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SIMPLIFEID APPROACH IN ANALYZING INELASTIC BEHAVIOR OF SHALLOW FOUNDATION SUBJECTED TO DYNAMIC LOADS

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Abstract

This paper introduces a new approach for seismic Soil-Structure Interaction (SSI) analysis of shallow foundations using computationally efficient yet practically accurate models. The study is inspired by the increasing interests amongst engineers and researchers in the nonlinear cyclic or rocking behavior of shallow foundation. Furthermore as the industry moves toward the performance based seismic design, the accurate modelling of the inelastic cyclic or rocking behaviour of shallow foundation has attracted considerable attention of researchers. There are two main characteristics of a shallow foundation that need to be considered in a numerical model: inelastic cyclic behaviour of near field soil and frequency-dependent characteristic of soil.

One of the rigorous and computationally expensive approaches which can capture both characteristics is to use a sophisticated constitutive model of soil materials, and developing a fine mesh of soil-foundation system using the finite element method. However, due to the extremely large modelling efforts and computational time, the FEM approach is not a feasible option in a routine engineering practice. To overcome the limitation, researchers have proposed macro-elements. In the macro elements, the inelastic cyclic behavior of soil-foundation interaction is captured at the lumped node where geometric nonlinearity such as uplift and sliding of the foundation as well as material nonlinearity are modelled simultaneously. The macro element's inelastic behaviour is defined by a plasticity law where its stiffness is updated at each computational step using the generalized force-displacement relationship with the bounding surface hypo-plastic model.

Macro-element is able to capture the inelastic behavior of the soil foundation system at the vicinity of the foundation. As the element replaces soil and foundation with hypo-plastic spring model, the soil mass and damping properties are missing in this element, which results in inability to capture wave propagation from the foundation to the structure. Soil exhibits frequency-dependent characteristic of stiffness and energy dissipation. Inclusion of the dynamic stiffness is pivotal in order to extend the application of this element to the domain of dynamic loading condition. In the current practices, the soil-foundation system is often modelled with elastic springs and dampers for simplicity which are inherently frequency-independent. In reality, soil exhibits frequency dependent characteristic at different frequencies of excitation.

In this paper, it is proposed to integrate a macro element which can capture the inelastic cyclic behaviour of soil-foundation system with a recursive parameter model which can model the frequency-dependent dynamic stiffness in time domain. By the integration of these two models, it can approximately capture the inelastic SSI effect of the soil foundation system and the frequency dependency of soil simultaneously. First part of the paper presents the introduction of the macro-element and the second part of the paper introduces the recursive parameter model. Then, the paper illustrates the combination of the two models using schematic diagrams and equations. The verification of the proposed method is provided with FEM model.

Keywords: Shallow foundation, Soil-structure interaction (SSI), Macro-element, and Frequency-dependency of soil.

1. Introduction

Shallow foundations exhibit inelastic behavior in soil-foundation interface where rocking of the foundation occurs upon excessive load. As the Performance Based Design (PBD) approach requires seismic evaluation of a structure with the target building performance objectives, explicit assessment of the response of structural components are recommended [1]. ASCE 41-13 [2] provides a new component action tables which provides modeling parameters and acceptance criteria for nonlinear and linear analysis of shallow foundation [3]. These values in the component action tables for nonlinear procedures are based on analysis of foundation performance in experimental model tests on rocking foundations. The acceptance criteria for linear analysis procedure is determined by the maximum allowable moment demand in a linear analysis divided by elastic yield strength, which is often referred to as m-factor. The m-factor includes empirical coefficient which reflects experimental data with rocking of the foundation in ASCE 41-13 and also are derived from the allowable rotation for nonlinear procedures [3]. In ASCE 41-13 [2], coupled foundation rocking and yielding at the soil-foundation interface are considered in the analysis procedure, unlike ASCE 41-06 [4] where both actions were decoupled and were separately considered [5]. This is more realistic since the failure of the foundation is governed by these two mechanism depending on the stiffness and yielding of the soil [5]. Thus, more realistic and accurate assessment of the foundation deformation effects can be obtained in the seismic evaluation of structure in PBD approach using ASCE 41-13 in a case of rocking foundation [6]. Kutter et al. provide a rationale for the revisions made in ASCE 41-13 regarding the rocking of the foundation [3] and validate the non-linear modeling parameters and acceptance criteria for rocking shallow foundation using extensive experimental results [6].

The continuum approach using finite element or boundary element method is the most rigorous approach yet computationally expensive [1, 6] for applications in engineering practice. There are many simplified models that are able to capture the inelastic behavior of shallow foundation, some of which are: lumped spring approach [3], beams on Winkler type foundation [7, 8], simplified spring model [1, 9], and macro-element [10, 11, 12]. Lumped spring provides the most simplistic approach where the foundation is represented by a linear spring and dashpot. This model does not capture the inelastic behavior of the soil, but it is a practical tool for conservative foundation design application [3]. Beams on Winkler type foundation is another simplified model where series of springs are used to represent the inelastic behavior of the foundation. However, there are many calibration parameter required for the model to capture the inelastic response of foundation. One study have found that 101 vertical nonlinear springs are required to accurately capture the nonlinear planar response of shallow foundation [8]. Another simplified spring model uses the rotational nonlinear spring to simulate the inelastic behavior of soil-foundation system for shallow foundation. This model is mainly focused on the sway-rocking motion of the foundation and provides satisfactory results compared to FEM model for pushover analysis [10]. Although this model is simple to use, there are limitations with coupled inelastic behavior of the soil-foundation system. Macro-element has been proposed to capture the inelastic behavior of shallow foundation with coupled behavior of uplift and sliding of the foundation in a lumped node. The foundation is represented with three DOFs in vertical, horizontal and rotational directions for two dimension (2D) analysis. The plasticity of the soil model is formulated based on the hypoplastic bounding surface. The quasi-static loading cases for macro-element are in good agreement with FEM results [10, 11].

Of these simplified models, all the models share one common limitation, where full frequency dependent property of soil is not captured. As the foundation is replaced by springs for some models, the mass and damping components of the soil are neglected. This leads to inaccurate wave propagation from foundation to the soil domain as the inertia and energy dissipation in the soil is missing. In some models, a constant dashpot has been used to tackle this problem. However, this method leads to frequency-independent behavior of soil which is not a realistic representation of the soil domain. Mylonakis et al. have compiled studies regarding the frequency-dependent characteristics of soil foundation system and have provided a guideline for engineers to include frequency-dependent properties of soil [14]. In their study, it is concluded that soil foundation system exhibit frequency-dependent characteristic even for low magnitude cyclic load. Neglecting the frequency-dependent characteristic of soil may lead to inaccurate representation of the soil-foundation system. Lesgidis et al. have investigated various case studies of RC bridge columns and have found that the error in calculated the fragility analysis curve is significant when considering frequency-independent soil properties [15]. Thus, accurate modeling of foundation with simplified models including frequency-dependency of soil is pivotal in capturing the accurate response of shallow foundation subjected to seismic loads. In this paper a combined macro-element and recursive-parameter model is used as a simplified model to incorporate frequency-dependent characteristics of the soil foundation. Due to its simplicity in framework with only a few calibrated parameter required from the user, this element has many attractive attributes in analyzing shallow foundation.

2. Introduction to the proposed method

Macro-element and a recursive parameter model have been used in this paper in order to capture dynamic nonlinear behavior of shallow foundations. Although there are various methods available for analyzing nonlinear behavior of soil-foundation systems, macro-element has simple template to work with lumped node at the foundation interface. A recursive parameter model introduced by Nakamura [14, 15] is adopted in this paper to present the frequency-dependent properties of soil in time domain. Nakamura's recursive parameter model has been chosen specifically in this proposed method as it provides stable algorithm in transforming impedance functions in frequency domain to a recursive parameter model for time domain analysis. More details on the stability of the recursive parameter models are provided by Duarte et al. [18]. The proposed model in this paper presents a general approach in integrating these two models. More details on the general concept of the proposed model are presented below.

2.1 General Concept

During seismic excitation of soil domain, both material damping and radiation damping occurs in soil domain [14]. Material damping describes energy dissipation occurring in soil material whereas radiation damping describes energy dissipation occurring from waves propagating away from the foundation to the infinite soil domain. In many simplified models including macro-element, the material damping is taken into consideration from the plasticity of the soil [19]. However, radiation damping is not taken into consideration in these simplified models as the energy dissipation away from the foundation to soil domain is neglected. In order to properly account for the radiation damping, frequency-dependent properties of soil, often referred to as dynamic impedance of soil, is required. However, as the nonlinear dynamic analysis of a structure and foundation system is performed in the time domain, it is challenging to incorporate dynamic impedance of soil which is represented in the frequency domain. Pecker et al. [11] and Chatzigogos et al. [12] have used a constant damping coefficient to include the radiation damping from foundation to the soil for macro-element. The value of the dashpot coefficient is determined based on the dynamic impedance of soil at the specific frequency. As the macro-element analyzes the nonlinear behavior of soil-foundation system near the vicinity of the footing, the constant dashpot is included to represent the far-field wave propagation of the soil domain. The Fig. 1 shows the schematic diagram of the overall soil domain represented with macro-element.

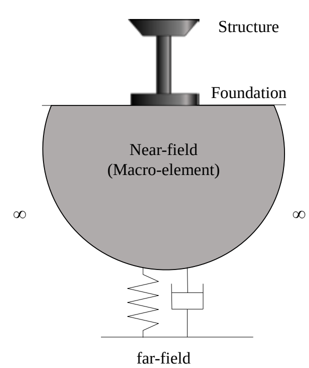

Fig. 1. Schematic diagram of macro-element with far-field soil domain

The choice of frequency for dynamic impedance is quite ambiguous as the user can define either predominant frequency of excitation or natural frequency of the soil-structure system [20]. Thus, in this paper, it is proposed to use recursive parameter model to include a full frequency-dependent properties of soil. The soil domain can be represented with restoring force occurring from macro-element as expressed in Eq. (1).

$$\boldsymbol{F}_{\mathbf{macroel}}(u,\dot{u}) = \boldsymbol{R} \tag{1}$$

This restoring force $\boldsymbol{R}$ is applied in direction opposite to the structure with applied force. In the combined model, the restoring force also contains reaction force from mass and damping of the soil which captures accurate wave propagation of soil. The governing equation of motion for soil is shown in Eq. (2).

$$\boldsymbol{M}_{soil} * \ddot{u} + \boldsymbol{C}_{radiation} * \dot{u} + \boldsymbol{F}_{macroel} = \boldsymbol{R} \tag{2}$$

In Eq. (2), the $\boldsymbol{M}_{soil}$ represents mass of the soil, $\boldsymbol{C}_{radiation}$ represents radiation damping of the soil, and $\boldsymbol{F}_{macroel}$ represents the restoring force occurring from macro-element. By re-arranging this equation, the overall equation is expressed as shown as Eq. (3):

$$\boldsymbol{M}_{soil} * \ddot{u} + \boldsymbol{C}_{radiation} * \dot{u} + \boldsymbol{K}_{soil} * u + (\boldsymbol{F}_{macroel} - \boldsymbol{K}_{soil} * u) = \boldsymbol{R} \tag{3}$$

From the Eq. (3), the terms $\boldsymbol{M}_{soil} * \ddot{u} + \boldsymbol{C}_{radiation} * \dot{u} + \boldsymbol{K}_{soil}\boldsymbol{u}$ can be represented with dynamic impedance of soil using recursive parameter model. Then, the overall soil restoring force can be formulated as shown in Eq. (4)

$$(\boldsymbol{F}_{macroel} - \boldsymbol{K}_{soil}\boldsymbol{u}) + \boldsymbol{F}_{recursive\ parameter} = \boldsymbol{R} \tag{4}$$

A recursive parameter model introduced by Nakamura [14, 15] can transform dynamic impedance functions of soil for time domain analysis. Duarte et al. [18] has worked extensively on stability analysis of the transformation of the dynamic impedance function functions from the frequency domain to the time domain. The authors have shown Nakamura's recursive parameter model to be the most stable algorithm over other recursive parameter models [18]. The recursive parameter model formulates the soil reaction force in terms of current and past response quantities such as displacement, velocity and acceleration values measured at the interface nodes. Detailed background information regarding the analysis of the components are described below.

2.2 Introduction to the adopted macro-element by Pecker et al. [11] and Chatzigogos et al. [13]

The proposed integration method for a macro-element and a recursive parameter model is general; thus it can be used with various macro-elements and recursive parameter models. In this study, the macro-element model by Chatzigogos et al. [13] is adopted. In the macro-element, a single node is placed at the centre of the rigid foundation. This node has horizontal, vertical, and rotational DOFs to represent the response of the footing in two-dimensional problems. The following force and displacement parameters are illustrated in Fig. 2.

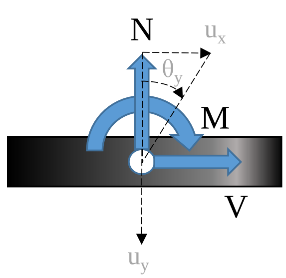

Fig. 2. Generalized force and displacement diagram for shallow foundation in macro-element

Using these parameters, the generalized force to displacement relationship can be represented with generalized stiffness matrix as shown in the Eq. (5).

$$\begin{bmatrix}\dot{Q}_N\\ \dot{Q}_V\\ \dot{Q}_M\end{bmatrix}=\begin{bmatrix}K_{NN} & K_{NV} & K_{NM}\\ K_{VN} & K_{VV} & K_{VM}\\ K_{MN} & K_{MV} & K_{MM}\end{bmatrix}\begin{bmatrix}\dot{q}_N\\ \dot{q}_V\\ \dot{q}_M\end{bmatrix} \tag{5}$$

For each respective DOFs and force-displacement relationship, macro-element calculates the response in increments denoted by the dots on each variables as shown in Eq. (5). Q_N , Q_V, Q_M, are the vertical, horizontal, and moment forces that are normalized with maximum bearing capacity of the foundation. The stiffness terms $K_{ij},\ i,j=N,V,M$ are the normalized elements of the stiffness matrix in each corresponding DOFs. The $\dot{q}_N,\dot{q}_V,\dot{q}_M$ are the vertical, horizontal, and rotational displacement terms normalized with dimension of the footing (width for strip foundation or diameter of the footing for circular foundation). This generalized relationship is used to describe the coupling effect of each DOFs at the foundation. More details on each stiffness terms for various soil and foundation types can be found in Malonakis et al. [14].

The mechanism of soil yielding in the vicinity of the footing is described by the bounding surface using hypoplastic model. For a simplified macro-element modelling, ellipsoidal shape bounding surface at the origin is used. This ultimate bounding surface defines the maximum bearing capacity of the foundation in each DOFs. In the interior of this bounding surface, a continuous plastic response is obtained as a function of the distance between the actual force Q to the image point I(Q) which lies along the bounding surface. The image point is the projection of the current force Q to the bounding surface as shown in Fig. 3.

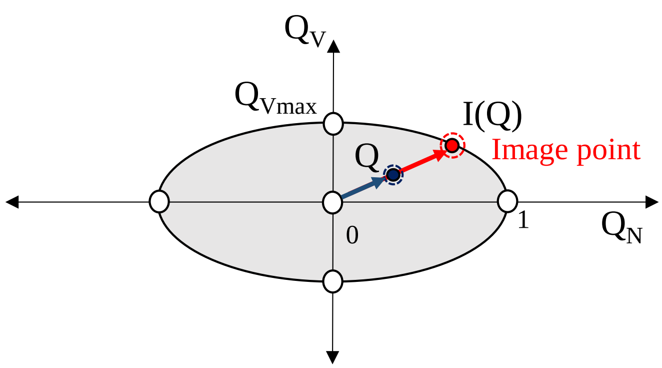

Fig. 3. Vertical and horizontal bounding surface with current force vector (Q) to image point (IQ)

This is also referred to as a mapping rule. As the actual force reaches close to the bounding surface, plasticity becomes more pronounced as defined in this mapping rule, where the ratio of current force to image point will get close to one. The image point defines the direction of plastic displacements and magnitude of plastic modulus. For this case, the plastic displacement is described by the inverse of the plastic modulus as shown in Eq. (6).

$$\dot{q}^{pl}=\bar{H}^{-1}\bar{\dot{Q}} \tag{6}$$

Then, the relationship can be written as the inverse of the plastic modulus in the form of Eq. (7).

$$\bar{H}^{-1}=\frac{1}{h}(n_g\otimes n_f) \tag{7}$$

This equation is used such that the multi-axial stress and load/unload direction are clearly defined. The variables n_g and n_f are used to distinguish the associative and non-associative flow rule which is discussed in detail in Chatzigogos et al. [19] and Pastor [21]. The magnitude of plasticity is defined by the scalar variable, h. The soil plasticity is defined by the combination of kinematic and isotropic hardening plasticity model, originally presented by Prevost in 1978 using clays [13]. For the case of loading history where the kinematic and isotropic hardening is applied, additional λ_{min} term is added to account for history of the maximum plasticity loading.

$$h = h_o \ln\left(\frac{\lambda^{p+1}}{\lambda^{p}_{min}}\right) \tag{8}$$

Where h_o is the numerical parameter determined by the particular foundation soil, and p is the numerical parameter which describes the plasticity extent of the response in reloading of the foundation soil. Both of the parameters are calibrated numerically and suggested values for range of various cases are provided in Chatzigogos et al. [13]. The λ_{min} is the minimum value obtained during the load history. The λ is ratio of the current force to the yield surface. When the maximum load is applied close to the bounding surface and reloaded, kinematic hardening effect is applied by the λ_{min} value. The variable h_o is the initial plastic stiffness defined by the user. The scalar factor matrix is used to multiply a factor to elastic term in order to define the initial plastic stiffness. There are lack of numerical or experimental results that is pertained to specific soil, thus, the user has limited soil types to work with the calibrated parameters [13]. After h is defined, then $\dot{q}^{pl}$ can be calculated as shown in Eq. (6). In the flow rule, the increment of strain is caused by two deformation components, i.e. elastic and plastic, which is expressed as the following in Eq. (9).

$$d\varepsilon = d\varepsilon^{e} + d\varepsilon^{p} \tag{9}$$

For the analysis of macro-element, elastic stiffness with uplift parameter is superimposed with plasticity of soil-foundation system as shown in Eq. (8). In this paper, the analysis of failure criterion was limited by the bounding surface of the foundation as shown in Fig. 3, i.e. the uplifting case is not considered. More details on the interface elements and the interaction between the foundation and the soil is provided in the literature for cohesive soil and frictional soil with general interface effect of sliding [11, 12].

2.3 Recursive parameter model in Nakamura [16, 17]

A recursive parameter is presented in this paper to illustrate the integration procedure of two simplified models, macro-element and Nakamura's coefficient terms, in order to include frequency-dependent properties of soil. In dynamic loading scenario, time domain analysis is used to analyze the inelastic behavior of structure. The frequency-dependent behaviour which is embedded in the concept of soil impedance is represented in the frequency domain. There have been many studies regarding the transformation method in order to convert the impedance of soil from frequency domain to time domain. Wolf [22] has studied extensively on this topic using impulse response with Fourier transformation. This method is widely used for its well established mathematical formulation. However, the method relies solely on superposition of linear elastic analysis. Also, the method is susceptible to numerical instability of the impedance functions that are causal [17]. Thus, another recursive parameter model has been studied by Nakamura [16] where the author has proposed a new transformation method which avoids this problem. Duarte et al. [18] compared the stability of various recursive parameter model and found Nakamura's recursive parameter model to be the most stable method. The frequency-dependent terms are converted in impulse response of displacement and velocity terms of the current and previous response.

In order to describe how Nakamura's coefficient terms are formulated from the soil impedance function, it is pivotal to review the concept of convolution integral. Convolution is a process which allows the output of the system to be calculated based on any arbitrary input signals with known impulse response of the system. This general definition of the term provides a powerful tool in which any signal process response could be calculated with summation of the delayed impulse response of the system. For instance, if there is an input signal *x(n)* and impulse response *h(n)*, then the output response, *y(n)*, can be summation of the delayed impulse response of the input signal *x(k)* at k^{th} term. Eq. (10) illustrates this relationship.

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k) * h(n-k) \tag{10}$$

Following this concept, Nakamura has formulated the following impulse response equation in the context of soil impedance function. The response of the restoring force from soil is expressed as Eq.(11).

$$F(t) = F_1(t) + \sum_{j=1}^{n} c_j' * F_1(t - T_j) \tag{11}$$

The response of the overall restoring force *F(t)* is the summation of the current impulse response $F_1(t)$ and the summation of the previous impulse response. In linear analysis, the impulse force response can be expressed with instantaneous mass, stiffness and damping terms. Then the corresponding restoring force can be expressed as Nakamura's recursive equation [17] which is expressed as the Eq. (12).

$$F(t) = \left(m_0 * \ddot{u}(t) + k_0 * u(t) + c_0 * \dot{u}(t)\right) + \left[\sum_{j=1}^{N-1} k_j * u(t - t_j) + \sum_{j=1}^{N-2} c_j * \dot{u}(t - t_j)\right] \tag{12}$$

Then, the corresponding impedance function can be expressed as shown in Eq. (13):

$$S(\omega) = -\omega^2 * m_0 + i\omega * c_0 + k_0 + \left[i\omega * \sum_{j=1}^{N-2} c_j * e^{-i\omega t_j} + \sum_{j=1}^{N-1} k_j * e^{-i\omega t_j}\right] \tag{13}$$

where parameters m_0, k_0 and c_0 represent instantaneous mass, stiffness and damping of the soil foundation, respectively. As the soil impedance function contains real and imaginary part, it can be described with the convolution terms as shown in Eq. (14):

$$S(\omega) = \begin{Bmatrix} Re[S(\omega_i)] \\ Im[S(\omega_i)] \end{Bmatrix} = \begin{Bmatrix} \sum_{j=0}^{N-1} cos\theta_{ij} * k_j + \omega_i \sum_{j=0}^{N-2} sin\theta_{ij} * c_j - \omega_i^2 m_0 \\ -\sum_{j=0}^{N-1} sin\theta_{ij} * k_j + \omega_i \sum_{j=0}^{N-1} cos\theta_{ij} * c_j \end{Bmatrix} \tag{14}$$

Finally, by transforming the coordinates to time domain by converting these terms to coefficient terms at each analysis step with matrix multiplication, a representation of soil impedance in time domain can be obtained. More details on the original derivation and formulation of this method are available in Nakamura [16]. After the coefficients are represented in the time domain, these parameters can be used to formulate the restoring force of the soil-structure interface resulting from soil impedance as expressed in Eq. (15):

$$\begin{aligned} \boldsymbol{R_{i+1}} &= m_0\ddot{u}_{i+1} + (C_0\dot{u}_{i+1} + C_1\dot{u}_{i+1-1} + C_2\dot{u}_{i+1-2} \dots) \\ &\quad + (K_0 u_{i+1} + K_1 u_{i+1-1} + K_2 u_{i+1-2} \dots) \\ &= m_0\ddot{u}_{i+1} + \sum_{j=1}^{N-1} C_j * \dot{u}_{i+1-j} + \sum_{j=1}^{N-1} K_j * u_{i+1-j} \end{aligned} \tag{15}$$

where $\boldsymbol{R_{i+1}}$ is the restoring force at time step i+1, and m_0 represents the instantaneous mass term of the soil. The time representation of the restoring force using recursive parameter model allows this element to be integrated into macro-element which calculates the nonlinear response of the soil foundation system at each time step of the analysis. By representing both simplified models (macro-element and Nakamura's recursive parameter model) as a restoring force occurring from soil, the proposed model captures the inelastic behavior of soil foundation system including the frequency-dependent properties of the soil. The proposed model is verified against FEM model for a quasi-static loading scenario as well as a dynamic harmonic excitation in order to compare the difference among the proposed method, the FEM model and the original macro-element approach. More details on the mathematical derivation of the proposed model can be found in Chai [23].

3. Numerical Verification

A case study including a rigid massless strip foundation resting on a soil domain model is selected for the verification analysis. The size of the soil domain is 100m by 100m and the strip rigid foundation has a length of 10m. The soil properties are presented in Table 1. Two different load cases including quasi-static and harmonic excitation at large amplitude with excitation frequency of 2Hz are considered for the analysis purpose. The results in terms of different response quantities are calculated using the proposed method and compared with those captured by FEM analysis with von Mises failure criterion material model.

Table 1. Material properties for homogeneous infinite soil domain

Parameters	Units	Values
v	Poisson's ratio	0.25
ρ	Density of soil (t/m3)	1.6
Co	Cohesion of soil (KN)	30
Vs	Shear wave velocity (m/s)	201.5
G	Shear modulus (KN/m3)	$G = \rho * Vs^2$ $G = 64963.6$
E	Elastic modulus (KN/m)	$E = 2 * G * (1 + v)$ $= 162409$
B	Foundation width (m)	2

Note: these values are used for strip foundation with unit width of 1m.

Firstly, the ellipsoidal bounding surface is formulated in macro-element based on the calibrated parameter of the soil model. These values are obtained based on maximum bearing capacity of the foundation in horizontal, vertical, and moment load. Then, in order to verify whether the approximation of bounding surface with ellipsoid is appropriate, a FEM model with the combined quasi-static loading scenario is analyzed until the failure of the model is captured (i.e. non-converging solution). Then, the failure surface generated using the FEM model is compared with the simplified bounding surface used in macro-element. The results are shown in Fig.4 on the left for 3D view of the bounding surface and right for 2D moment and vertical load view.

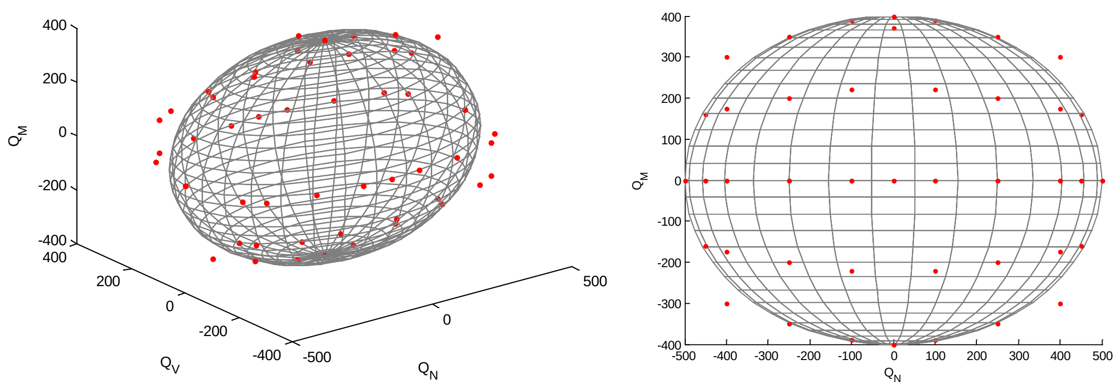

Fig. 4. Bounding surface generated from FEM and macro-element

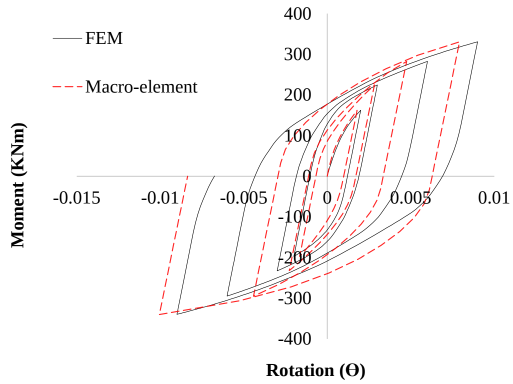

Fig. 5. Quasi-static load for moment vs. rotation at the foundation

As shown in Fig. 5, the response of foundation subjected to cyclic moment load is presented for both FEM and macro-element models. Macro-element requires two calibration parameters to define the plasticity model of the soil-foundation system. More details on the calibration procedure and range of suggested values are provided in Chatzigogos et al. [13] and Chai [23]. As shown in Fig. 5, the results are in good agreement for the soil model of 100 m by 100 m with a rigid beam width of 10 m for the soil properties defined from Table 1. Once the quasi-static loading scenario is verified, then sinusoidal excitation with 2000 KNm of moment is applied to the foundation with 2 Hz frequency of excitation. The analysis results among different methods including macro-element, Nakamura's recursive parameter model, and FEM are compared. For the FEM analysis, commercial FEM package *RS*^{2} and *OpenSees* has been used for verification. Fig. 6 illustrates the captured displacement time-history analysis results using different models. Fig. 7 presents the generated hysteretic loop for different numerical approaches.

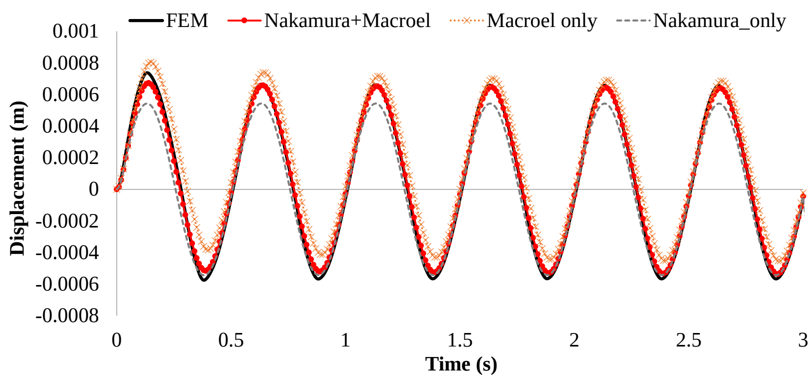

Fig. 6. Displacement history results for the simplified models and FEM model

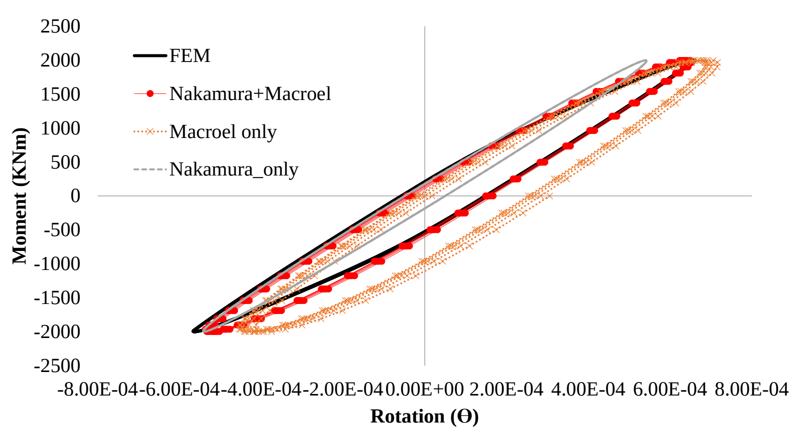

Fig. 7. Hysteretic loop for simplified models and FEM model with cyclic moment load

As shown in Fig. 6 and Fig. 7, it is found that the proposed model agrees well with the FEM result for the case with cyclic moment excitation applied at the foundation. The macro-element without frequency-dependent properties of soil shows some difference with the FEM results. In addition, the single recursive parameter model does not capture the overall behavior of the inelastic response of the foundation and a significant difference is observed compared to FEM results as shown in Fig. 6.

As a conclusion, it is shown that the macro-element model without considering frequency-dependent properties of soil can result in different response from the FEM model. By incorporating the frequency-dependent properties of soil into macro-element using recursive parameter model, the model captures inelastic behavior of foundation with accurate wave propagation of soil and the corresponding results are in good agreement with the FEM results.

For seismic application, one can apply a seismic excitation directly to the structure and record the response of the shallow foundation using this method, similar to the analysis procedure carried out in the example above. Future studies regarding the verification analysis using different soil models and various loading conditions can be carried out to further improve this modeling approach.

4. Conclusion

There are various methods to analyze the response of shallow foundation subjected to dynamic loads. Due to the benefits of rocking foundation, more studies are focusing towards the analysis of shallow foundation with rocking of the foundation. Although FEM modeling technique has been extensively used in research fields for its reliable results verified against the experiments, it takes enormous amount of computation time to analyze the model. As a result, various simplified and practical models have been presented by researchers where the inelastic behavior of the shallow foundation is captured.

The simplified models however, do not have the capability to capture accurate wave propagation away from the foundation to soil domain. In this paper, a revised method is proposed where the frequency-dependent properties of soil is integrated into the simplified model using recursive parameter model. For this study, macro-element and recursive parameter model introduced by Nakamura have been employed. Through verification analysis, it is shown that the results from the proposed method agree well with the ones captured by FEM model. The proposed methodology simplifies the modeling approach, and also greatly reduces the analysis computation time.

However, there are some limitation of this approach in practical application. Firstly, finding the realistic frequency dependent behavior of the soil domain may be a challenging task due to the complexity of soil domain in nature. In addition, the proposed method may not yield the most accurate results compared to FEM results when the wave propagation and nonlinear SSI effect are considered. This is due to the simplistic approach in defining the inelastic behavior of the soil foundation system in the proposed model.

5. Future studies

There are some future studies that may cover some of the limitations related to the proposed methodology. The macro-element is a simplified element where the ellipsoidal bounding surface has to be pre-defined. This limits the proposed method to only consider the specific soil types with pre-defined bounding surfaces. By providing the analysis with different bounding surface for other types of soil could expand the applicability of this method. Also, additional work is required to verify the model against different loading scenarios with varying magnitudes of excitation in order to investigate the accuracy of the proposed model to capture the inelastic response of shallow foundation with wave propagation away from the foundation. Finally, investigating the capability of this element to simulate the seismic behaviour of structure mounted on a shallow foundation using realistic earthquake load will further build confidence of this element to be used for practical application example.

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