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A Simplified Evaluation Method for the Seismic Fragility of Subway Stations

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Abstract

A simplified evaluation method for the seismic fragility of subway stations is presented in this paper. Pushover-method, as an effective method, is wildly used and has been evaluated by many researchers. Recognizing that the nonlinear history analysis of underground structures is extremely computational demanding, this paper presents a pushover-based evaluation method for the seismic fragility of subway stations in the following steps: 1. determining the structural damage index and classifying the structural performance levels; 2. conducting pushover analysis of the subway station; 3. selecting seismic records; 4. calculating the target displacement of every seismic record; 5. conducting probabilistic analysis and expressing the structural motion-damage relationship by means of fragility curves. Additionally, the pushover-based seismic fragility analysis of Daikai subway station, which was greatly destroyed in 1995 Great Hanshin Earthquake, is introduced as an example.

Keywords: subway station; seismic fragility; pushover; risk assessment; soil structure interaction

1. Introduction

Underground structures had been thought to be relatively safe during earthquakes until the 1995 Great Hanshin earthquake. [1] This earthquake brought about catastrophic damage to Daikai subwy station. During this strong ground shaking, the central columns of Daikai subwy station collapsed, leading to the collapse of the roof slab. The collapse mechanism of Daikai subway station was extensively studied by several researchers[1-3].

Nowadays, many underground structures such as subways, underground malls and tunnels have been constructed actively. At the same time, the 21st century has experienced a high incidence of earthquakes, with both the frequency and magnitude increasing sharply. Scholars investigated the reasons causing seismic damage of underground structures, and hereby built analysis theories and put forward design methods. The efforts brought an upsurge to seismic research of underground structures, which turned into an important direction in the earthquake engineering field.

Structural seismic fragility analysis quantitatively describes the structure's conditional probabilities of sustaining different degrees of damage under given levels of ground motion. Damage and disruption of underground structures may have severe effect on civil life since it may lead to loss of vital services, communications and transportation systems. As a consequence, the seismic fragility assessment of these structures should been investigated.

A large number of methods have been proposed to compute fragility functions in the last 20 years, ranging from expert judgment [4], data analysis on observed damages [5], to fully analytical approaches [6-10]. In present, there are two basic approaches for the seismic design of underground structures. One approach is to carry out dynamic, time-history analysis using finite element (FE) or different method. The second approach assumes the seismic ground motions to induce a pseudo-static loading condition on the structure.[11] Recognizing that the nonlinear history analysis of underground structures is extremely computational demanding, current engineering practitioners prefer to use the nonlinear static analysis (pushover analysis). Reference [12] introduced the pushover method over underground structures. References [11, 13-15] improved the pushover method. Nishioka et al.[16-17], by positing underground structures as free structures without the restraint of surrounding soil, applied pushover analysis to evaluate their seismic performance.

This paper presents a pushover-based evaluation method for the seismic fragility of subway stations. In this method, by building the relationship between the structural performance level and the target displacement through nonlinear pushover analysis, the time-consuming computation process of nonlinear time-history analysis is transferred to the calculation of the target displacement through one dimensional free field analysis. After introducing the evaluation method step-by-step, the seismic fragility of Daikai subway station, which was greatly destroyed in 1995 Great Hanshin Earthquake, is given as an example.

2. Implementation procedure for the pushover-based seismic fragility evaluation method

The outline of the method is as follows. First, by conducting the pushover analysis of the subway station, the capacity curve of structural transversal shear deformation which is defined as the structural damage index later is obtained. The relationship between the structural transversal shear deformation and the target displacement is accordingly built. Based on that, the structural performance level is classified according to the target displacement. Meanwhile, a number of seismic records whose central values and measures of dispersion can be assumed to reflect the uncertainty of the seismic force are selected. The acceleration value of each individual seismic record is scaled to match different ground motion intensity. For each individual seismic record, the target displacement is calculated through one dimensional free field analysis. After that, through statistical analysis, the probability distributions of those target displacements under the given seismic intensity are obtained. Finally, based on the target-displacement-based classification of structural performance levels and the probability distributions of the target displacement, the structure's conditional probabilities of sustaining different degrees of damage under given levels of ground motion are obtained. Correspondingly, the structural fragility curves are drawn.

The step-by-step introduction of the simplified seismic fragility analysis method is as follows.

2.1 The determination of structural damage index and the structural performance level

The determination of the structural damage index and the classification of structural performance levels have been discussed by many researchers. Since the seismic deformation of underground structures is mainly the shear deformation, the structural shear deformation γ_s as Fig. 1 shows is taken as the structural damage index in this paper.

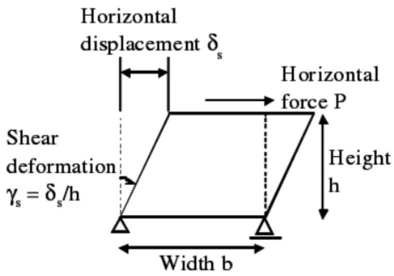

Fig. 1 – The shear deformation

2.2 Pushover analysis of the subway station

In the pushover methodology, the analysis model, lateral force distribution, and target displacement are the three key parameters. In this paper, the pushover method proposed in reference [18] is adopted. The brief introduction is as follows.

2.2.1 Analysis model

The soil-structure system is proposed as the analysis model. Additional to the soil-structure system FE model, an additional free-field FE column in which the soil properties, boundary conditions, and force distribution are as the same as the soil-structure system are built, in order to determine the target displacement. The analysis model of soil-structure system with the additional free field column is shown in Fig. 2.

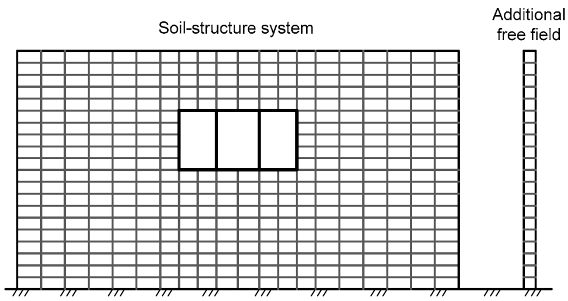

Fig. 2 – Analysis model of soil-structure system with the additional free field column

2.2.2 The distribution of lateral forces

The monotonically increasing horizontal inertial body forces is applied on the soil layers, the subway structure, and the additional free field column according to their depths. The three lateral force distribution patterns (i.e., the distribution based on the shear stress of the soil layers, the distribution of the peak accelerations in all soil layers, and an inverted triangular distribution) are discussed ın reference[18].

2.2.3 Target displacement

Target displacement in pushover analysis refers to the maximum displacement that the structure can reach during an earthquake. İt denotes the ultimate status of the structure. During an earthquake, the peak ground displacement relative to the peak bedrock displacement is called PGD which denotes the peak relative displacement between the ground and bedrock. Available researches [19-20] have proved that the PGD is an effective and reasonable design parameter for underground structures than the peak ground acceleration (PGA). Thus, the parameter PGD is regarded as the target displacement in this paper.

2.3 The PGD-based classification of structural performance levels

Through the pushover analysis, the capacity curve of the structural transversal shear deformation which is defined as the structural damage index in the fragility analysis can be obtained. Thus, the relationship between the structural transversal shear deformation and the target displacement PGD is built. According to this, the classification of the structural performance levels that based on the structural transversal shear deformation can be transferred to the classification that based on PGD.

2.4 Selection of seismic records

Aim to considerate the high uncertainty of seismic motions, the selected seismic records should distribute in a wide range of earthquakes rupture type, epicentral distance, and site type, etc.. The sets of acceleration seismic records should represent the seismic hazard at different return periods, describe intensity, frequency content, and duration with sufficient comprehensiveness so that central values and measures of dispersion of the demand parameters can be determined with confidence and efficiency. [21] There are several seismic motion databases worldwide, such as PEER (Pacific Earthquake Engineering Research) Center Ground Motion Database, VDC (Strong-motion Virtual Data Center) database, and Strong-motion Seismograph Networks (K-NET, KIK-net), etc.

Reference [21] designated four magnitude-distance bins for selecting seismic records from. Those four record bins are: (1) Large Magnitude-Short Distance Bin, LMSR, ($6.5<M_w<7.0$, 13 km$<R<$30km); (2) Large Magnitude-Long Distance Bin, LMLR, ($6.5<M_w<7.0$, 30 km$\leq R \leq$60km); (3) Small Magnitude-Short Distance Bin, SMSR, ($5.8<M_w<6.5$, 13 km$<R<$30km); (4) Small Magnitude-Long Distance Bin, SMLR, ($5.8<M_w<6.5$, 30 km$\leq R \leq$60km).

Among the many parameters that measuring the ground motion intensity, the record's peak acceleration is the most popular one. In this paper, the record's peak acceleration is used to measure the ground motion intensity. To obtain the probability characteristics of PGDs under different given ground motion intensity levels, the acceleration value of each individual seismic record is scaled according to the given ground motion intensities.

2.5 Calculation of the target displacement for every selected seismic record

The PGD which denotes the peak relative displacement between the ground and bedrock can be obtained by one dimensional free field seismic analysis, conducting by procedures such as SHAKE91. By repeating the one dimensional free field seismic analysis for each individual seismic record, the PGDs for all selected seismic records with defferent given peak accelerations can be obtained.

MATLAB can host the ActiveX control, i.e., controlling the extra-program using ActiveX. Thus, the batch computing process can be conducted by MATLAB: using MATLAB to call the seismic analysis procedure and loop the computation process.

2.6 Probabilistic analysis

By statistical analysis, the probability characteristics of PGD under each given ground motion intensity level can be obtained. Recalling the PGD-based classification of structural performance level in section 2.2, the structure's conditional probability of sustaining a certain damage degree under the given ground motion intensity can be calculated. Thus, the seismic fragility curve of the underground structure can be drawn.

According to paper [18], firstly, N seismic waves with the same design PGA must be selected according to the design requirement. Then, the N values of PGD can be obtained by SRGA. It is assumed that the N values of PGD follow a normal distribution. Thus, the expected value of PGD and the maximum value of PGD with confidence probability P can be computed as follows:

$$E(PGD) = \frac{1}{N}\sum_{i=1}^{N} PGD_{(i)} \tag{1}$$

$$C(PGD) = E(PGD) + Z \cdot \sigma \tag{2}$$

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}[PGD_{(i)} - E(PGD)]^2} \tag{3}$$

Where $E(PGD)$ and $C(PGD)$ are, respectively, the expected value of PGD and the maximum value of PGD with confidence probability P; $PGD_{(i)}$ is the value of PGD obtained by SRGA due to seismic wave i; z is the z score that corresponds to confidence probability P (this can be obtained in a z-score table of standard normal distribution); and σ is the standard deviation. Thus, either E(PGD) or C(PGD) can be taken as the target displacement, according to the design requirement, and then connected with PGA, which is still used as the design parameter in other design methods.

3. Example — the seismic fragility of Daikai subway station

3.1 The Daikai subway station

Daikai subway station is located in Kobe city, Japan. The Hyogoken - Nanbu Earthquake, also known as the the Great Hanshin Earthquake, accured on 17 January 1995 with a magnitude of 7.2, brought catastrophic damage to Daikai subway station whose central pillars are totally collapsed over a length of 80m, with settlements up to 2.5m at surface.

The Daikai subway station is a close reinforced concrete frame structure. The construction of Daikai station began in August 1962. Since the 1995 Hyogoken - Nanbu Earthquake, a group of studies focus on the damage pattern of Daikai subway stations during an earthquake. [2, 23-26]. Reference [2] presents that, along the Kobe subway line, a lot of damage was caused to the underground structures. In almost all sections, diagonal shear cracks were clearly observed. In some sections, few diagonal shear cracks in the column are observed. For another section, the diagonal shear cracks were associated with the splitting of concrete volume, but still the column has a load-carrying mechanism against the vertical loads. As for the completely damaged sections, the columns with diagonal shear crack loose loadcarrying mechanism against the vertical forces were associated with dead loads applied on the top slab and soil overlayer. As a result, the top slab was completely collapsed.

3.2 The FE model

The control cross section considered in this paper is shown in Fig. 3. İt is a typical cross section of Daikai subway station with a total width 17m and a total height 7.17m. The thickness of side walls is 0.7m and its reinforcement ratio is 0.8%. The thicknesses of the top and bottom plates are 0.8m and 0.85m respectively, and the reinforcement ratio are both 1.0%. The size of the middle column is 0.4 m×1 m with a middle-to-middle space 3.5m and the reinforcement ratio is 6.0%. [14, 27-28] Round steel bars with diameters from 16mm to 25mm were used as reinforcing for the walls and slabs, and 32mm diameter bars were used in the central column. [26] The schematic diagram of the bars arrangement is shown in Fig. 4.

The ABAQUS FE model of the soil-structure system with the additional free field model is shown in Fig. 5. The FE model of the subway station is shown in Fig. 6. The FE model of bars is shown in Fig. 7.

The concrete solid is simulated by elements C3D8R. The bars are simulated by elements T3D2. The bars are considered as embedded into the concrete solid. The soil is simulated by elements C3D8R. The material model of concrete is the Plastic Damage Concrete Model. The material model of bars is the Bilinear Plastic Model. The material proprieties of soil is simulated by the Isotropic Hardening Elastoplastic Model and the Mohr–Coulomb criterion.[26] Physical properties of soil layers are illustrated in Table 1. [14] The damping of both soil and structure are not included in the pushover analysis.

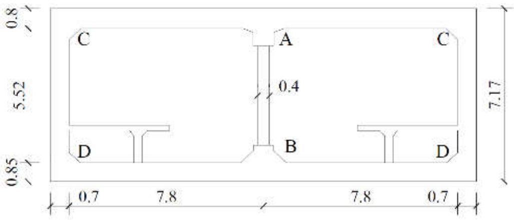

Fig. 3 – Typical cross section of Daikai subway station (unit: m)

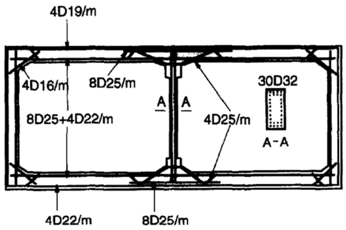

Fig. 4 – The schematic diagram of the bars arrangement [22]

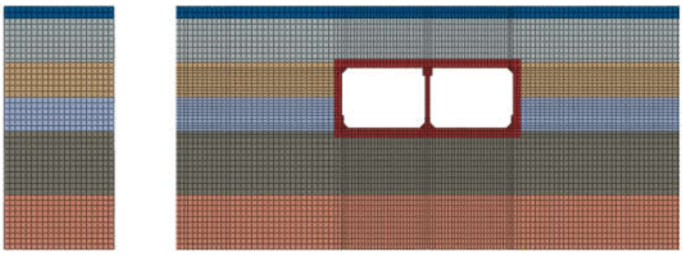

Fig. 5 – The FE model of the soil-structure system

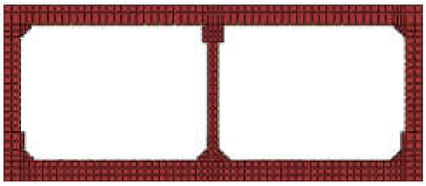

Fig. 6 – The FE model of the subway station

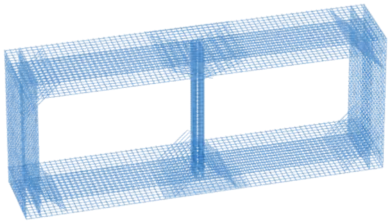

Fig. 7 – The FE model of bars

Table 1 – Soil profile properties

No.	Type	Depth/ m	Density/ $\left(kg/m^3\right)$	v_s / $\left(m/s\right)$	G_{max} (MPa)	Poisson's ratio
1	Artificial filled soil	0~1.0	1.9	140	38.00	0.33
2	Sand in Pleistocene	1.0~5.1	1.9	140	38.00	0.32
3	Sand in Pleistocene	2.1~8.3	1.9	170	56.03	0.32
4	Clay in Pleistocene	8.3~11.4	1.9	190	69.99	0.40
5	Clay in Pleistocene	11.4~17.2	1.9	240	111.67	0.30
6	Sand in Pleistocene	17.2~22.2	2.0	330	222.24	0.26

3.3 Pushover analysis

The inverted triangular distribution horizontal body force is applied on the soil-structure system and the additional free-field model. The capacity curve of the structural shear deformation which is defined as Fig. 1 is shown in Fig. 9. The classification of structural performance levels based on the structural shear deformation is listed in Table 2. Four damage conditions defined by the five performance levels (from fully operational to collapse) are: NO, IO, LF, and CP, respectively. According to Fig. 9, the maximum elastic shear deformation of Daikai subway station is 0.00078.

The relationship curve between the structural shear deformation and the target displacement PGD is shown in Fig. 10. We can see that, PGD is 0.017 m when the structural shear deformation is 1/1000 (i.e., the structural damage condition NO); PGD is 0.024 m when the structural shear deformation is 1/600 (i.e., the structural damage condition IO); PGD is 0.042 m when the structural shear deformation is 1/400 (i.e., the structural damage condition LF); PGD is 0.114 m when the structural shear deformation is 1/200 (i.e., the structural damage condition CP). Thus, the PGD-based classification of structural performance is listed in Table 3.

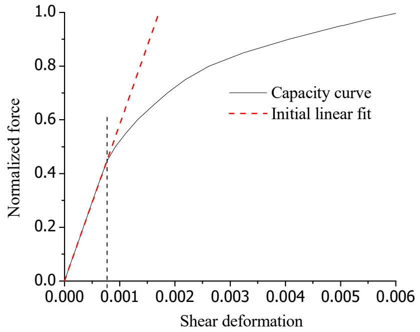

Fig. 9 – The capacity curve of structural shear deformation

Table 2 – The classification of structural performance levels based on structural Shear deformation

Performance levels	Fully operational	Slight damaged	Medium damaged	Serious damaged	Collapse
Structural shear deformation θ	<1/1000	1/1000~1/600	1/600~1/400	1/400~1/200	>1/200

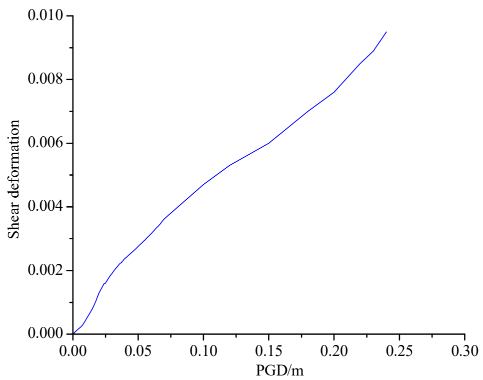

Fig. 10 – The relationship curve between the structural shear deformation and the target displacement PGD

Table 3 – The classification of structural performance levels based on PGD

Performance levels	Fully operational	Slight damaged	Medium damaged	Serious damaged	Collapse
PGD (m)	0.017	0.017~0.024	0.024~0.042	0.042~0.114	>0.114

3.3 Selection of seismic records and the one dimensional free field analysis

The selection of seismic records is based on the principles given in reference [21]. From each magnitude-distance bin (LMSR, LMLR, SMSR, SMLR), 20 seismic motion records are selected. Thus, there are totally 80 seismic motion records. According to the given ground motion levels, the peak acceleration of each seismic record is scaled to 0.05g, 0.1g, 0.2g, 0.3g, 0.4g, 0.5g 0.6g, 0.7g and 0.8g, respectively,

With each single seismic record as the input record, the one dimensional free field analysis is conducted through the procedure SHAKE91. In SHAKE91, the analysis for any set of properties is linear. An equivalent linear procedure is used to account for the nonlinearity of the soil using an iterative procedure to obtain values for modulus and damping that are compatible with the equivalent uniform strain induced in each sublayer. [29]

The batch computing process is realized by MATLAB. Thus, the PGDs of all seismic motions with every given peak acceleration are obtained. The results are shown in Fig. 11~Fig. 19. Through the statistical analysis, the probability distributions of PGD under the given seismic intensity levels are respectively obtained. The box figure of PGD is shown in Fig. 20.

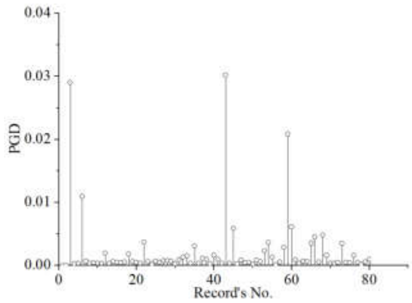

Fig. 11 – The PGDs of the 80 seismic records with the peak acceleration of 0.05g)

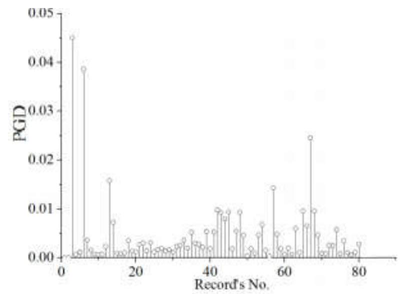

Fig. 12 – The PGDs of the 80 seismic records with the peak acceleration of 0.1g)

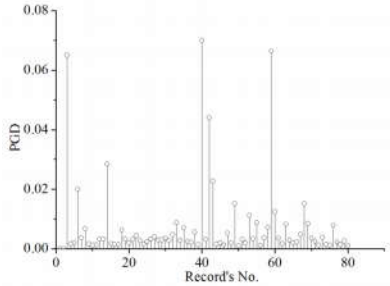

Fig. 13 – The PGDs of the 80 seismic records with the peak acceleration of 0.2g)

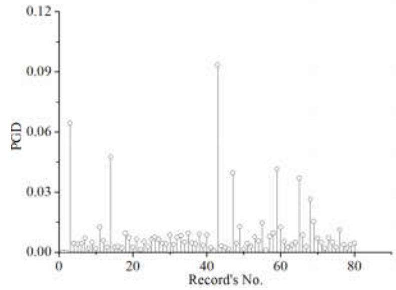

Fig. 14 – The PGDs of the 80 seismic records with the peak acceleration of 0.3g)

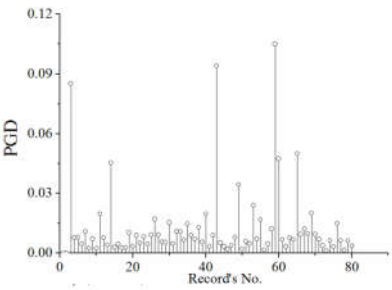

Fig. 15 – The PGDs of the 80 seismic records with the peak acceleration of 0.4g)

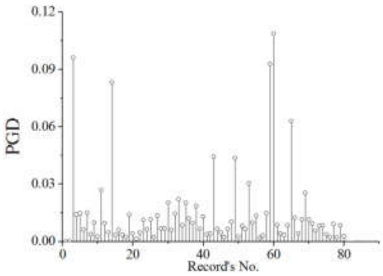

Fig. 16 – The PGDs of the 80 seismic records with the peak acceleration of 0.5g)

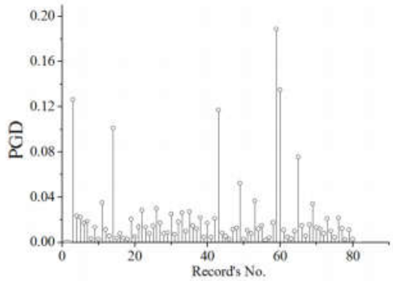

Fig. 17 – The PGDs of the 80 seismic records with the peak acceleration of 0.6g)

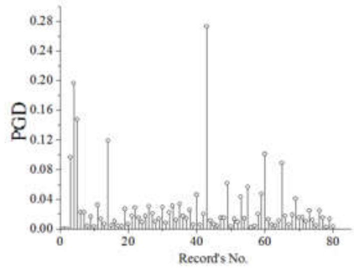

Fig. 18 – The PGDs of the 80 seismic records with the peak acceleration of 0.7g)

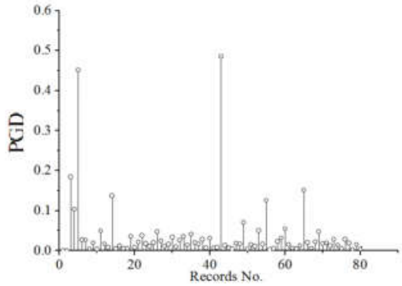

Fig. 19 – The PGDs of the 80 seismic records with the peak acceleration of 0.8g)

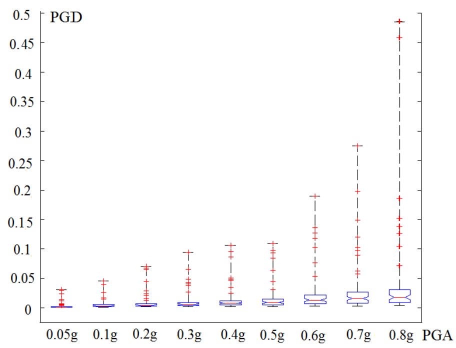

Fig. 20 – The box figure of PGDs with respect to PGAs

3.4 Seismic fragility

Based on the PGD-based classification of structural performance levels in Table 3 and the distributions of PGDs, the structure's conditional probabilities of sustaining different degrees of damage under given seismic intensity levels are calculated. The seismic fragility curves of Daikai subway station are plotted in Fig. 21.

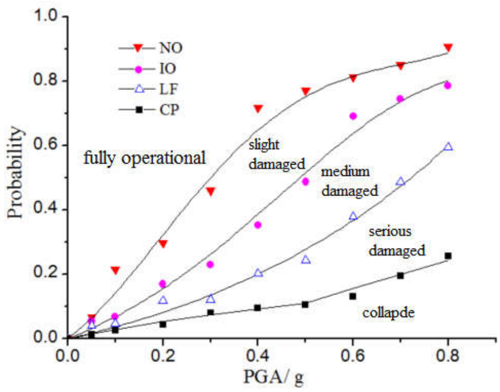

Fig. 21 – The Seismic fragility curves of Daikai subway station

Conclusion

This paper presents a pushover-based evaluation method for the seismic fragility of subway stations. In this method, through the relationship between the structural damage index and the target displacement which is obtained from the pushover analysis, the structural damage-index-based classification of structural performance level is transferred to the target-displacement-based classification. Thus, the time-consuming computation process of the structural nonlinear response is transferred to the calculation of the target displacement through one dimensional free field analysis. By this way, the time costs is reduced enormously.

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5. References

[1] Hiroomi Iida, Toshio Hiroto, Nozomu Yoshida, Masahiko Iwafuji (1996): Damage to Daikai subway station. Soils and foundations, 283-300.

[2] Xuehui An, Ashraf-A Shawky, Koichi Maekawa (1997): The collapse mechanism of a subway station during the Great Hanshin Earthquake. Cement and concrete composites, 19(3), 241-257.

[3] H Huo, A Bobet, G Fernández, J Ramirez (2005): Load transfer mechanisms between underground structure and surrounding ground: evaluation of the failure of the Daikai Station. Journal of Geotechnical and Geoenvironmental Engineering, 131(12), 1522-1533.

[4] Applied-Technology Council (1985): Earthquake Damage Evaluation Data for California. Applied Technology Council, 1st editon.

[5] Ajay Singhal, Anne-S Kiremidjian (1998): Bayesian updating of fragilities with application to RC frames. Journal of structural Engineering, 124(8), 922-929.

[6] C-A Cornell, F Jalayer, R-O Hamburger, Foutch Da (2001): The probabilistic basis for the 2000 SAC/FEMA steel moment frame guidelines. Journal of Structural Engineering.
[7] Paolo Gardoni, Armen Der Kiureghian, Khalid-M Mosalam (2002): Probabilistic capacity models and fragility estimates for reinforced concrete columns based on experimental observations. Journal of Engineering Mechanics.
[8] S-K Au, J-L Beck (2003): Subset simulation and its application to seismic risk based on dynamic analysis. Journal of Engineering Mechanics.
[9] Mij Schotanus, P Franchin, A Lupoi, P-E Pinto (2004): Seismic fragility analysis of 3D structures. Structural Safety, 26(4), 421-441.
[10] Giorgio Lupoi, Paolo Franchin, Alessio Lupoi, Paolo-E Pinto (2006): Seismic fragility analysis of structural systems. Journal of Engineering Mechanics, 132(4), 385-395.
[11] Huiling Zhao, Yong Yuan (2010): An Improved Pseudo-Static Seismic Analysis for Underground Frame Structures. Deep and Underground Excavations.
[12] Kyriazis Pitilakis, Grigorios Tsinidis (2014): Performance and seismic design of underground structures. Earthquake Geotechnical Engineering Design, 279-340.
[13] T Nishioka, Shigeki Unjoh (2003): A simplified evaluation method for the seismic performance of underground common utility boxes. Proceedings 2003 Pacific Conference on Earthquake Engineering, Melbourne, Australia.
[14] Jingbo Liu, Xiangqing Liu (2008): Pushover analysis of Daikai subway station during the Osaka-Kobe earthquake in 1995. The 14th world Conference on earthquake engineering, Beijing, China.
[15] Zhi-Yi Chen, Wei Chen, Wei Zhang (2014): Seismic Performance Evaluation of Multi-Story Subway Structure Based on Pushover Analysis. Advances in Soil Dynamics and Foundation Engineering.
[16] T Nishioka, Shigeki Unjoh (2003): A simplified evaluation method for the seismic performance of underground common utility boxes. Proceedings 2003 Pacific Conference on Earthquake Engineering, Melbourne, Australia.
[17] Jingbo Liu, Bin Li (2005): A unified viscous-spring artificial boundary for 3-D static and dynamic applications. Science in China Series E Engineering & Materials Science, 48(5), 570-584.
[18] Jingbo Liu, Wenhui Wang, Gautam Dasgupta (2014): Pushover analysis of underground structures: Method and application. Science China Technological Sciences, 57(2), 423-437.
[19] Liu-Jingbo-Liu-Xiangqing Bin (2008): A pushover analysis method for seismic analysis and design of underground structures [J]. China Civil Engineering Journal, 4(11), 73-80.
[20] Jingbo Liu, Bin Li, Yin Gu (2005): Seismic response analysis of shielded subway tunnels. Journal of Tsinghua University, 45(6), 757.
[21] Ricardo-A Medina, Helmut Krawinkler (2004): Seismic demands for nondeteriorating frame structures and their dependence on ground motions, PEER Report, 5, Blume Earthquake Engineering Center Department of Civil and Environmental Engineering Stanford University, Stanford, California.
[22] N Takewaki, A Ohtsuki, Y Ohnishi (1996): An examination on failure mechanism of a subway station due to the 1995 Hyogoken-Nanbu Earthquake. Proceedings of Technical Conference on the Great Hanshin-Awaji Earthquake, JSCE, Japan.
[23] T Yamato, T Umehara, H Aoki, S Nakamura, J Ezaki, I Suetomi (1996): Damage to Daikai subway station, Kobe rapid transit system and estimation of its reason. Proceedings, technical conference on the great Hanshin-Awaji Earthquake, JSCE, Tokyo, Japan.
[24] H Huo, A Bobet, G Fernández, J Ramirez (2005): Load transfer mechanisms between underground structure and surrounding ground: evaluation of the failure of the Daikai Station. Journal of Geotechnical and Geoenvironmental Engineering, 131(12), 1522-1533.
[25] David-Lourenço Brito, Collapse analysis of the Daikai station in Kobe Metropolitan.

[26] Xuehui An, Ashraf-A Shawky, Koichi Maekawa (1997): The collapse mechanism of a subway station during the Great Hanshin Earthquake. Cement and concrete composites, 19(3), 241-257.
[27] Koji Uenishi, Shunsuke Sakurai (2000): Characteristic of the vertical seismic waves associated with the 1995 Hyogo-ken Nanbu (Kobe), Japan earthquake estimated from the failure of the Daikai Underground Station. Earthquake Engineering and Structural Dynamics, 29(6), 813-822.
[28] I.M Idriss, Josephi. Sun (1992): User's manual for SHAKE91, Center for Geotechnical Modeling, Department Of Civil And Environmental Engineering, University Of California, Davis, California.