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DAMAGE ANALYSIS METHOD FOR MULTIPLE DAMAGED ELEMENTS USING FRREQUENCY RESPONSES

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Abstract

This paper develops the 'Damage Analysis Method' to detect, locate and quantify multiple damages in civil structures. Damage extent coefficients are proposed to be used to locate and quantify damaged elements in structures altogether in one stage. They are obtained by solving a series of equations simultaneously. These equations are related to the degrees-of-freedom and to the summation of the product of the changes in frequency responses and stiffness of each element. A numerical truss structure is used to test and compare the proposed method with the 'Two-Stage Method' presented by Guo and Li [17] for different damage cases ranging from single damage to multiple damages. The results demonstrate that the proposed method locates and quantifies damages accurately. It also allows structures with multiple damages to be analyzed with high accuracy while the Two-Stage Method cannot be performed for all multiple damages scenarios. Moreover, the method works in one stage which removes the need for manual analysis and knowledge of the geometry of the structure, which are required for the Two-Stage Method.

Keywords: Damage detection; frequency responses; structural health monitoring; multiple damages

1. Introduction

Civil structures of the likes of bridges and buildings are the fundamentals of any urban fabric. However, due to the increase in population demand as well as the severe environmental conditions the structures are faced to, they are in constant degradation, which as a result, increases the risk of catastrophic failures. Also, after the occurrence of natural disasters such as earthquakes and tsunamis, these structures are left with severe damages which make them prone to failure. Therefore, Structural Health Monitoring (SHM), which aim is to monitor the structural condition of structures, is essential for the safety of the society. It allows the authorities to take appropriate measures to maintain and replace damaged structural components before more severe damages occur, hence increasing the safety and lifetime of these structures.

The traditional approach used in SHM consists of labor-intensive visual inspection and several non-destructive local damage detection methods such as acoustic emission and ultrasound [1]. This approach usually requires the damage locations to be known a priori, and accessible for inspections, which makes it not practical [2]. Moreover, only damages on or near the surface of the structures can be detected which limits the reliability of the traditional approach [3]. As a result, in the past decades, researchers have been active in developing methods for global damaged detection which monitor the changes occurring in the vibration properties of the structures under consideration [2]. The principle behind this vibration-based approach is that, any change occurring in the physical properties (mass, stiffness and damping) of the system due to damage (e.g. cracks and lose connections), will alter its vibration properties (natural frequency, mode shape, etc.) and frequency responses (displacement, velocity and acceleration). If the change of vibration characteristics is captured, damages in the structure may be located and quantified via the processing of the measured data.

One of the strategies used in the vibration-based approach is to monitor the changes of natural frequency which occur as a result of damage [2-7]. Natural frequency can be readily obtained using only a few sensors and is not highly affected by the presence of noise, which makes its use attractive in SHM [6]. However, only large damages can be detected due to the fact that small damages will not change the natural frequency substantially [3]. Also, finding the exact location and severity of damage is not always possible due to the non-uniqueness of the natural frequency obtained from different damage scenarios [6]. It is obvious that for a symmetrical structure, damage with the same severity at the same symmetrical location will result in the same natural frequency. Also, different damage severities at different damage locations can also produce the same natural frequency. Moreover, system identification is required to extract the natural frequency from the raw measured data, which might introduce errors. Thus, due to these constrains, the use of natural frequency is limited.

Another strategy proposed in the literature is to use mode shape of the structure and its derivatives as key parameters for damage detection [2, 5-10]. As mode shape is sensible to local damages, this feature can therefore be used to locate and quantify multiple damages in a structure [6]. However, similar to natural frequency, the mode shape needs to be extracted from the raw measurement, which as a result can introduce errors. This approach also has the disadvantage that the measured data is easily contaminated by noise which therefore decreases its accuracy [1]. In addition, a large amount of sensors are required and the placement of these sensors is subject to the expertise of the engineers [6]. Hence, the use of mode shape for damage detection is restricted.

Both of the strategies mentioned above have the common disadvantage that preprocessing of the raw measured data is required to extract these natural frequency and mode shape; therefore, devising an approach that uses the raw measurements directly becomes more appealing. In this regard, one approach is to use the frequency responses and frequency response function of the structures directly for damage detection [11-16]. The use of frequency responses and frequency response function eliminate the computational complexities and associated numerical errors in extracting the modal parameters [11]. Thus, the use of this approach in SHM is appealing. The ‘Two-Stage Method’ proposed by Guo and Li [17], which is the subject of a comparison with the proposed method in this paper, works in two stages using frequency responses. In the Two-Stage Method, a damage index is firstly proposed to locate potentially damaged elements, followed by determining the damage extent using a reduced system of equations which is related to the degrees-of-freedom (DOFs) of the potentially damaged elements. The method performs well for single damaged cases; however, for multiple damaged cases, it requires that the amount of available DOFs is equal to or greater than the number of potentially damaged elements. Also, the method works in two stages which makes it less favorable in implementation.

In this paper, a method to analyze structures with multiple damaged elements using frequency responses is proposed. This method detects, locates and quantifies multiple damages in structures accurately, and works for cases where the amount of DOFs of the structure is less than the amount of damaged elements. Moreover, it works in one stage which makes it advantageous, unlike conventionally been done where two stages of analysis are required [17].

2. Formulation of Damage Analysis Method

Consider a structure with n number of DOFs subjected to a forced harmonic excitation. The mathematical model representing the motion of the structure can be given by the equation of motion as follows

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{C}\dot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{f}e^{j\omega t} \tag{1}$$

Where

M, **C** and **K** are the n × n mass, damping and stiffness matrices of the structure, respectively;

X(t) is the n × 1 displacement response vector of the structure;

f is the n × 1 amplitude vector of the forced harmonic excitation;

j is the imaginary unit ($=\sqrt{-1}$);

ω is the angular excitation frequency of the forced harmonic excitation.

For simplicity, the damping matrix **C** is neglected. Thus, Eq. (1) becomes

$$\mathbf{M}\ddot{\mathbf{X}}(t) + \mathbf{K}\mathbf{X}(t) = \mathbf{f}e^{j\omega t} \tag{2}$$

Structural damage here is referred to as a reduction in the stiffness property of the structure with no change in the mass property. Hence, the equation of motion of the structure in its damaged state under the same forced harmonic excitation can be given as

$$\mathbf{M}\ddot{\mathbf{X}}_d(t) + \mathbf{K}_d\mathbf{X}_d(t) = \mathbf{f}e^{j\omega t} \tag{3}$$

Where

$\mathbf{K}_d$ is the n × n stiffness matrix of the damaged structure;

$\mathbf{X}_d(t)$ is the n × 1 displacement response vector of the damaged structure.

The stiffness matrix and the displacement response vector of the damaged structure can be expanded and rewritten as

$$\mathbf{K}_d = \mathbf{K} - \delta\mathbf{K} \tag{4a}$$

$$\mathbf{X}_d = \mathbf{X} - \delta\mathbf{X} \tag{4b}$$

Where

$\delta\mathbf{K}$ represents the small change of the stiffness matrix due to damage;

$\delta\mathbf{X}$ represents the small change of the displacement response vector due to damage.

Substituting Eq. (4) into Eq. (3) and noting Eq. (2) leads to

$$\mathbf{M}\delta\ddot{\mathbf{X}}(t) + \mathbf{K}\delta\mathbf{X}(t) + \delta\mathbf{K}\mathbf{X}_d(t) = 0 \tag{5}$$

Since the input excitation is harmonic, the steady-state output will also be harmonic. Therefore, at steady-state

$$\mathbf{X}(t) = \mathbf{Y}e^{j\omega t} \tag{6a}$$

$$\delta\mathbf{X}(t) = \delta\mathbf{Y}e^{j\omega t} \tag{6b}$$

$$\mathbf{X}_d(t) = \mathbf{Y}_d e^{j\omega t} \tag{6c}$$

In which $\mathbf{Y}$, $\delta\mathbf{Y}$ and $\mathbf{Y}_d$ represent the n × 1 steady-state vibration amplitudes of $\mathbf{X}(t)$, $\delta\mathbf{X}(t)$ and $\mathbf{X}_d(t)$, respectively.

Substituting Eq. (6) into Eq. (5) leads to Eq. (7) which is only valid for a specific excitation frequency ω.

$$(\mathbf{K} - \omega^2\mathbf{M})\delta\mathbf{Y} + \delta\mathbf{K}\mathbf{Y}_d = 0 \tag{7}$$

Rearranging yields

$$\delta\mathbf{K}\mathbf{Y}_d = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y} \tag{8}$$

In which the left-hand side of the equation has an unknown term $\delta\mathbf{K}$ which represents the change in global stiffness of the structure as a result of damage. This change is due to the reduction in elemental stiffness of all the damaged elements, and it can be represented by

$$\delta\mathbf{K} = \sum_{s=1}^{NE} c_s\mathbf{K}_s \qquad (0 \le c_s \le 1) \tag{9}$$

Where

NE is the total number of elements in the structure;

$\mathbf{K}_s$ is the elemental stiffness matrix of the s^{th} element;

c_s is the damage extent coefficient of the s^{th} element with two extreme cases being $c_s = 0$ for no damage and $c_s = 1$ for fully damaged.

Substituting Eq. (9) into Eq. (8) yields

$$\left(\sum_{s=1}^{NE} c_s\mathbf{K}_s\right)\mathbf{Y}_d = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y} \tag{10}$$

Rearranging results into

$$\sum_{s=1}^{NE} c_s\mathbf{K}_s\mathbf{Y}_d = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y} \tag{11}$$

Eq. (11) can be decomposed into n number of equations, with each being expressed as

$$\sum_{s=1}^{NE} c_s \mathbf{K}_{s,p} \mathbf{Y}_d = v_p \tag{12}$$

Where

$\mathbf{K}_{s,p}$ is the pth row in the elemental stiffness matrix of the sth element;

v_p is the pth row of the n × 1 vector **v**, where $\mathbf{v} = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y}$.

Expanding Eq. (12) yields

$$c_1\mathbf{K}_{1,p}\mathbf{Y}_d + c_2\mathbf{K}_{2,p}\mathbf{Y}_d + c_3\mathbf{K}_{3,p}\mathbf{Y}_d + \cdots + c_{NE}\mathbf{K}_{NE,p}\mathbf{Y}_d = v_p \tag{13}$$

Hence, similar to Eq. (13), the system of the n number of equations due to the decomposition of Eq. (11) can be expressed as

$$\begin{aligned} c_1\mathbf{K}_{1,1}\mathbf{Y}_d + c_2\mathbf{K}_{2,1}\mathbf{Y}_d + c_3\mathbf{K}_{3,1}\mathbf{Y}_d + \cdots + c_{NE}\mathbf{K}_{NE,1}\mathbf{Y}_d &= v_1 \\ c_1\mathbf{K}_{1,2}\mathbf{Y}_d + c_2\mathbf{K}_{2,2}\mathbf{Y}_d + c_3\mathbf{K}_{3,2}\mathbf{Y}_d + \cdots + c_{NE}\mathbf{K}_{NE,2}\mathbf{Y}_d &= v_2 \\ &\vdots \\ c_1\mathbf{K}_{1,n}\mathbf{Y}_d + c_2\mathbf{K}_{2,n}\mathbf{Y}_d + c_3\mathbf{K}_{3,n}\mathbf{Y}_d + \cdots + c_{NE}\mathbf{K}_{NE,n}\mathbf{Y}_d &= v_n \end{aligned} \tag{14}$$

The system of equation above, which is strictly valid for a specific excitation frequency ω, can be solved simultaneously to obtain the damage extent coefficient of each element present in the structure. In order to solve all unknown damage extent coefficients, the amount of DOFs in the structure must be equal to or greater than the number of elements present in the structure. However, this is not always the case. To tackle this problem, several sets of equations may be generated using different excitation frequencies (ω_1, ω_2, …, ω_e), where the subscript e represents the amount of excitation frequency used. Following this approach, a new set of equations can be formed by selecting NE equations from the generated sets of equations, followed by solving the new set of equations simultaneously to obtain the damage extent coefficients. Note that the undamaged elements will have damage extent coefficient of zero while the damaged elements will have values between zero and one with one representing fully damaged.

The proposed method allows the damage locations and the damage extent to be obtained altogether in one stage. The method is also applicable in detecting damages of a structure, regardless of one or multiple damaged elements are present.

3. Formulation of Two-Stage Method

Since the incentive of this research is to improve the Two-Stage Method proposed by Guo and Li [17], their method is briefly described below for convenience. The Two-Stage Method works in two stages by first locating the potentially damaged elements, followed by giving the damaged extent of the damaged elements.

The mathematical model under consideration is the same as that described in Eq. (1); therefore, formulation of damaged elements is exactly the same as that listed in Eqs. (2) - (7). In this method, to locate the potentially damaged elements, the left hand-side of Eq. (8) which has an unidentified term δ**K** due to the reduction in stiffness caused by damage, is represented by a damage index η so that

$$\eta = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y} \tag{15}$$

After determining the potentially damaged elements using Eq. (15), the second phase is to quantify the damage. Damage is assumed to be a reduction in global stiffness of the structure which can be given by the summation of the product of each elemental stiffness matrix and an associated damage coefficient as

$$\delta\mathbf{K} = \sum_{j=1}^{NC} \delta c_j \mathbf{K}_j \qquad (0 \le \delta c_j \le 1) \tag{16}$$

Where

$\mathbf{K}_j$ is the elemental stiffness matrix of the j^{th} element;

δc_j is the damage coefficient of the j^{th} element;

NC is the total number of elements in the structure.

Since only the damaged elements will contribute to the change of the global structural stiffness, Eq. (16) becomes

$$\delta\mathbf{K} = \sum_{r=1}^{ND} \delta c_r \mathbf{K}_r \qquad (0 \le \delta c_r \le 1) \tag{17}$$

Where

$\mathbf{K}_r$ is the elemental stiffness matrix of the r^{th} potentially damaged element;

δc_r is the damage coefficient of the r^{th} potentially damaged element;

ND is the total number of potentially damaged elements in the structure.

Substituting Eq. (17) into Eq. (8) yields

$$\left(\sum_{r=1}^{ND} \delta c_r \mathbf{K}_r\right)\mathbf{Y}_d = (\omega^2\mathbf{M} - \mathbf{K})\delta\mathbf{Y} \tag{18}$$

Eq. (18) which is an n-dimensional system of equation, can be reduced to an ND-dimensional system of equation which is related to the DOFs correlating to the suspected damage elements as

$$\left(\sum_{r=1}^{ND} \delta c_r \overline{\mathbf{K}}_r\right)\mathbf{Y}_d = (\overline{\omega^2\mathbf{M} - \mathbf{K}})\delta\mathbf{Y} \tag{19}$$

Where

$\overline{\mathbf{K}}_r$ is the ND × n reduced elemental stiffness matrix of the r^{th} element;

$(\overline{\omega^2\mathbf{M} - \mathbf{K}})$ is the ND × n reduced matrix of $(\omega^2\mathbf{M} - \mathbf{K})$.

Eq. (19) can be re-written as

$$\boldsymbol{\beta}\delta\mathbf{c} = (\overline{\omega^2\mathbf{M} - \mathbf{K}})\delta\mathbf{Y} \tag{20}$$

In which $\delta\mathbf{c}$ and $\boldsymbol{\beta}$ represent the damage coefficient vector of the suspected damage elements and a ND × ND matrix with each column represented by $\boldsymbol{\beta}_{ri}$, respectively. Each component of $\delta\mathbf{c}$ represents the individual damage extent of each potentially damaged element expressed as δc_r, where the subscript r indicates the r^{th} potentially damaged element. $\boldsymbol{\beta}_{ri}$ can be expressed as

$$\boldsymbol{\beta}_{ri} = \overline{\mathbf{K}}_r \, \mathbf{Y}_d \tag{21}$$

Rearranging Eq. (20) and using Eq. (21), the damage coefficient vector can be obtained from

$$\delta\mathbf{c} = \boldsymbol{\beta}^{-1}(\overline{\omega^2\mathbf{M} - \mathbf{K}})\delta\mathbf{Y} \tag{22}$$

The method works in two stages which makes it less desirable when compared to the Damage Analysis Method which performs in only one stage. It is also worth noting that, to quantify the damage extent, the amount of available DOFs should be equal to or greater than the amount of potentially damaged elements present in the structure. However, this is not always possible and this premise makes the method not feasible for cases where many elements are damaged. Manual analysis and knowledge of the geometry of the structure are also required to select the potentially damaged elements, which increase the complexity of the method.

4. Numerical Verification

4.1 Model description

A two-dimensional truss structure model shown in Fig. 1 is used to test and compare the Damage Analysis Method with the Two-Stage Method. The finite-element model of the structure consists of 31 elements and 28 DOFs. All the elements are made of the same steel material with Young's Modulus of 200 GPa, density of 7850 kg/m^3, and cross-sectional area of 0.001 m^2. A harmonic excitation force is applied horizontally at the 27th DOF to excite the structure. For the Damage Analysis Method, since the number of elements is greater than the number of DOFs in the structure, two different excitation forces with frequencies ω_1 = 2000 rads/sec and ω_2 = 2450 rads/sec are used to generate two sets of frequency responses. For the Two-Stage Method, only one excitation force with ω = 2000 rads/sec is used. The forced harmonic excitation amplitude used in both methods is 1N.

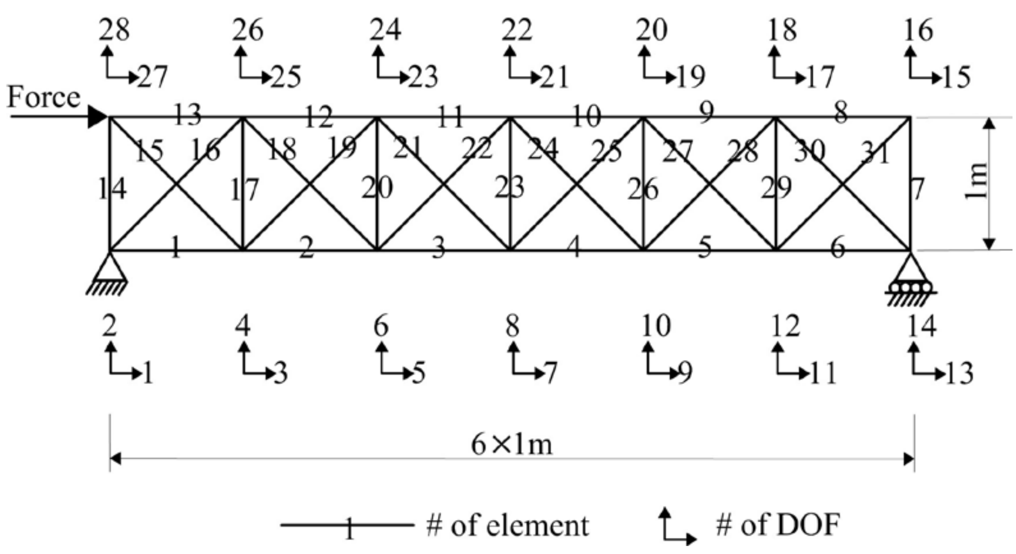

Fig. 1 – Two-dimensional truss structure model

4.2 Case studies

Damage in the structure is simulated as a reduction in elemental axial stiffness of the members. Three different damage cases ranging from single damage to multiple damages are used to test and compare the two methods. The first case is a single damage case with element 8 having a reduction in stiffness of 10%. The second case is a multiple damage case with elements 10 and 21 having a reduction in stiffness of 30% and 20%, respectively. The last case is an extreme case where all the diagonal elements in the structure have their stiffness being reduced by 20%.

4.2.1 Case 1: 10% reduction in stiffness of element 8

The Two-Stage Method is first applied to the structure. The damage index is presented in Fig. 2, and it can be seen that DOFs 15 and 17 are affected. Since only element 8 is related to both affected DOFs, it is concluded that only element 8 is potentially damaged. To obtain the damage extent, only one DOF is required and DOF 17 is selected. By selecting DOF 17 in the damage quantification formula in Eq. (22), the damage extent is calculated as $\delta c_8 = 0.10$ which represents a reduction in stiffness of 10% in member 8.

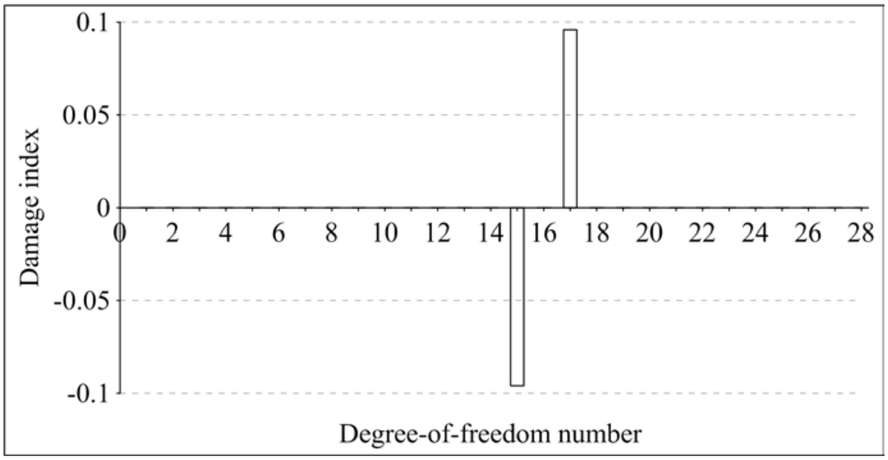

Fig. 2 – Damage index for case 1

Now, by using the proposed Damage Analysis Method, the location of the damage elements with their damage extent can be obtained directly in one stage. The results are presented in Fig. 3, and it can be seen that only element 8 is damaged with a reduction in stiffness of 10% ($c_8 = 0.10$).

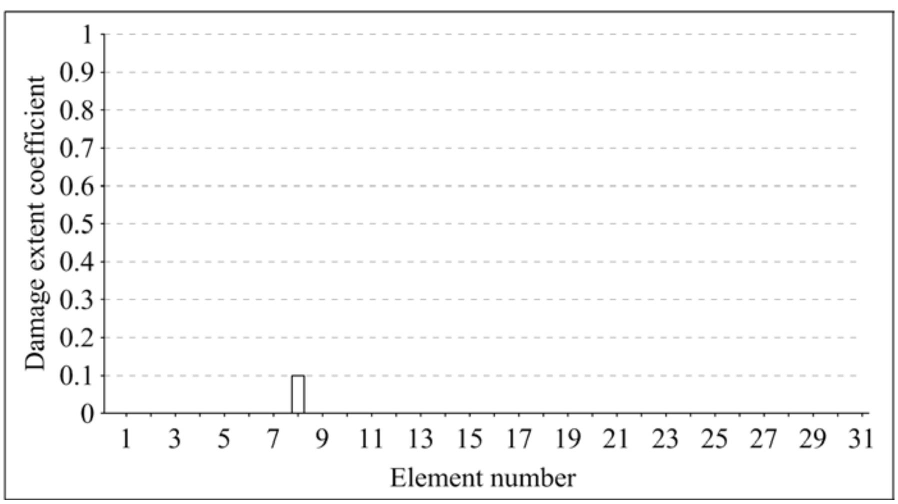

Fig. 3 – Damage extent coefficient for case 1

The true damage element is identified by both methods. The results from both methods show that the damage location and extent can all be accurately obtained. Therefore, it can be concluded that both methods perform well for the case of a single damage in the structure. However, the Damage Analysis Method requires less processing than the Two-Stage Method, which makes it advantageous. Moreover, the Two-Stage Method requires manual analysis and knowledge of the geometry of the structure to select the potentially damaged elements depending on which DOFs are affected, which are not required to implement the Damage Analysis Method.

4.2.2 Case 2: 30% and 20% reduction in stiffness of elements 10 and 21, respectively

For the second case, the damage index obtained from the Two-Stage Method is presented in Fig. 4. It is observed that DOFs 19 and 21 are highly affected while DOFs 7, 8, 23 and 24 are slightly affected when compared to other DOFs. Since only element 10 correlates to both DOFs 19 and 21, element 11 to DOFs 21 and 23, and element 21 to DOFs 7, 8, 23 and 24, these three elements are suspected to be damaged. By selecting DOFs 7, 19 and 21 into stage 2 of the Two-Stage Method, the damage coefficient formula calculates the damage coefficients as $\delta c_{10} = 0.30$, $\delta c_{11} = 0.00$ and $\delta c_{21} = 0.20$. These indicate that element 11 is not damaged while elements 10 and 21 are damaged with a reduction in stiffness of 30% and 20%, respectively.

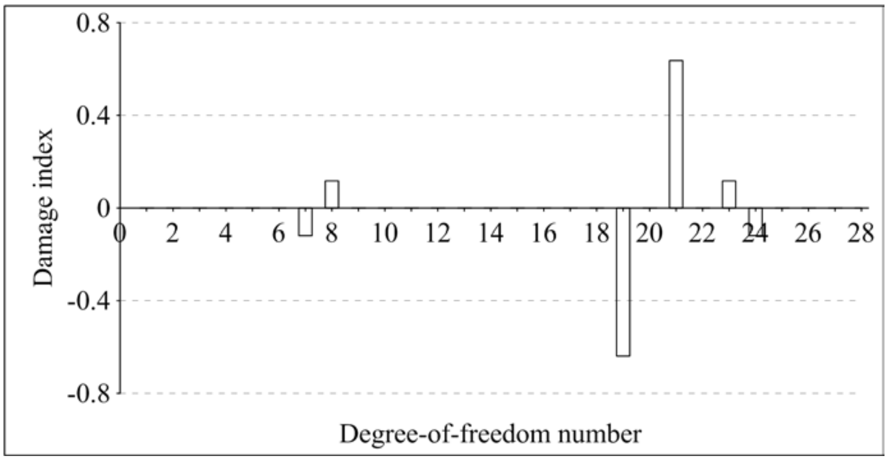

Fig. 4 – Damage index for case 2

For the Damage Analysis Method, the results obtained are presented in Fig. 5. It can be seen from the figure that elements 10 and 21 have a reduction in stiffness of 30% and 20%, respectively.

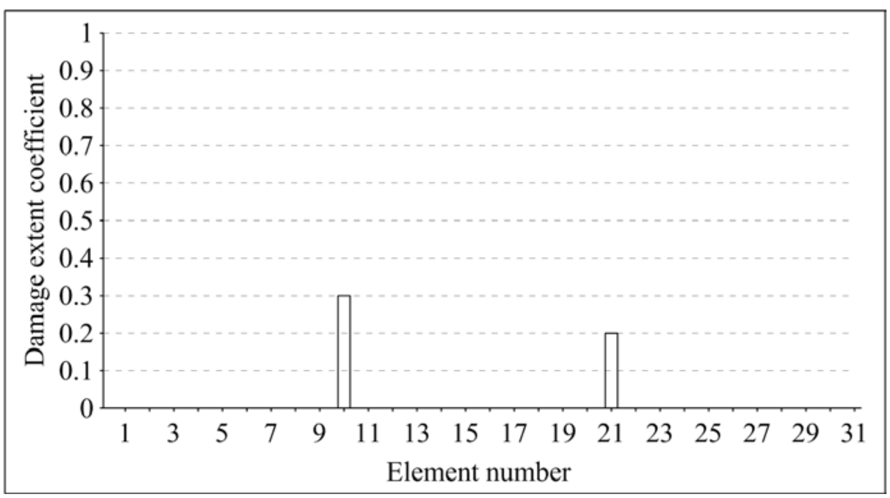

Fig. 5 – Damage extent coefficient for case 2

From the damage index of the Two-Stage Method, three elements are suspected to be damaged. After calculating the damage coefficients, it is found that only elements 10 and 21 are damaged. Element 11 is suspected to be damaged because its associated DOFs are also associated to elements 10 and 21 which are damaged. However, the damage coefficients are used to validate the exact damage scenario. For multiple damages cases, there is a risk that the Two-Stage Stage Method suspects elements that are not damaged to be damaged. Therefore, the damage index cannot be performed alone even for cases where only the damage locations are required. The damage coefficients should be calculated to validate the exact damage case.

For the Damage Analysis Method, the results show that the true damage case is found directly and in one stage with high precision.

4.2.3 Case 3: 20% reduction in stiffness of all diagonal elements

For case 3, the damage index is given in Fig. 6. It can be seen that all the DOFs are affected and therefore, all the elements are suspected to be damaged. Thus, to validate this, the damage quantification formula needs to be employed. However, since there are more potentially damaged elements than DOFs available, the damage quantification formula cannot be employed to obtain the damage extent of the elements. The damage locations without their severities also cannot be obtained as was shown in case 2. As a result, it can be concluded that for this case, the Two-Stage Method is not applicable.

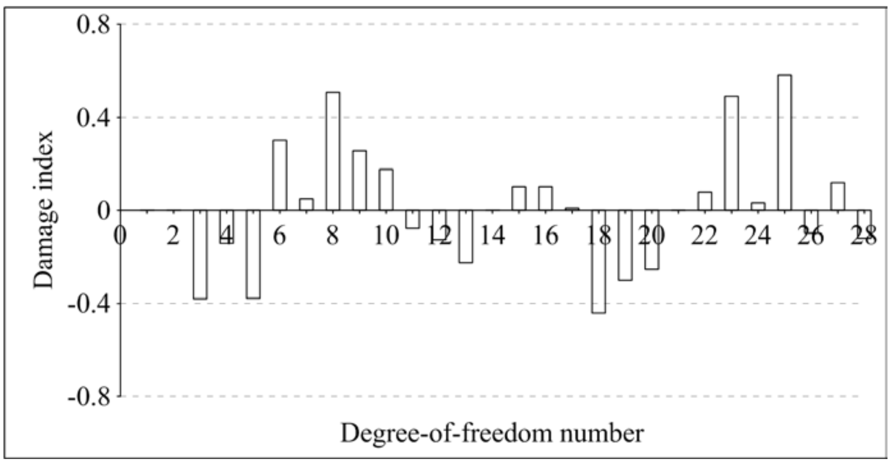

Fig. 6 – Damage index for case 3

For the Damage Analysis Method, the results are presented in Fig. 7. It can be seen that for this extreme case, the damage locations and extent are well determined. The elements 15, 16, 18, 19, 21, 22, 24, 25, 27, 28, 30 and 31 are all identified to have identical reduction in stiffness of 20%.

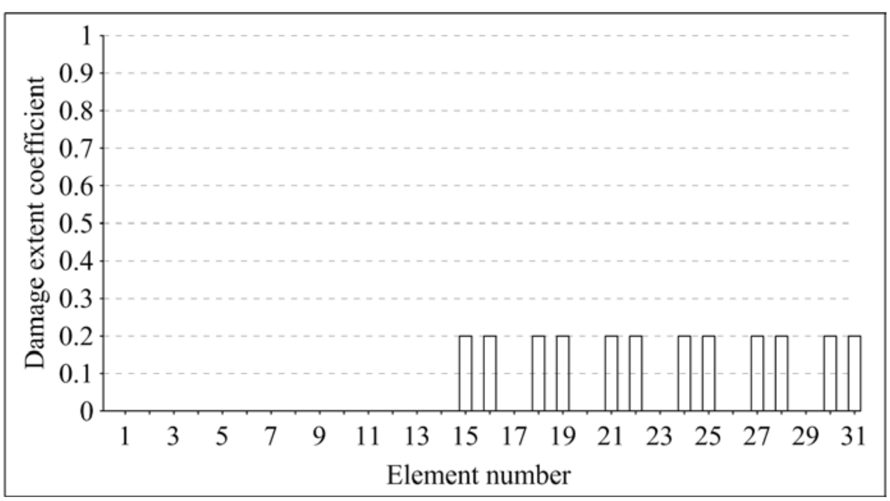

Fig. 7 – Damage extent coefficient for case 3

From the results obtained, it can be concluded that the Two-Stage Method cannot be employed when the amount of suspected damaged elements is more than the available amount of DOFs associated to them. This is because to employ this method, each suspected damaged element should have at least one correlated DOF which can only be used for that element. This, therefore, makes this method not practical for damage localization and quantification of structures containing multiple damages.

For the Damage Analysis Method, it can be seen that it performs well for this extreme case due to the fact that two excitation forces are applied one at a time to generate two different sets of frequency responses, and therefore equations. In general, even if all the elements in the structure are damaged, this method would give their damage locations and extent accurately. This is because to apply this method, a minimum of NE number of equations is required, and can be generated by applying different excitation forces one at a time to the structure.

5. Discussion and conclusion

A method using frequency responses is proposed in this paper to locate and quantify multiple damages in civil structures. Damage extent coefficients are proposed to be used to locate and quantify the damaged elements all together in one stage. They are obtained by solving a set of equations simultaneously. The method is tested on a truss structure and is compared to the method proposed by Guo and Li [17]. From the results obtained, it is found that multiple damages in civil structures are well located and quantified using the proposed method. Extreme cases where the damaged elements affect all DOFs are also well located and quantified using the proposed method which is not able using the Two-Stage Method. The method works in one stage and does not require manual analysis and knowledge of the geometry of the structure after the equations have been formulated, which are required for the other method to select the potentially damaged elements. This, therefore, eliminates confusion in implementing the method. However, it is not always possible to use only one excitation force to generate the equations to apply the proposed method. Depending on the structure to be analyzed, the amount of different excitation forces required varies.

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