EFFECT OF ACTUATOR DELAY ON UNCERTAINTY QUANTIFICATION FOR REAL-TIME HYBRID SIMULATION

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Abstract

Analysis of uncertainties in numerical substructure is necessary to estimate the variance of structural response obtained from structural experiments through the hybrid simulation technique. In a real-time hybrid simulation, the servo-hydraulic actuators apply the desired responses to the experimental substructures in a real-time manner. It has been observed that actuator delay could be reduced but cannot be eliminated. In this study, the effect of actuator delay on uncertainty quantification is investigated for real-time hybrid simulation through polynomial chaos expansion. Responses of a single-degree-of-freedom system are projected on a series of orthogonal stochastic functions then represented using simple polynomials. The mean and variance from polynomial chaos expansion are compared between the cases with and without delay. Actuator delay in a real-time hybrid simulation is observed to result in increases of mean and variance for the structural responses. Sensitivity analysis also indicates actuator delay can affect the variance on uncertainty quantification in real-time hybrid simulation.

Keywords: real-time hybrid Simulation, polynomial chaos expansion, uncertainty quantification

1. Introduction

Real-time hybrid simulation divides the prototype structure under investigation into numerical and experimental substructures, where the former is simulated in finite element program and the latter are physically tested in laboratory [1-2]. The substructure technique helps reduce experiment cost and makes large and even full-scale structural test possible for earthquake engineering research. Different from traditional quasi-static hybrid simulation, the desired structural response in real-time hybrid simulation is imposed onto the experimental substructure in a real-time manner, which makes it more suitable to accommodate rate dependency in size limited laboratories. Under the recent developments in unconditionally stable explicit integration algorithms [3-5] and servo-hydraulic actuator delay compensation methods [6-8], real-time hybrid simulation has become a promising experimental technique to evaluate performances of civil engineering infrastructures subjected to earthquake excitations [9-11].

The experimental substructure in real-time hybrid simulation often behaves exactly the same as its counterpart in the prototype structure under investigation. The computational model for numerical substructures however may not truthfully represent the analytically modeled structural components due to the lack of knowledge, which could lead to uncertainties in numerical modeling. Moreover accounting for the uncertainties in numerical substructures could help estimate the mean and variance of structural response measured from real-time hybrid simulation. It is therefore important to account for model uncertainties in numerical substructure when conducting real-time hybrid simulation for earthquake engineering research.

The solution of a stochastic system has been extensively studied and existing methods can be classified as statistical or non-statistical [12]. Monte Carlo simulation [13] is one of the most well-known statistic methods, which the behavior of a random variable can be studied by the empirical process of actually drawing lots of random samples and observing this behavior. Since the accuracy of this method highly depends on the sample size, the limited computational time and memory make simulation extremely expensive. The most widely used non-statistical method is perturbation method due to the mathematical simplicity. The random quantities are expanded around its mean values via a Taylor series. High order terms are truncated for the stability and computation reasons [14]. The main drawback of this method is that the variance of the random elements should be much smaller than their mean values. Besides, higher order statistics are not available by this method. Other non-statistical methods are based on discretizing the random process in random space. Karhunen-Loeve expansion utilizes a set of orthogonal functions and constant values determined by the spectral expansion of the correlation function to represent a random process [15]. This expansion has the smallest mean-square error resulting from a finite representation of the random process among other possible decompositions, which is also called error minimizing property. However, this method is not suitable when the covariance function is not known.

Polynomial chaos expansion is an alternative method to Karhunen-Loeve expansion, which projects the model output on a basis of orthogonal stochastic polynomials. This method was first developed by Wiener [16], which the framework is based on series expansions of Hermite polynomials of Gaussian random variables. The convergence rate of this method depends on the shape of the orthogonal functions. The exponential convergence can be achieved for Gaussian random variables with Hermite polynomials, which is faster than other methods. This method is successfully applied by Ghanem and Spanos by a finite element method [14]. For other distribution of the random variables than Gaussian, the convergence rate of traditional polynomial chaos expansion is not so optimal. Xiu and Karniadakis [17] developed a generalized polynomial chaos expansion based on Askey scheme. By this method, each family of orthogonal polynomials corresponds to a given choice of distribution. For example, a function with uniform distribution is optimally represented by Lengendre polynomials and a gamma distributed random variable is better to use Laguerre polynomials. After that, polynomial chaos expansion was further developed to represents an arbitrary probability of the stochastic processes [18].

Polynomial chaos expansion has been used to analyze the stochastic processes in civil engineering [19]. Results show that polynomial chaos expansion is a promising method to solve stochastic problems in structure engineering. Abbiati *et al.* [20] introduced polynomial chaos expansion to study uncertainties in numerical substructures for quasi-static hybrid simulation. It was demonstrated that a small number of tests can achieve good estimates of mean and variance for structural responses when uncertainties exist in numerical substructures. In this paper, polynomial chaos is utilized to quantify the effect of uncertainties in numerical substructure on the statistical characteristics of structural responses when time delay occurs in a real-time hybrid simulation. The polynomial chaos expansion for the system without delay is first discussed to select the most suitable degree of the polynomial chaos expansion and the number of sampling points. The polynomial chaos expansion is then applied for real-time hybrid simulation with different delay. Comparing between real-time hybrid simulations with and without time delay, the effect of delay is assessed for real-time hybrid simulation with uncertainties in numerical substructures.

2. Analysis methods for model uncertainty

2.1 Karhunen-Loeve expansion

The Karhunen-Loeve (KL) expansion is used to represent a random process based on the spectral expansion of the correlation function of the process [15]. The KL expansion can be expressed as,

$$\omega(x,\theta)=\bar{\omega}(x,\theta)+\sum_{i=0}^{\infty}\sqrt{\lambda_i}\,f_i(x)\xi_i(\theta) \tag{1}$$

where $\omega(x,\theta)$ is a random process based on position vector x and random vector θ, $\bar{\omega}(x,\theta)$ is the expected value of the random process, $\xi_i(\theta)$ are a set of uncorrelated random variables. $f_i(x)$ and λ_i are the eigen functions and eigenvalues of the correlation function,

$$\int R_{hh}(x,y)\,f_i(y)dy=\lambda_i f_i(x) \tag{2}$$

where R_{hh}(x,y) donates the correlation function of the random process.

In practice, it is impossible to calculate the infinite series in Eq. (1), and the summation is truncated at finite number n,

$$\omega(x,\theta)=\bar{\omega}(x,\theta)+\sum_{i=0}^{n}\sqrt{\lambda_i}\,f_i(x)\xi_i(\theta) \tag{3}$$

2.2 Homogeneous chaos

The original polynomial chaos, also termed as Winer-Hermite expansion or the homogeneous chaos, employs the Hermite polynomials for variables in Gaussian distribution [9]. Suppose $X(\omega)$ is a general random process determined by infinite independent standard Gaussian random variables $\xi=(\xi_{i1}, \xi_{i2,\ldots}\ \xi_{in},\ldots)$, then $X(\omega)$ can be represented as:

$$X(\omega)=a_0H_0+\sum_{i_1=1}^{\infty}a_{i_1}H_1(\xi_{i_1})+\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i1}a_{i_1,i_2}H_2(\xi_{i_1},\xi_{i_2})+\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2}a_{i_1,i_2,i_3}H_3(\xi_{i_1},\xi_{i_2},\xi_{i_3})+\ldots \tag{4}$$

where coefficients a_0, a_{i1}, $a_{i1,i2}$, $a_{i1,i2,i3}\ldots$ represent determined parameters, $H_n(\xi_{i1}, \xi_{i2,\ldots}\ \xi_{in})$ represents nth Hermite polynomial.

2.3 Generalized polynomial chaos

When the types of the random variables are non-Gaussian, exponent convergence cannot be reached. To solve this problem, Askey-scheme is used to derive the generalized polynomial chaos [17]. Similar to homogeneous chaos, a general random process $X(\omega)$ can be represented as

$$X(\omega)=a_0I_0+\sum_{i_1=1}^{\infty}a_{i_1}I_1(\xi_{i_1})+\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i1}a_{i_1,i_2}I_2(\xi_{i_1},\xi_{i_2})+\sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2}a_{i_1,i_2,i_3}I_3(\xi_{i_1},\xi_{i_2},\xi_{i_3})+\ldots \tag{5}$$

where I_n donates Askey polynomials. Table 1 presents the commonly used random variables and the polynomial chaos. In Eq. (5), a_0 is regarded as the mean value of the random process $X(\omega)$, while the sum of square of a_i (i>0) is considered as the variance.

Table 1 – Commonly used polynomial chaos

Type of random variable	Type of polynomials chaos
Uniform	Legendre
Gaussian	Hermite
Gamma	Laguerre
Beta	Jacobi

2.4 Sobol index

The convergence of the polynomial chaos expansion can be determined by the Sobol index. Sobol index is a global index used for estimating the influence of individual variable or groups of variables on the model output [21]. The Sobol index can also be used to determine the accuracy of the polynomial chaos expansion, which compared with the Sobol index calculated by Monte Carlo simulation.

The integral function

$$f(x) = f_0 + \sum_{s=1}^{n} \sum_{i_1 < \cdots < i_s}^{n} f_{i_1 \cdots i_s}(x_{i_1}, \cdots, x_{i_s}) \tag{6}$$

which defined in I^n is called ANOVA-representation of $f(x)$ if

$$\int_0^1 f_{i_1 \cdots i_s}(x_{i_1}, \cdots, x_{i_s}) dx_k = 0 \quad (k = i_1, \cdots, i_s) \tag{7}$$

where f_0 is a constant value. Defining total variance D and partial variance D_{is} as

$$D = \int_I f^2(x)dx - f_0^2 \tag{8a}$$

$$D_{i_1 \cdots i_s} = \int_I f_{i_1 \cdots i_s}{}^2(x_{i_1}, \cdots, x_{i_s}) dx_{i_1 \cdots i_s} \tag{8b}$$

The Sobol index is then defined as

$$S_{i_1 \cdots i_s} = D_{i_1 \cdots i_s} / D \tag{9}$$

where $S_{i_1 \cdots i_s}$ describes the influence of the input parameter on the amount of the total variance. The larger the Sobol index is, the more influence the parameter has on the amount of total variance. However, the computational effort increase exponential with the input parameters, so only first order Sobol index and total Sobol index are mainly used in practice. The first order Sobol index $S_i(i=i_1,i_2\ldots)$ represents the influence of i-th input parameter alone on the total output variance. On the other hand, total Sobol index S_{Ti} represents the influence of i-th input parameter combined with other parameters on the total output variance.

3. Polynomial chaos expansion for system without delay

3.1 DDE model with uncertainty parameters

Consider a mass-spring oscillator system with an excitation input $F(t)$ in Fig.1, the equation of motion can be expressed as

$$m \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + k \cdot x(t) = F(t) \tag{10}$$

where m, c and k are the mass viscous damping and stiffness of the SDOF structure, respectively; t is time function; $\dot{x}(t)$ and $\ddot{x}(t)$ are the velocity and acceleration responses of the SDOF structure, respectively; and $F(t)$ is the external excitation force. In worst case of real-time hybrid simulation, all stiffness belongs to experiment substructure while all mass and damping belongs to numerical substructure [3]. Thus, the stiffness is isolated as experimental substructure while the rest of the SDOF structure is modeled as analytical substructure including the inertial mass and the viscous damping. When time delay exists in the response of servo-hydraulic actuator, Eq. (17) can be revised as

$$m \cdot \ddot{x}(t) + c \cdot \dot{x}(t) + k \cdot x(t-\tau) = F(t) \tag{11}$$

where τ is time delay. It can be observed that Eq. (11) is a delay differential equation (DDE) which takes derivatives of the unknown function based on the values of the function at previous times.

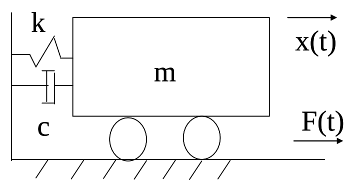

Fig. 1 The mass-spring oscillator system

The DDE model is very useful to understand the influence of the delay on the emulated response from real-time hybrid simulation. The delay effects on model uncertainty can be calculated when DDE model built as stochastic system with non-deterministic parameters. The maximum displacement is a key control parameter in structure response, and can be easily calculated when the mass, damping and stiffness are deterministic for given time delay. However, when the mass, damping and stiffness become random variables, traditional methods have difficulty in calculating its distribution. Alternatively, polynomial chaos expansion can easily establish the relationship between the maximum displacement and random variables from which the statistics of maximum displacement response can be calculated.

For simplicity but without loss of generality, the mass, damping and stiffness in Eqs.(10) and (11) are assumed to follow lognormal distribution. The mean values of the mass, viscous damping and stiffness are 1 kg, 0.2513 Nsec/m and 39.48 kN/mm, respectively. The variances of the three variables are assumed to be one tenth of their mean values. The MU2035 component recorded at Beverly Hills – Mulhol station during the 1994 Northridge earthquake is selected as ground motion input with the peak ground acceleration of 0.617g. When the structural mass, damping and stiffness take mean values, the maximum displacement response is 0.1007m.

3.2 Monte Carlo simulation

Before calculating polynomial chaos expansion, one should consider: the method to calculate polynomial chaos, the order of the polynomial chaos expansion and the number of sampling points. For this reason, 50000 Monte Carlo simulations are conducted to calculate the mean and variance of the maximum displacement as well as Sobol indices for each of the random variables for mass, damping and stiffness. The results of the 50000 Monte Carlo simulations are considered as reference values for this DDE model. Simulink model is used to calculate the structure responses for given mass, damping and stiffness as shown in Fig.2. The maximum displacement response is shown in Fig.3 for all 50000 simulations, where the mean and variance are 0.1094 m and 2.376×10^{-4} m^2, respectively. The first order and total Sobol indices are presented in Table 2 for all three random variables.

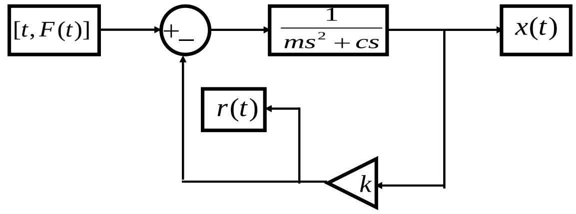

Fig. 2 Block diagram representation for computational simulation of structural response

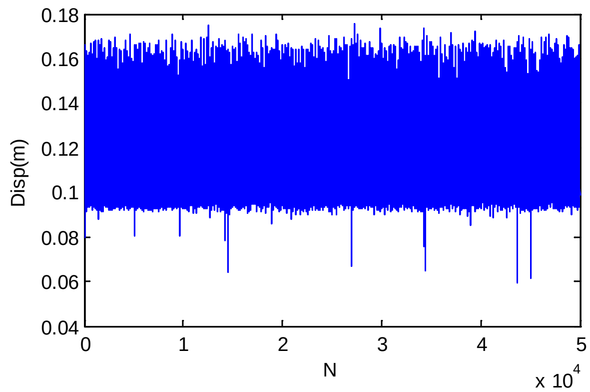

Fig. 3 Maximum displacement response from Monte Carlo simulation

Table 2 – Sobol indices of the random variables

	m	*C*	*K*
First order	0.4383	0.0108	0.2682
Total	0.7106	0.0180	0.6358

Compared with the maximum displacement of deterministic case, the mean of maximum displacements from Monte Carlo simulations is 8.6% larger. In other words, the mean of maximum displacement is not necessarily equal to the maximum displacement when stochastic variables equal to their mean values. This demonstrates the need for uncertainty quantification when model contains random variables.

3.3 Methods to calculate coefficients

UQlab is a MATLAB based frame work for uncertainty quantification [22]. The coefficients can be calculated by projection method or regression method through this software. To compare the performance of two methods, analysis using the first order, third order and eighth order polynomial chaos expansions is conducted using UQlab. The numbers of coefficients are 4, 20, 156 for first, third and eighth order polynomial chaos expansions, respectively. For the projection method, the numbers of sample are 7, 83 and 2541 for first order, third order and eighth order polynomial chaos expansions, respectively. On the other hand, the number of samples for the regression method can be determined by researchers. In this study, the same number of sample points is selected for both the projection method and the regression method. The LARS method is used in regression, which only part of the coefficients is calculated.

The mean and variance from different polynomial chaos expansions are presented in Table 3. The differences between values from polynomial chaos expansion and Monte Carlo simulations are also listed in the table. From Table 3, the mean values for projection method are 0.1074, 0.1056 and 0.1107 with the corresponding error 1.79%, 3.51% and 1.3% when using first order, third order and eighth order polynomial chaos expansions, respectively. For regression method, the mean values are 0.1160, 0.1095 and 0.1093 with the corresponding error 6.06%, 0.13% and 0.06% when using first order, third order and eighth order polynomial chaos expansions. The performance of the regression method in mean is worse than projection method for first order polynomial chaos expansion. However, the error quickly decreases with the increase in the order of polynomial chaos expansion for regression method. The error can be as small as 0.06% for the eighth order polynomial chaos expansion using regression method, while the error for projection method with same order is 1.3%. The variance for projection method are 1.2393×10^{-4}, 4.4722×10^{-4} and 6.2859×10^{-4} with the corresponding errors of 47.84%, 88.23% and 164.52% when using first order, third order and eighth order polynomial chaos expansions. For regression method, the variance are 2.2814×10^{-4}, 2.3921×10^{-4} and 2.5421×10^{-4} with the corresponding error 3.98%, 0.68% and 6.99% when using first order, third order and eighth order polynomial chaos expansions. The regression method shows much better performance in variance than projection method especially for high order polynomial chaos expansions.

Table 3 – The mean values and variances from different polynomial chaos expansions

Method	Projection (Quadrature)			Regression (LARS)		
Polynomial order	1	3	8	1	3	8
Number of sampling	7	83	2541	7	83	2541
Mean	0.1074	0.1056	0.1107	0.1160	0.1095	0.1093
Error of the mean (%)	1.79	3.51	1.3	6.06	0.13	0.06
Variance (10^{-4})	1.2393	4.4722	6.2859	2.2814	2.3921	2.5421
Error of the variance (%)	47.84	88.23	164.52	3.98	0.68	6.99

The summation and square summation of the error between the simulated maximum displacements and those estimated from polynomial chaos expansions are shown in Table 4. The summation and square summation of the error for projection method grow with the polynomial order while regression method decreases with the polynomial order. The errors for regression method are much smaller than projection method except first order polynomial chaos expansion. In conclusion, the projection method is better than regression method for low order polynomial chaos expansion in mean and Sobol indices. However, the variance from using regression method is much better than that from projection method. In high order, the regression is better than regression method in all indexes. Thus, the regression method is used to calculate the polynomial chaos expansion when time delay exists in real-time hybrid simulation.

Table 4 – Summation and square summation of the error

Method	Projection (Quadrature)			Regression (LARS)		
Polynomial order	1	3	8	1	3	8
Summation	0.8592	0.8638	1.4145	1.6684	0.2683	0.068
Square of summation	0.0154	0.0171	0.0367	0.0440	0.0036	0.0002

3.4 Order of the polynomial chaos expansion

For regression method, the optimal number of regression points would be the (M-1) multiply P. It can be recognized from Table 3 that the higher order polynomial expansion chaos using regression method can get better results. However, higher order polynomial chaos expansion always mean more sampling points and more computational time. So it is important to select a suitable order for polynomial chaos expansion that can balance accuracy and calculation efficiency for DDE model.

The orders of the polynomial chaos expansion are selected from 1 to 15. For each order, the sampling points meet (*M*-1) multiply *P*. The mean and variance of each polynomial chaos are shown in Fig.4. From Fig.4, the mean and variance both become convergence at third order polynomial, which are 0.1094 and 1.2393×10^{-4} respectively. Since polynomial chaos expansion is a sample based method, different sample points can lead to different results. Thus, the mean and variance for third order polynomial chaos expansion in this place has a little difference from the mean value and variance in Table 2. The errors between the mean and variance calculated by third order polynomial and 50000 Monte Carlo simulations are 0% and 12.7%, respectively.

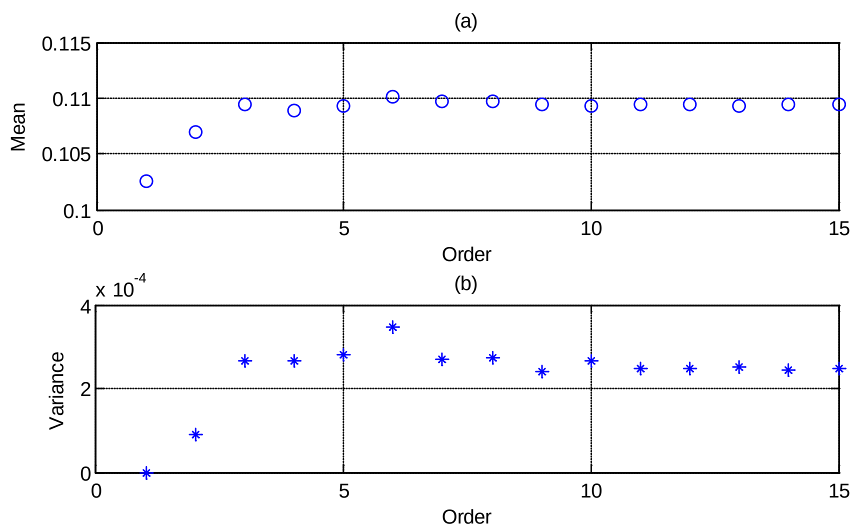

Fig. 4 Performance of different order polynomial chaos expansions for (a) mean (b) variance

4. Polynomial chaos expansions for system with delay

The third order polynomial chaos expansion is used with regression method to calculate the influence of the time delay in DDE model when mass, damping and stiffness are random variables. The number of the sample is selected as (*M*-1) multiply *P*, which is 40 sampling points. Since the mean value of the mass and stiffness are 1kg and 39.47KN/mm, the time delay is selected from 0.5ms to 5ms for every 0.5ms.

4.1 Coefficients of the polynomial chaos expansion for system with delay

The third order polynomial chaos expansion with random mass, damping and stiffness can be written as,

$$\begin{aligned} y_{\max}(m,c,k) &= a_0H_0 + a_1H_1(m_i) + a_2H_1(c_i) + a_3H_1(k_i) + a_4H_2(m_i,m_i) + a_5H_2(c_i,c_i) + \\ & a_6H_2(k_i,k_i) + a_7H_2(m_i,c_i) + a_8H_2(m_i,k_i) + a_9H_2(c_i,k_i) + a_{10}H_3(m_i,m_i,m_i) + a_{11}H_3(c_i,c_i,c_i) \\ & + a_{12}H_3(k_i,k_i,k_i) + a_{13}H_3(m_i,m_i,c_i) + a_{14}H_3(m_i,m_i,k_i) + a_{15}H_3(m_i,c_i,c_i)a_{16}H_3(m_i,k_i,k_i) + \\ & a_{17}H_3(m_i,c_i,k_i) + a_{18}H_3(c_i,k_i,k_i) + a_{19}H_3(k_i,c_i,c_i) \end{aligned} \quad (12)$$

where y_{max} represents the maximum displacement in the simulation, a_i are the coefficients to be determined, H_i are the Hermit polynomials which follow

$$H_0 = 1 \quad (13a)$$

$$H_1(\omega) = \omega \qquad (13b)$$

$$H_2(\omega,\omega) = \omega^2 - 1 \qquad (13c)$$

$$H_2(\omega,\xi) = \omega\xi \qquad (13d)$$

$$H_3(\omega,\omega,\omega) = \omega^3 - 3\omega \qquad (13e)$$

$$H_3(\omega,\omega,\xi) = \xi(\omega^2 - 1) \qquad (13f)$$

$$H_3(\omega,\xi,\varsigma) = \omega\xi\varsigma \qquad (13g)$$

The coefficients for different time delay are shown in Fig.5. In Fig.5, the first coefficient for each polynomial chaos expansion represents the mean of the maximum displacement, while the summation square of rest coefficients represents the variance. From Fig.5, the mean and variance of the maximum displacement vary with time delay.

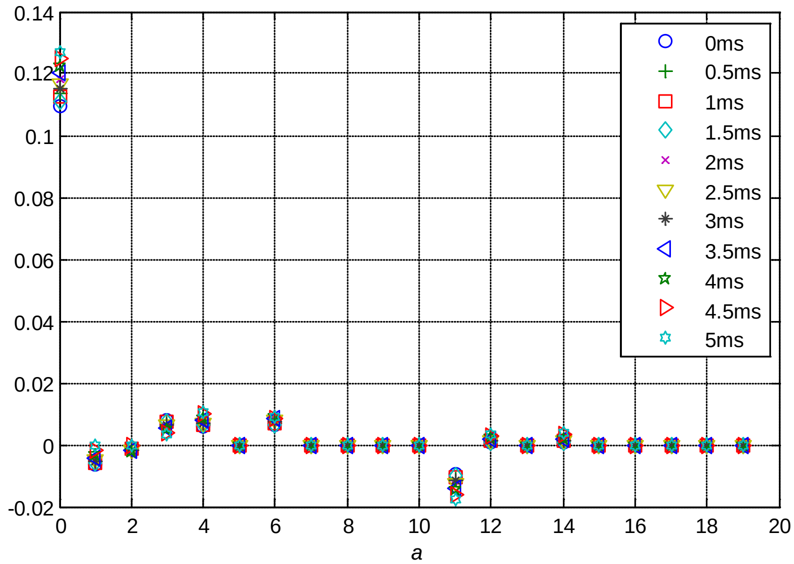

Fig. 5 Coefficients of the polynomial chaos expansion for system with delay

4.2 Delay effect on the mean of the maximum displacement response

The mean of maximum displacement and its corresponding time delay is shown in Table 5 for cases with stochastic variables. Meanwhile, the maximum displacements for deterministic mass, damping and stiffness when equal to the mean are also presented in the figure. For stochastic case, the maximum displacement increases with respect to time delay. When the time delay is less than 3.5ms, the slope is 0.003m/ms, then the slope increases to 0.004m/ms after 3.5ms. When compared with the results for constant variables, the maximum displacements for random variables are about 10% larger, which is consistent with 50000 Monte Carlo simulations. The difference between the maximum displacements for stochastic variables and deterministic variables increases with time delay, which shows the need for uncertainty quantification again.

Table 5 –Relationship between time delay and the mean of maximum displacement

Delay (ms)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
deterministic (m)	0.101	0.102	0.103	0.105	0.106	0.107	0.109	0.111	0.112	0.114	0.115
stochastic (m)	0.110	0.112	0.113	0.114	0.115	0.117	0.118	0.120	0.122	0.125	0.127

4.3 Delay effect on variance of the maximum displacement

The relationship between the variances of the maximum displacement and time delay for random variables is presented in Table 6. It can be observed that the variance of the maximum displacement increases with time delay exponentially. When DDE model contains random variables, the time delay could enhance the uncertainty of the model output. The variance of the maximum displacement time delay is 5ms is more than twice of that when than no delay case.

Table 6 –Relationship between time delay and variance

Delay (ms)	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
Variance (10^{-4})	2.4	2.6	2.9	3.0	3.2	3.4	3.5	3.8	4.1	4.7	5.4

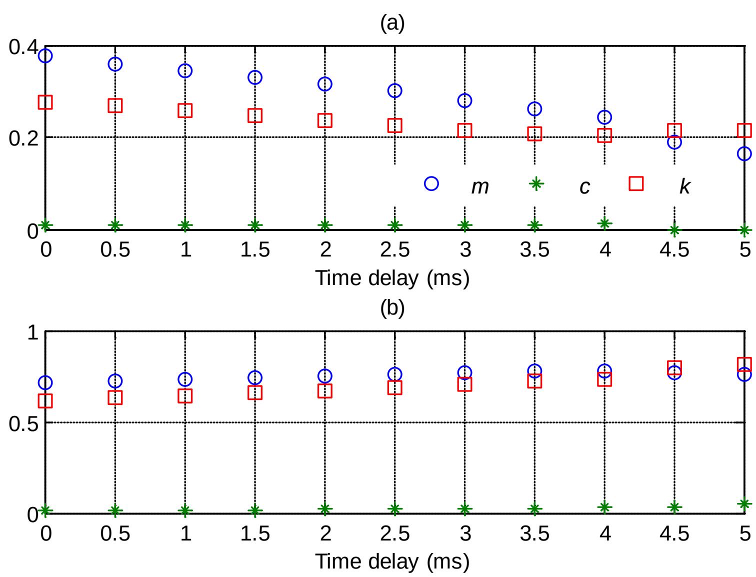

Fig.6 Relationship between time delay and Sobol indices of the stochastic variables for (a) First order Sobol indices and (b) Total Sobol indices

4.4 Delay effect on Sobol indexes of the random variables

The relationships between the Sobol indexes of the random variables and time delay are shown in Fig.6. From Fig.6, the first Sobol indexes for all mass, damping and stiffness decrease with the time delay, while the total Sobol indexes increase with time delay for all variables. Since the first Sobol index represents the influence of single parameter alone on the total output variance, the decrease of the first Sobol index with time delay means the ability to influence on the variance the maximum displacement is decreasing for one parameter alone with increase of the time delay. On the other hand, total Sobol index S_{Ti} represents the influence of one parameter combined with other parameters on the total output variance. The increase of the total Sobol index with respect to time delay means time delay can increase the coupling effect of the random variables.

In Fig. 6, the most important parameter which affects the variance of the maximum displacement is the mass for the case of no delay. However, when the time delay is larger than 4.5ms, the most influential parameter becomes the stiffness in both first order Sobol and total Sobol indices. Thus, time delay can change the variable which has the most influence on the variance of the maximum displacement from real-time hybrid simulation.

5. Summary, conclusion and future work

Uncertainties exist for both numerical and experimental substructures in real-time hybrid simulation. The uncertainties may results in different structural responses even for the same prototype structure and the same servo-hydraulic control in real-time hybrid simulation. In this paper, DDE model with random variables is selected to study the effect of the delay when model contains uncertainty parameters. Polynomial chaos expansion is used to build the relationship between uncertainty variables and the maximum displacement response. The mean, variance and Sobol indices of 50000 Monte Carlo simulations are used as standard values to select the best coefficient calculation method, degree and number of sampling when conduct polynomial chaos expansion. The polynomial chaos expansion then applied to DDE models with different time delay.

For DDE model, third order polynomial chaos expansion using regression method (LARS) is good enough to build the meta-model between of the maximum displacement and stochastic variables. The mean and variance of the maximum displacement increase with time delay linearly and exponentially, respectively. The difference between the mean of the maximum displacement and the maximum displacement when random variables equal to their mean values grows with time delay. Sensitivity analyses show the influence of the single random variable is decreasing while the coupling effect is increasing with the time delay. Meanwhile, time delay can change the variable which has the most influence on the variance of the maximum displacement in DDE model. Only the linear elastic structure is considered in this study. Future study will consider the effect of structural nonlinearity as well as different ground motions.

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