translating, or twisting). Therefore, there is a critical need for accurate prediction of the seismic response of freestanding structural systems.

Housner presented the first significant effort to predict the seismic response of freestanding structural systems in 1963 [2]. In this work, a two-dimensional simple rocking model was proposed and remains a widely studied and utilized model for this system. This classical rocking model is characterized by piecewise equations of motion, strictly rocking motion with sliding and free-flight neglected, and the assumption that both the two-dimensional block and the foundation are rigid. All energy is further assumed to be dissipated only at instantaneous, perfectly inelastic impacts. This is shown schematically and in equation form in Fig. 2, where the rectangular block is rotated with angle, θ, and the equation of motion and coefficient of restitution are nonlinear with respect to the defined geometric parameters. This classical model has served as the basis for a number of studies including the study and prediction of slide-rocking modes [3, 4], probabilistic assessments of overturning [5], derivation of rocking modes for flexible bodies [6], and many others. However, this classical rigid body rocking model has been shown to poorly predict the experimental response of symmetric and eccentric freestanding systems [7, 8, respectively]. Furthermore, these classically derived equations of motion are not easily extensible to three-dimensions and systems of multiple freestanding bodies [9, 10, respectively]. Therefore, alternative models have been proposed. These models leverage the observation that modified coefficients of restitution improve agreement of the classical model with experimental results; and, as such, the models relax the assumption of a rigid interface. For example, Chatzis and Smyth enhanced the classical model with concentrated and distributed springs at the interface in both two and three dimensions, which allowed for rotational and translational motion as well as complete separation of the structure from its foundation [11, 12]. A similar approach incorporating springs and dashpots at the interfaces of rigid or flexible components is the discrete or distinct element method (DEM), as first introduced by Cundall [13]. DEM is typically incorporated within specialized software geared towards the simulation of large particle groups, such as soil. Nonetheless, this method has shown good agreement with experimental results for freestanding structural systems, namely classical multi-drum columns [e.g. 14].

Given the need to accurately predict the seismic response of freestanding structural systems, it is the objective of this paper to detail the development and preliminary experimental validation of a multi-physics numerical model that is easily implemented in a widely available and heavily utilized platform. Specifically, the proposed model is developed within LS-DYNA and incorporates spring and dashpot elements to represent contact forces between distinct bodies in the simulation through a penalty-based contact algorithm [15]. To this end, this paper initially details a shake table testing program of single and dual-body freestanding systems over a range of size, slenderness, and asymmetry of the individual structures. Key observations are then discussed in the context of numerical model development; and, ultimately, the experimental results are utilized to validate the numerical model for the seismic response of a wide variety of geometrically-unique freestanding structural systems.

Fig. 1 – Post-earthquake reconnaissance of freestanding structural systems: (a) statue that overturned and twisted from its pedestal, and (b) red-tagged electrical transformer that twisted and translated during the 2014 South Napa Earthquake. Images from [1].



Equation of motion to characterize the rotational acceleration, $\ddot{\theta}$, when body is subject to horizontal ground acceleration, $\ddot{x}_g$:

$$(I + mR^2)\ddot{\theta} = m\ddot{x}_g R\, cos(\alpha - |\theta|) - sgn(\theta)mRg\, sin(\alpha - |\theta|)$$

Coefficient of restitution at impact:

$$r = 1 - \frac{3}{2} sin^2\alpha$$

From Housner (1963)

Fig. 2 – Schematic of the two-dimensional classical model with piecewise equation of motion and coefficient of restitution defining instantaneous inelastic impacts, as originally presented by Housner [2].

2. Experimental Program

In an effort to develop and validate a numerical model for the seismic response of arbitrary freestanding structural systems, an extensive shake table testing campaign was conducted at the University of California, San Diego between January 2013 – June 2014. The primary variables included: size, slenderness, and eccentricity of the individual freestanding bodies, number of bodies in a stacked system, material at the interface, and base excitation. The specimen design, testing protocol, and summary of results and implications are summarized in the following subsections.

2.1 Specimen Design

The experimental campaign was largely modeled after the human-form statue-pedestal system, as it is one of the most geometrically complex freestanding systems. Given the results of a geometric survey of culturally significant statues, a range of extreme geometries was determined and incorporated in the design of the experimental specimen [16]. This statue-like structure, shown in Fig. 3a, consists of a stiff steel tower with moveable weight plates able to shift the center of mass horizontally and vertically. This geometrically-variable tower was capable of representing freestanding bodies with rocking radii, *R*, ranging from 0.7 m – 1.6 m and slenderness, α, ranging from 9° to 35°, where these parameters are shown schematically in Fig. 2. The specimen was further verified to be classified as rigid as its natural frequencies are in excess of 16.67 Hz [17]. Furthermore, this tower was outfitted with a bonded marble base resting freely atop a fixed marble slab on a uniaxial shaking table to account for the high-friction stone-stone interfaces anticipated for statue-pedestal systems. A total of fifteen unique tower configurations were tested as single-body systems (Fig. 3a), while a subset of four of these configurations were tested in dual-body systems (Fig. 3b). A total of two geometric variations of the 'pedestal' or bottom body were incorporated into the dual-body systems with heights to the center of mass ranging from 0.38 m – 0.76 m. In addition, the material at the base of this pedestal was varied between marble and a low-friction steel plate. These geometric and material variations yielded a total of twenty-seven unique freestanding structural systems, as detailed in [8, 18].

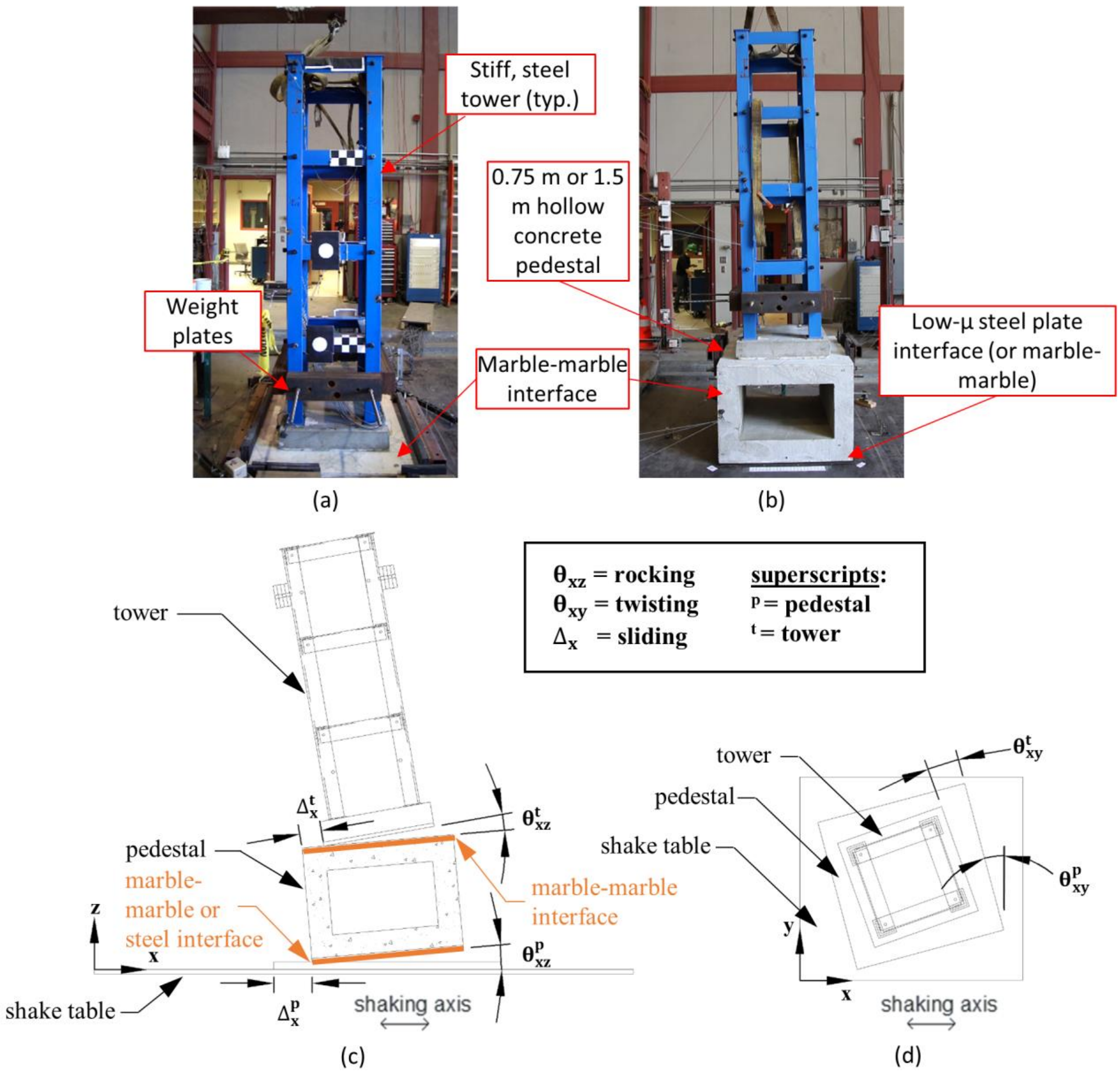


Fig. 3 – Shake table testing of freestanding structural systems: (a) Geometrically-reconfigurable stiff, steel tower with marble-marble interface, (b) stiff steel tower atop a freestanding pedestal in a dual-body test, (c) elevation view schematic of displaced shape including rocking of both tower and pedestal, and (d) plan view schematic of displaced system including twisting of both tower and pedestal.

2.2 Test Protocol

Each of the twenty-seven unique configurations of freestanding structural systems was tested on the uniaxial shaking table in UC San Diego's Powell Laboratory. The tests consisted of either a single-body or a stacked dual-body system resting unattached atop the table platen. The test protocol consisted of: 1) recorded or simulated earthquake motions, 2) variable velocity sliding tests, and 3) free rocking tests. Each of the configurations was subjected to at least five earthquake motions, including both near-fault and far-fault motions to explicitly account for pulse effects. A detailed list of these motions can be found in [18]. The variable velocity sliding tests were conducted by restraining the tower or pedestal specimen and inducing slip at the interface by displacing the table platen. This was conducted at velocities ranging from 10 mm/s – 300 mm/s, and yielded coefficients of friction of 0.4 – 0.68 for the marble-marble interfaces and 0.15 – 0.22 for the coated steel plate interface. The free rocking tests were induced by a single sinusoidal pulse excitation, where the free rocking was measured after the motion of the table concluded. The fully three-dimensional motion of the individual specimens was measured through a network of string potentiometers and high-resolution cameras. The calculated motion parameters relevant for freestanding systems are presented schematically in Fig. 3c-d, namely rocking (θ_{xz}), sliding (Δ_x), and twisting (θ_{xy}). In this paper, the displacements and rotations associated with the tower and pedestal are denoted with superscripts 't' and 'p', respectively. After each dynamic, sliding, or free rocking test, the interfaces were cleared of any dust and the specimens placed in their original positions and orientations.

2.3 Implications on Numerical Modeling

While a detailed discussion of the results of the single- and dual-body tests is out of the scope of this paper [8, 17], three primary conclusions are presented in the context of their implications on numerical modeling, as summarized in Table 1. The first of which indicates that eccentric bodies can respond significantly differently than their symmetric counterparts, even when the eccentricity is quite small ($\alpha_1/\alpha_2 \approx 0.8$, where α is defined in Fig. 2). Specifically, it was observed that certain eccentric bodies can respond with greater or lesser magnitude of response. In addition, the mode of response (i.e. rocking, sliding, and twisting) can be entirely different for eccentric and symmetric bodies. This was most noticeable for smaller, squatter bodies (e.g. $R \approx 0.75$, $\alpha \approx 26°$). As a result, numerical modeling needs to be conducted in a three-dimensional framework capable of representing asymmetric and arbitrary geometries.

Table 1 – Primary experimental findings and implications on numerical modeling

Finding No.	**Experimental Conclusion**	**Implication on Modeling**
1	Eccentric bodies may respond with greater or lesser magnitude as well as in different modes than symmetric counterparts	Model must uniquely represent asymmetric and three-dimensional geometries
2	Multi-modal behavior, including the simultaneous response in one or more modes, may be observed for symmetric, eccentric and stacked configurations	Modeling scheme must allow primary (rock, slide, twist) and interactive modes within a single simulation
3	Single-body systems can be less stable than dual-body counterparts due to complex interactions at impact	Multi-body systems must be solved accounting for the distinct motion of each freestanding body.

The second primary experimental conclusion regards the multi-modal response of freestanding systems. This multi-modal response took the form of the simultaneous response in two or more modes as well as the transition from one mode to another in the same response history, termed modal interaction and modal transitioning, respectively. Specifically, modal interaction was observed in over 30% of all dynamic tests conducted. An example response incorporating both modal interaction and modal transition is that of a taller, eccentric specimen (e.g. $R \approx 1.5$ m, $\alpha_1 \approx 9°$, $\alpha_2 \approx 18°$), in which the body rocks about the more slender edge and slide-rocks upon impact with the squatter edge. Therefore, an appropriate numerical modeling scheme must allow primary and interactive response modes to freely transition within a single simulation.

The third and final experimental conclusion emphasizes the complexities of stacked or dual-body systems. It was observed that certain single-body systems were less stable, or more prone to overturning, than the same tower structure placed atop a pedestal in a dual-body system. This was largely attributed to the complex interactions at impact, during which two moving bodies are colliding and can result in significantly more energy dissipation that a single-body with a rigid foundation. In addition, the response of certain dual-body systems was dominated by the response of the tower (i.e. top body) with fairly negligible response of the pedestal (i.e. bottom body). In these cases, the response of the tower atop the pedestal was still considerably different than the response of the same tower atop the shaking table when subject to the same base excitation. These two example situations emphasize the complex interactions between the individual bodies of a multi-body system. As a result, multi-body systems must be simulated accounting for the distinct motion, including complete separation, of each freestanding body comprising the system.

3. Model Development

3.1 Numerical Modelling Platform

The numerical model was developed in the explicit, multi-physics solver LS-DYNA [15]. LS-DYNA is a robust platform which incorporates the finite element method, rigid body dynamics, and advanced contact-impact algorithms, which are particularly attributable to freestanding structural systems. This software has both two-dimensional and three-dimensional capabilities, with a broad library of material models, element types, and contact algorithms. Furthermore, LS-DYNA is a readily-available and widely-used engineering software allowing for ease of implementation and further analyses of freestanding systems, including fluid-structure interaction and other multi-physics simulations.

3.2 Geometric and Material Modelling

Individual bodies of the freestanding structural systems are modeled as distinct, three-dimensional parts with unique meshes. Each body consists of a solid mesh of 8-node brick elements utilizing the rigid material model, characterized by bulk modulus, K, and density, ρ (i.e. *MAT_RIGID). While the external surface of the body is defined by this mesh, the critical geometric and mass properties including the inertia tensor, mass, and relative location of the center of mass are explicitly defined by the user to avoid numerical errors associated with a coarse finite element mesh. This three-dimensional framework, which accounts for arbitrary and asymmetric geometric configurations, directly addresses the first primary experimental conclusion, which emphasized the differences in the dynamic responses of symmetric and asymmetric geometries.

The treatment of rigid body dynamics, as defined by the use of *MAT_RIGID in LS-DYNA, is originally detailed by Benson and Hallquist [19]. This treatment is similar to that incorporated within DEM simulations, shown schematically in Fig. 4a. Specifically, the rigid body accelerations of each distinct body in the simulation are found through Newton's 2nd Law following a summation of forces on each body. The rigid body velocities and displacements are explicitly solved by way of the central difference method; and, ultimately, the updated positions of the bodies are used in the determination of contact forces for the subsequent time step.

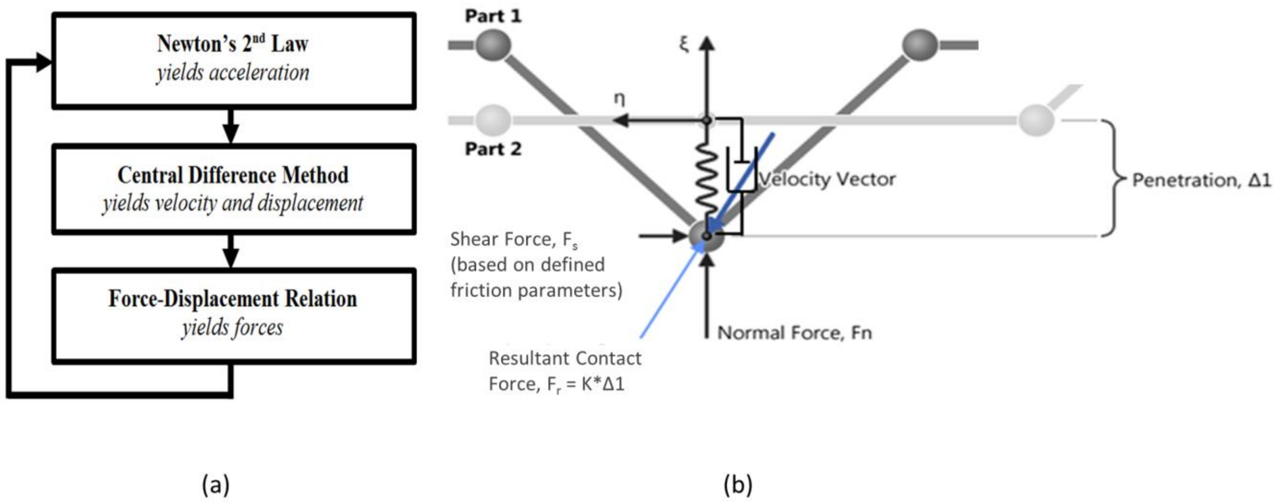


Fig. 4 – (a) Schematic of penalty-based contact algorithm and (b) numerical scheme for time history response as implemented in LS-DYNA (modified from LSTC [15]).

3.3 Contact Modelling

The interaction between individual rigid bodies in the simulation is modeled with a penalty-based contact algorithm (i.e. *CONTACT_AUTOMATIC_ONE_WAY_SURFACE_TO_SURFACE). This method searches for the penetration of a node into one of the bodies of the simulation, indicating contact. When penetration is detected, spring and dashpot elements are generated to resist the penetration and produce forces of contact, which are proportional to the magnitude of penetration. Specifically, normal and tangential forces allow the resistance of penetration as well as the modelling of Coulomb friction, as shown schematically in Fig. 4b. The contact parameters, namely the spring stiffness, damping ratio, and coefficients of static and dynamic friction, are specified by the user for a given simulation. In an effort to calibrate the numerical model, the contact parameters were fitted to the results of the variable velocity sliding tests and free rocking tests of the single-body configurations. The arithmetic mean over the range of best-fit parameters for the individual configurations was then utilized in an 'average' model. This average model is validated in the next section by comparing the experimental results with the model's prediction in terms of both multi-modal and multi-body response.

4. Model Validation

A numerical model, based on rigid body dynamics and penalty-based contact, was calibrated with the results of both free rocking tests and variable-velocity sliding tests. This model, which consists of average fitted parameters, is presented in this section and then used to predict the experimental response of single and dual-body systems with respect to multi-modal and multi-body behavior.

4.1 Multi-Modal Behavior

The second primary experimental conclusion outlined the multi-modal behavior of both symmetric and eccentric bodies in single- and dual-body systems. As a result, the numerical modeling scheme must allow for the free transitioning between primary (i.e. rocking, sliding, and twisting) and interactive (i.e. rock-sliding, rock-twisting, and slide-twisting) modes within a single simulation. The penalty-based contact algorithm and rigid body dynamics of the numerical model address this need and allow for the full three-dimensional response of each of the individual bodies in the simulation, including translation, rotation, and complete separation. This is shown via time histories of rocking, sliding, and twisting, which are plotted against one another as a function of time in Fig.

5a-c. In each of these plots, the response of an experimental configuration is overlaid with the numerical prediction, utilizing the average contact parameters and corresponding user-defined geometric and mass properties. The presented modal interaction plots emphasize the versatility of the numerical model in sufficiently reproducing the three interactive modes, where modal interaction is observed as a sloped line indicating the simultaneous response in multiple modes and horizontal or vertical lines indicate response in a single primary mode. In addition, the ability of the model to transition between modes is evidenced in Fig. 5a, which shows the interaction of rocking and sliding for a given configuration. Specifically, the model is able to capture the initial rocking response, transition to slide-rocking, and ultimate transition to a rocking response.

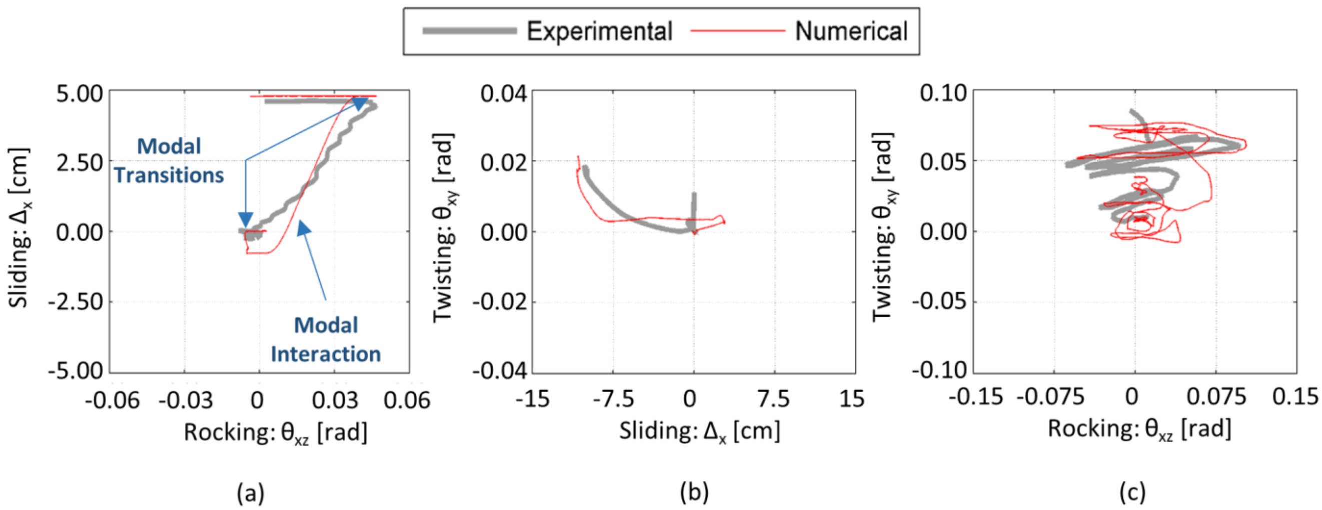


Fig. 5 – Experimental and numerical modal interaction diagrams for (a) slide-rocking, (b) slide-twisting, and (c) rock-twisting.

4.2 Multi-Body Behavior

The third experimental conclusion detailed in this paper is related to the interaction of individual bodies that comprise a freestanding structural system. It was observed that even fairly negligible motion of the pedestal, or bottom body, could cause significant differences in the response of the tower, or upper body. To address this in the numerical model, the motion of each individual body is simulated as discrete entities allowing for large displacements and rotations subject to contact forces generated due to the interaction with other bodies in the simulation. The impact of this within a multi-body simulation is presented in Fig. 6, in which the experimental response of a configuration is overlaid with its numerical prediction for the dual-body and single-body system. The presented configuration is a tall, symmetric tower atop a tall tower (i.e. $R \approx 1.55$ m, $\alpha \approx 13.5°$ for the tower, and $R \approx 0.91$ m, $\alpha \approx 33°$ for the pedestal). It can be seen that the response of the tower as a single-body is dominated by a significant rocking response. As it is a slender body, it is anticipated that this tower would rock atop the pedestal in a dual-body configuration, rather than initiating into a rocking mode at the base of the pedestal. As anticipated and observed in Fig. 6, the rocking of the tower dominates the response of the dual-body system with minimal participation of the pedestal. However, it is emphasized that the time history of rocking for the same tower, subject to the same motion, is significantly different when tested and simulated as a single-body and a dual-body. Therefore, the numerical model is able to sufficiently capture the complex interactions between the individual bodies and account for the effects of fairly minimal motion of the pedestal or bottom bodies in a simulation.

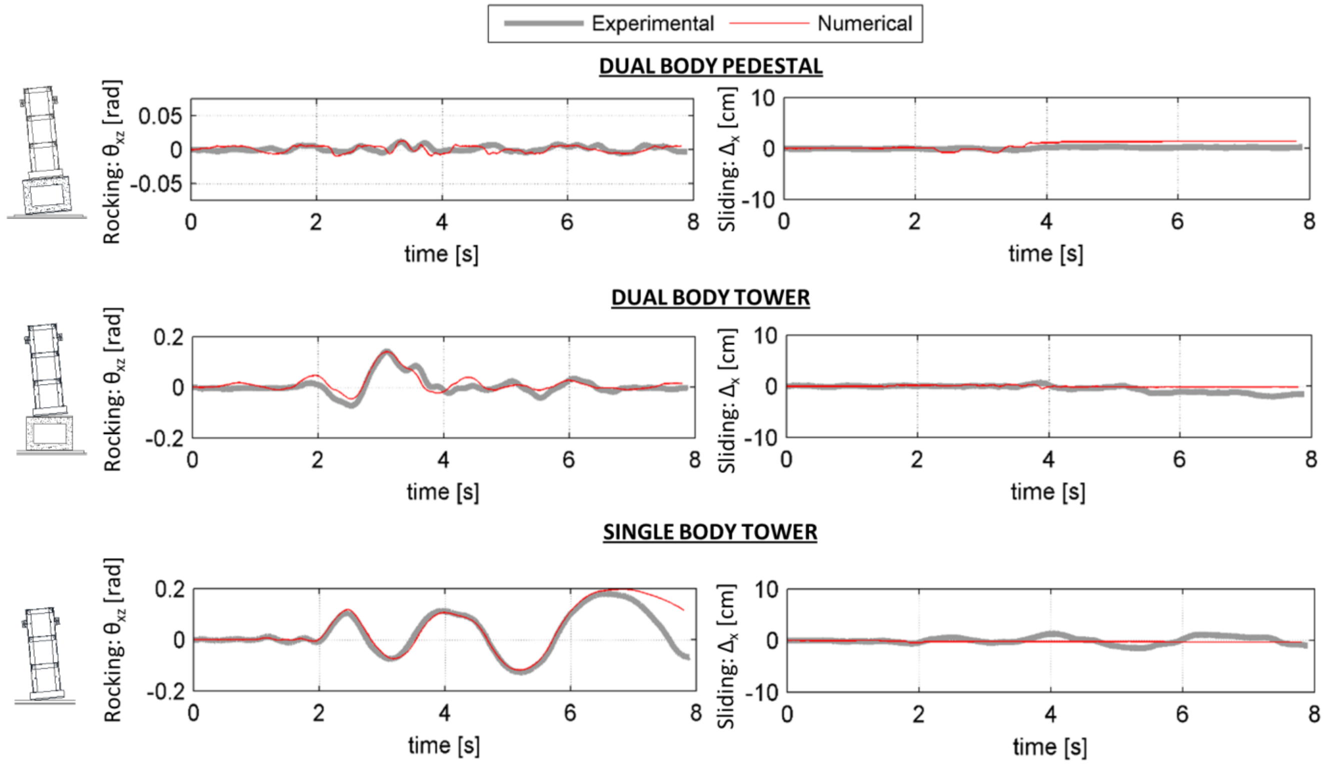


Fig. 6 – Overlaid time histories of experimental and numerical response of rocking and sliding for the pedestal and tower of a dual-body test as well as that of the tower in a single-body test. Note: Input motion is 1999 Duzce Earthquake at Bolu Station (unscaled).

5. Conclusions

Freestanding structural systems include all unanchored structures, and contain a wide variety of everyday, critical, and unique systems such as mechanical and electrical equipment, building contents, unreinforced masonry walls, and culturally significant statue-pedestal systems. These systems tend to respond poorly to earthquake excitation and may observe a wide variety of failure modes, including overturning and excessive translation. However, existing analytical methods are limited with respect to three-dimensional effects, number of bodies in the system, and number of modes accounted for in the analysis (i.e. rocking, sliding, twisting). Therefore, there is a critical need to develop tools to accurately predict the seismic response of arbitrary freestanding systems. To address this need, extensive shake table testing of freestanding systems was conducted varying geometric and material parameters. The results of this campaign emphasized: 1) asymmetric bodies can respond with varying magnitude and in different modes than symmetric counterparts; 2) symmetric, asymmetric, and dual-body systems tend to respond with significant multi-modal behavior including the simultaneous response in two or more modes as well as the transitioning from one mode to another in a single response; and, 3) dual-body systems respond significantly differently than their single-body counterparts, even when the motion of lower bodies appears negligible.

In an effort to develop predictive tools for freestanding systems, a numerical model was developed in the widely-available, explicit, multi-physics solver LS-DYNA, which allows the simulation of three-dimensional and asymmetric geometric configurations. Each body in the simulation of the freestanding system is modeled as a distinct rigid entity, and the interactions between each body are modeled with a penalty-based contact algorithm. As a result, the model allows for the individual rotation, translation, and separation of each body, an important facet observed during the experiments. The contact parameters of the numerical model were fit to the results of sliding and free rocking tests conducted within the shake table testing campaign. The multi-modal interaction, modal transitions, and complex interactions between multiple bodies were validated utilizing various results from the dynamic portion of the shake table testing campaign. Further study of this modeling technique is anticipated with respect to its ability to represent the fundamental rocking dynamics and the sensitivity of the model parameters.

6. Acknowledgements

This investigation was supported by the National Science Foundation under IGERT Award #DGE-0966375, "Training, Research and Education in Engineering for Cultural Heritage Diagnostics," and award #CNS-1338192, "MRI: Development of Advanced Visualization Instrumentation for the Collaborative Exploration of Big Data." Additional support was provided by Achievement Rewards for College Scientists, the UC San Diego Academic Senate, the Qualcomm Institute at UC San Diego, the Friends of CISA3, and the World Cultural Heritage Society. The assistance and support of the Charles Lee Powell Laboratory staff is greatly appreciated, particularly Mr. Darren McKay. The continuous support of this project and guidance of Professor Falko Kuester is greatly valued. The opinions, findings, and conclusions expressed in this report are those of the authors and do not necessarily reflect those of the sponsoring organizations.

7. References

[1] Wittich CE, Hutchinson TC, Lo E, Meyer D, Kuester F (2014): The South Napa earthquake of August 24, 2014 – drone-based aerial and ground-based LiDAR imaging survey. *Structural Systems Research Project Report SSRP-2014/09*, Department of Structural Engineering, University of California, San Diego, La Jolla, CA, USA.

[2] Housner GW (1963): The behavior of inverted pendulum structures during earthquakes. *Bulletin of the Seismological Society of America*, **53**(2), 403-417.

[3] Ishiyama Y (1982): Motion of rigid bodies and criteria for overturning by earthquake excitations. *Earthquake Engineering and Structural Dynamics*, **10**(5), 636-650.

[4] Shenton H, Jones N (1991): Base excitation of rigid bodies. 1: formulation. *Journal of Engineering Mechanics (ASCE)*, **117**(10), 2286-2306.

[5] Yim CS, Chopra AK, Penzien J (1980): Rocking response of rigid blocks to earthquakes. *Earthquake Engineering and Structural Engineering*, **8**(6), 565-587.

[6] Acikgoz S, DeJong MD (2014): The rocking response of large flexible structures to earthquakes. *Bulletin of Earthquake Engineering*, **12**(2), 875-908.

[7] Lipscombe PR, Pellegrino S (1993): Free rocking of prismatic blocks. *Journal of Engineering Mechanics (ASCE)*, **119**(7), 1387-1410.

[8] Wittich CE, Hutchinson TC (2015): Shake table tests of stiff, unattached asymmetric structures. *Earthquake Engineering and Structural Dynamics*, **44**(14), 2425-2443.

[9] Konstantinidis D, Makris N (2007): The dynamics of a rocking block in three dimensions. *8th Hellenic Society for Theoretical and Applied Mechanics International Congress*, Patras, Greece.

[10] Psycharis IN (1990): Dynamic behaviour of rocking two-block assemblies. *Earthquake Engineering and Structural Dynamics*, **19**(4), 555-575.

[11] Chatzis MN, Smyth AW (2011): Robust modeling of the rocking problem. *Journal of Engineering Mechanics (ASCE)*, **138**(3), 247-262.

[12] Chatzis MN, Smyth AW (2012): Modeling of the 3D rocking problem. *International Journal of Non-Linear Mechanics*, **47**(4), 85-98.

[13] Cundall PA, Strack OD (1979): A discrete numerical model for granular assemblies. *Geotechnique*, **29**(1), 47-65.

[14] Papantonopoulos C, Psycharis IN, Papastamatiou DY, Lemos JV, Mouzakis HP (2002): Numerical prediction of the earthquake response of classical columns using the distinct element method. *Earthquake Engineering and Structural Dynamics*, **31**(9), 1699-1717.

[15] Livermore Software Technology Corporation (2013): *LS-DYNA*. Livermore Software Technology Corporation, Version LS-DYNA_971 Revision 78769, Livermore, CA.

[16] Wittich CE, Hutchinson TC, Wood RL, Seracini M, Kuester F (2015): Characterization of full-scale, human-form, culturally important statues: case study. *Journal of Computing in Civil Engineering*, **30**(3), 05015001.

[17] International Code Council (2007): Acceptance criteria for seismic qualification by shake table testing of nonstructural components and systems. *Report No. AC156*, International Code Council, Whittier, CA.

[18] Wittich CE, Hutchinson TC (2016): Shake table tests of unattached, asymmetric, dual-body systems. *Earthquake Engineering and Structural Dynamics* (in review).

[19] Benson DJ, Hallquist JO (1986): A simple rigid body algorithm for structural dynamics programs. *International Journal for Numerical Methods in engineering*, **22**(3), 723-749.