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Definition of an absolute acceleration response spectrum

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Abstract

After an introduction on the response spectrum calculation, this article sets a comparison between the pseudo acceleration spectra, the Eurocode-8 spectrum and the structure acceleration spectra. It shows that the Eurocode-8 procedure gives results which are not conservative concerning damped and isolated structures. In consequence, a formula is introduced to fix the Eurocode-8 spectra and ensure a safe design.

Keywords: Response spectrum, pseudo acceleration, absolute acceleration, Eurocode-8 spectrum corrections

1. Introduction

A spectral analysis is proposed by most seismic codes for damping ratios below 30 %. This kind of analysis outputs a pseudo acceleration, assuming that it is a good assumption of the absolute acceleration. The purpose of this paper is to show that the use of the pseudo acceleration response spectrum can lead to under-designed isolation systems. A correction of the pseudo acceleration response spectrum is proposed in order to obtain absolute acceleration.

2. Definition of the response spectrum

This section briefly sets out the steps used to define the expression of the displacements, velocities and accelerations, and their corresponding spectral values. The calculation details are given in [1]. The equation of motion (1) for a system with one degree of freedom made up of a mass, a spring and a linear viscous damper can be written in its reduced form (2).

$$M\ddot{x} + C\dot{x} + Kx = -Ma_{soil} \tag{1}$$

$$\ddot{x} + 2\xi\omega\dot{x} + \omega^2 x = -a_{soil} \; with \; \xi = \frac{C}{2m\omega} \; and \; \omega^2 = \frac{K}{M} \tag{2}$$

Duhamel's integral (3) gives an analytical expression of the relative displacement of the system subjected to seismic excitation.

$$x(t) = -\frac{1}{\omega_d}\int_0^t a_{soil}\,(\tau)e^{-\xi\omega(t-\tau)}.\sin[\omega_d(t-\tau)]\,.\,d\tau \; with \; \omega_d = \omega\sqrt{1-\xi^2} \tag{3}$$

The first derivative of Duhamel's integral gives the relative velocity $\dot{x}(t)$.

$$\dot{x}(t) = \int_0^t a_{soil}\,(\tau)e^{-\xi\omega(t-\tau)}.\cos[\omega_D(t-\tau)]\,.\,d\tau - \frac{\xi}{\sqrt{1-\xi^2}}\int_0^t a_{soil}\,(\tau)e^{-\xi\omega(t-\tau)}.\sin[\omega_D(t-\tau)]\,.\,d\tau \tag{4}$$

The absolute acceleration $a(t) = \ddot{x}(t) + a_{soil}(t)$ is deduced from equation (2) and equals $-(2\xi\omega\dot{x} + \omega^2 x)$. Hence:

$$a(t) = \omega\frac{2\xi^2-1}{\sqrt{1-\xi^2}}\int_0^t a_{soil}\,(\tau)e^{-\xi\omega(t-\tau)}.\sin[\omega_D(t-\tau)]\,.\,d\tau - 2\xi\omega\int_0^t a_{soil}\,(\tau)e^{-\xi\omega(t-\tau)}.\cos[\omega_D(t-\tau)]\,.\,d\tau \tag{5}$$

The definition of the spectral values takes place through several simplifications:

- Assimilate the pulsation ω_d to ω, which is reasonable for damping rates ξ less than 10%.
- For velocity and acceleration, eliminate the terms for which damping is factorised. These terms are considered negligible by [1] for damping rates less than 20%.
- For velocity, in the remaining term that is expressed with a cosine, replace the cosine by a sine. In [3], Hudson shows that the error introduced by this substitution is negligible for small periods and low damping rates. This is not the case for damped, isolated systems.

The spectral displacement S_d, pseudo velocity S_{pv} and pseudo acceleration S_{pa} are defined as follows.

$$S_d(\xi,\omega) = \max_t |x(t)| \tag{6}$$

$$S_{pv}(\xi,\omega) = \omega S_d(\xi,\omega) \tag{7}$$

$$S_{pa}(\xi,\omega) = \omega S_{pv}(\xi,\omega) = \omega^2 S_d(\xi,\omega) \tag{8}$$

A response spectrum is the representation of one of the three quantities S_d, S_v or S_a as a function of period or frequency.

By analogy, a velocity response spectrum and an acceleration response spectrum can be constructed that are respectively a representation of the relative velocity S_v and the absolute acceleration S_a as a function of period or frequency. S_v and S_a are defined by the following expressions.

$$S_v(\xi,\omega) = \max_t |\dot{x}(t)| \tag{9}$$

$$S_a(\xi,\omega) = \max_t |a(t)| \tag{10}$$

In the spectral modal analysis, the response of structures subjected to an earthquake is calculated, for each of the significant modes, on the basis of equations (6), (7) and (8). For isolated systems, the first vibration mode corresponds to the deformation of the isolation devices and a quasi-rigid body movement of the superstructure. The spectral modal analysis is applicable in accordance with Eurocode 8 up to a damping rate of 28%. This article documents the error obtained with the spectral method for damping rates greater than 10%.

The displacement response spectrum is conventionally calculated on the basis of Duhamel's integral, given by analytical formula (3). The pseudo velocity and pseudo acceleration response spectra are deduced from this by multiplying by the pulsation, as shown in equations (7) and (8).

3. Comparing acceleration and pseudo acceleration at different damping rates

3.1. Comparison method

A normative response spectrum is chosen, referred to as the target spectrum in this paper. Accelerograms are selected with a response spectrum that corresponds to the target spectrum. The pseudo acceleration and absolute acceleration response spectra are calculated for three damping values: 5%, 16% and 28%. The relative differences are then examined between pseudo acceleration and absolute acceleration, and between target spectrum and absolute acceleration. The influence of the damping, the type of target spectrum and the type of accelerogram will be analysed. Three target spectra are examined and their characteristics are given in Table 1. The main differences between the spectra are as follows:

- The intensity of the earthquake is different for the three spectra.
- The spectra have offset plateaus.
- The ASCE spectrum has a falling limb at 1/T that starts from the plateau and extends beyond 6 s, whereas the Eurocode spectrum has a falling limb at 1/T up to 2 s, then at 1/T² beyond 2s.

Table 1 – Definition of target response spectra

Standard	Eurocode 8	Eurocode 8	ASCE
Type	1	1	-
Earthquake zone	5	4	-
Importance coefficient γ_I	1.4	1.3	1
Rock acceleration a_{gr} (m/s^2)	3	1.6	3.45
Soil class	D	A	-
Soil parameter S	1.35	1	1
T_B (s)	0.2	0.15	0.063
T_C (s)	0.8	0.4	0.315
T_D (s)	2	2	8

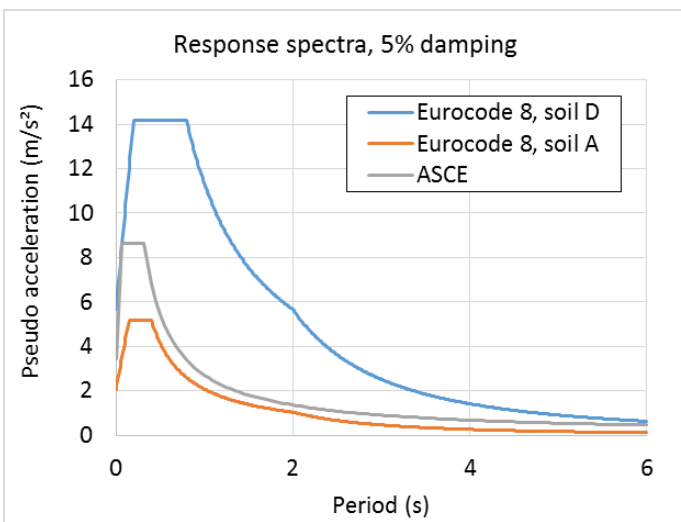

Fig. 1 – Comparison of the three target spectra

Accelerograms are generated with a response spectrum that corresponds to these three target spectra. Table 2 gives the general characteristics of these accelerograms.

Table 2 – *Definition of accelerograms*

Spectrum	Eurocode 8, soil D	Eurocode 8, soil D	Eurocode 8, soil A	ASCE
Accelerogram type	Natural	Artificial	Artificial	Artificial
Number	20	9	15	9
Generation tool	PEER database	SIMQKE	SIMQKE	SIMQKE

The natural signals selected from the PEER (Pacific Earthquake Engineering Research) accelerogram database are summarised in Table 3. For each accelerogram, the components of the two horizontal directions were analyzed.

Table 3 – Main characteristics of the 20 accelerograms selected from the PEER database

Record number	**Amplification factor**	**Earthquake name**	**Year**	**Station name**	**Magnitude**
70	4.4441	San Fernando	1971	Lake Hughes #1	6.61
78	4.237	San Fernando	1971	Palmdale Fire Station	6.61
164	2.5246	Imperial Valley-06	1979	Cerro Prieto	6.53
289	2.6913	Irpinia Italy-01	1980	Calitri	6.9
313	2.2281	Corinth Greece	1981	Corinth	6.6
755	1.8596	Loma Prieta	1989	Coyote Lake Dam - Southwest Abutment	6.93
864	1.5601	Landers	1992	Joshua Tree	7.28
881	2.3799	Landers	1992	Morongo Valley Fire Station	7.28
1083	3.138	Northridge-01	1994	Sunland - Mt Gleason Ave	6.69
3757	3.0722	Landers	1992	North Palm Springs Fire Sta #36	7.28
4013	4.0421	San Simeon CA	2003	San Antonio Dam - Toe	6.52
4844	3.4969	Chuetsu-oki Japan	2007	Tokamachi Matsunoyama	6.8
4850	1.373	Chuetsu-oki Japan	2007	Yoshikawaku Joetsu City	6.8
4872	3.1189	Chuetsu-oki Japan	2007	Sawa Mizuguti Tokamachi	6.8
5274	3.9551	Chuetsu-oki Japan	2007	NIG028	6.8
5275	3.8404	Chuetsu-oki Japan	2007	NIGH01	6.8
5284	3.1543	Chuetsu-oki Japan	2007	NIGH11	6.8
5806	2.0702	Iwate Japan	2008	Yuzawa Town	6.9
5818	0.9159	Iwate Japan	2008	Kurihara City	6.9
6971	2.4982	Darfield New Zealand	2010	SPFS	7

The artificial signals, generated using the SIMQKE method [2], have the following characteristics:

- duration: 40 s,
- duration of the strong part 15 s,
- envelope shape: exponential.

3.2. Comparison of the response spectra at 5%

Fig. 2 shows that the difference between pseudo acceleration and absolute acceleration remains below 3% over the entire period range between 0 and 6 seconds, whatever the type of spectrum and whether the accelerograms are natural or artificial. It is therefore reasonable to consider the pseudo acceleration and the acceleration of the structure as equal for 5% damping. The natural accelerograms show a greater difference from the target spectrum than the artificial accelerograms. This factor must be taken into account in the analysis.

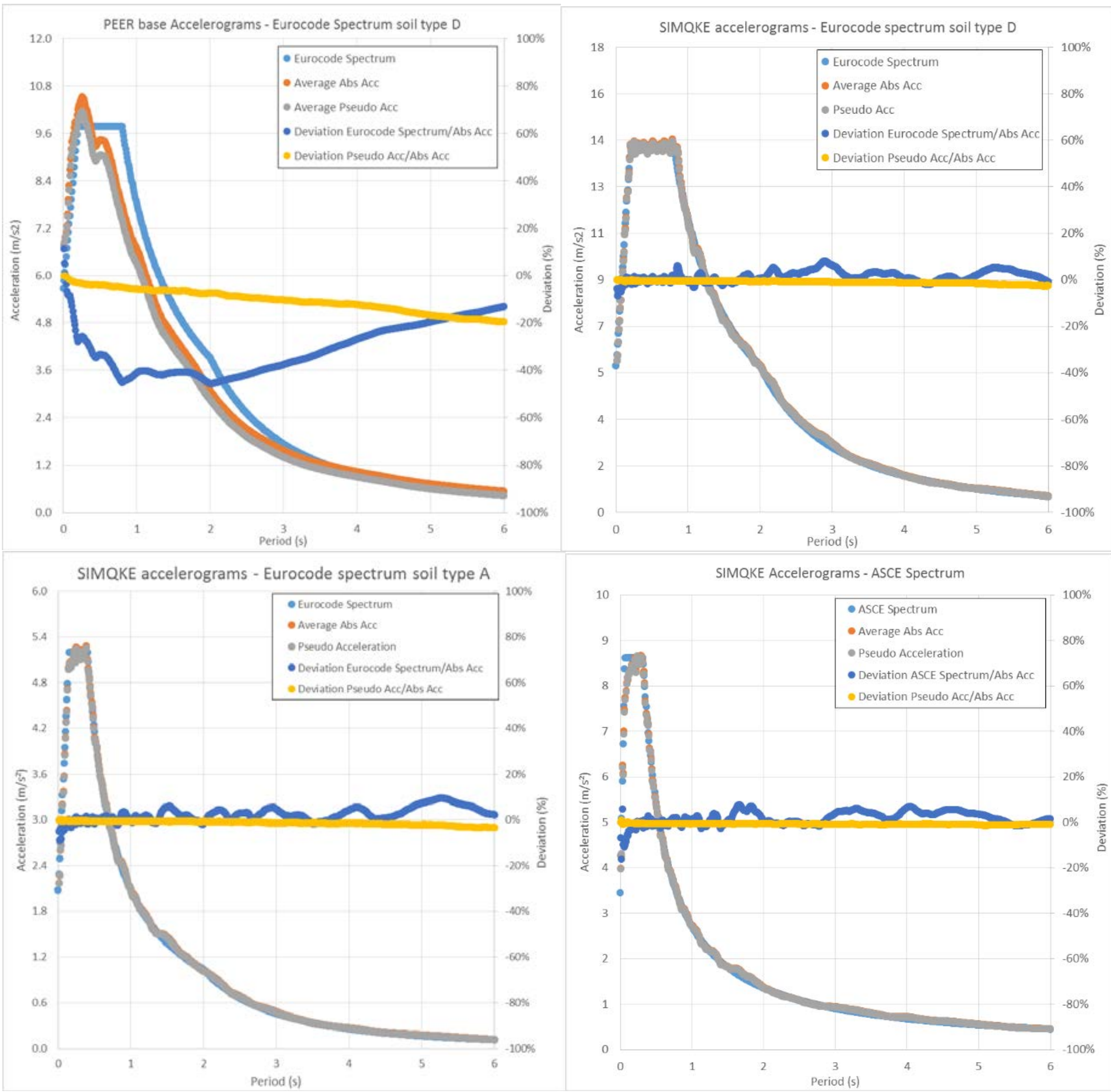

Fig. 2 – Comparison of pseudo acceleration and absolute acceleration at 5% damping

3.3. Comparison of response spectra at 16%

Fig. 3 shows that the ratio between pseudo acceleration and mean absolute acceleration decreases consistently when the period increases. The difference between absolute acceleration and the target spectrum depends on the period. For periods less than T_D, i.e. before the falling portion at $1/T^2$, the difference is relatively constant. The difference increases considerably for periods greater than T_D. The value of the T_D period of the ASCE spectra is more than 8 s. The difference between absolute acceleration and the target spectrum is therefore almost constant up to T = 6 s.

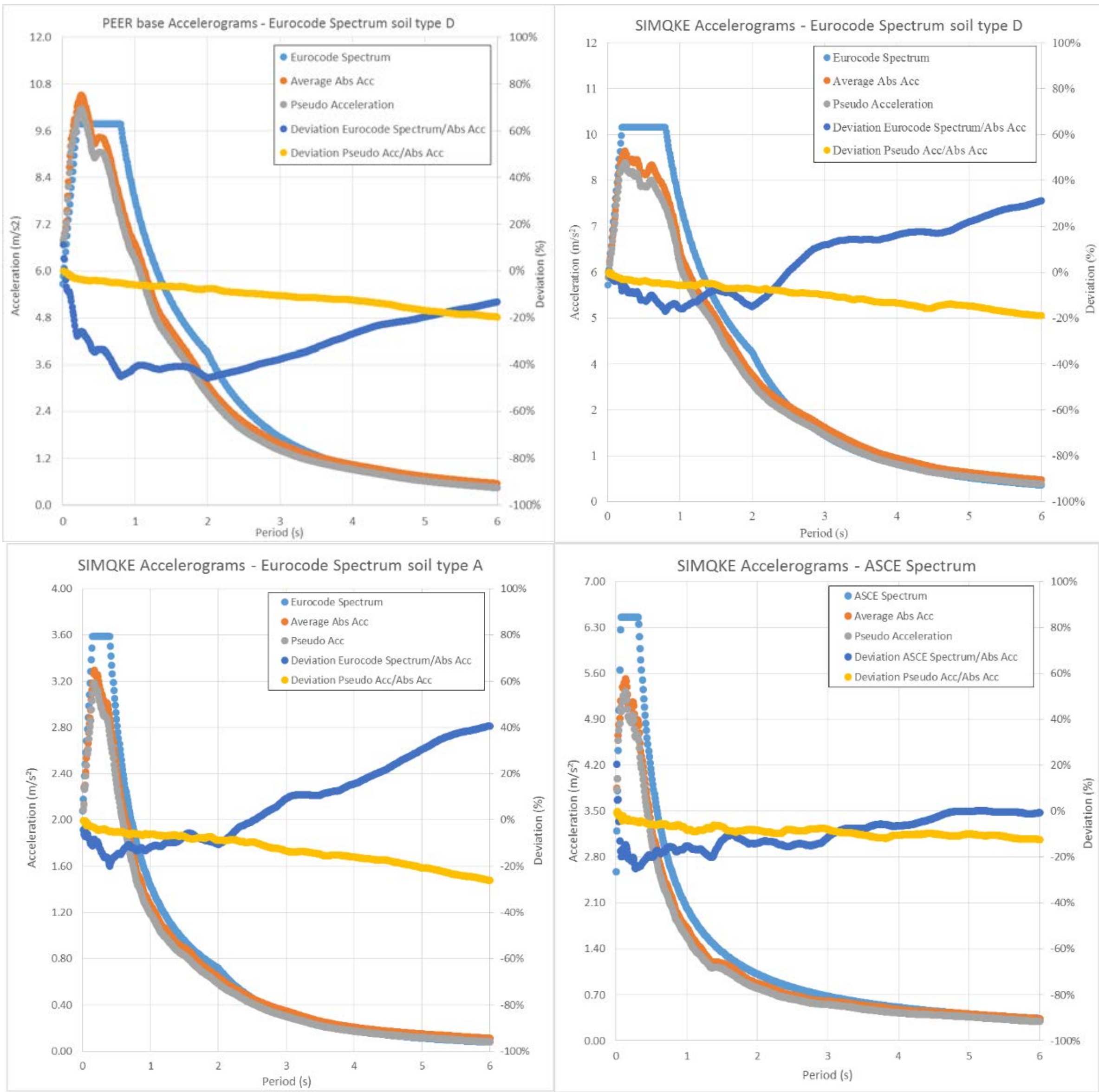

Fig. 3 – Comparison of pseudo acceleration and absolute acceleration at 16% damping

3.4. Comparison of the response spectra at 28%

Fig 4 confirms the analysis carried out for 16% damping. The same trends can be seen evenly more markedly. The 1/T part of the Eurocode spectrum gives a good approximation of the absolute acceleration, whereas a the $1/T^2$ part provides a good correlation with the pseudo acceleration.

It is also observed that the variability due to the fact that the accelerograms are natural is significantly reduced.

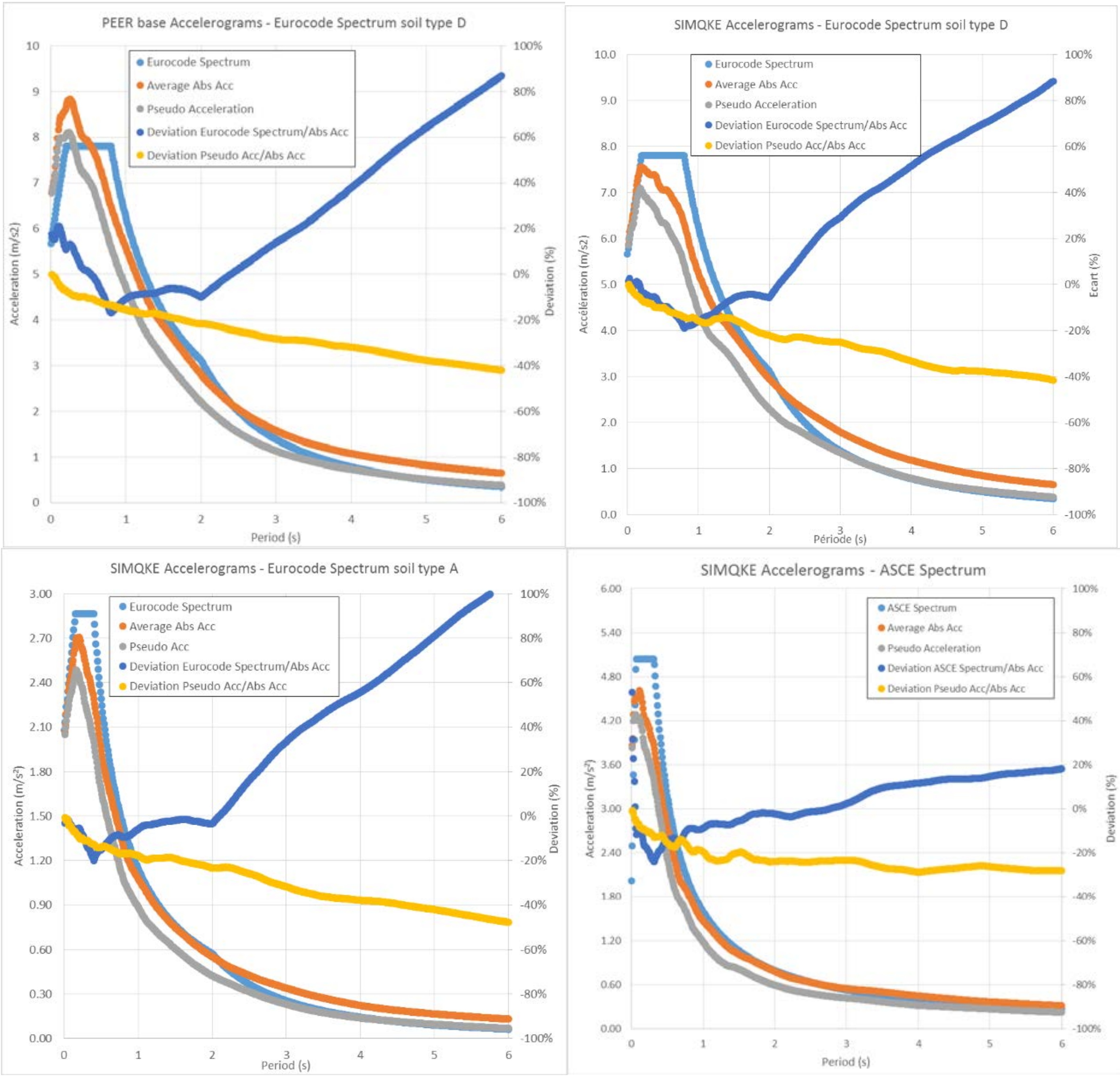

Fig 4 – Comparison of pseudo acceleration and absolute acceleration at 28% damping

4. Analysis

4.1. Difference between pseudo acceleration and absolute acceleration

A growing relative difference is observed between pseudo acceleration and absolute acceleration when the period increases. This difference increased with the damping rate. Section 2 showed that the absolute value of the absolute acceleration is equal to $\omega^2 x + 2\xi\omega v$. This is the sum of the pseudo acceleration and an acceleration due to the damping forces. For 28% damping, the share of acceleration due to the damping forces equals almost half of the total acceleration.

4.2. Difference between target spectrum and absolute acceleration

The Eurocode spectrum is the envelope of absolute accelerations for periods less than T_D. Beyond T_D, the Eurocode spectrum underestimates absolute acceleration. This deviation increases with the damping.

It appears that the differences observed depending on the nature of the accelerograms are only significant for low damping rates. Whether the accelerograms are natural or artificial therefore has no influence on the conclusions.

Furthermore, the trend is the same for the two types of Eurocode spectrum examined. It therefore appears reasonable to conclude that this analysis remains valid whatever the Eurocode spectrum.

4.3. Impact of the analysis on isolation systems

For isolated systems, it appears that pseudo acceleration is not an appropriate measurement for calculating the acceleration of the structure.

Different isolator technologies exist, described in NF EN 15129 [6]:

- **Elastomeric isolator**. This isolator is similar to an elastomeric bearing, with special arrangements made during their design to ensure stability under seismic displacement. It is made up of a series of elastomeric and steel layers. The elasticity and damping of the device are linked to the properties of the elastomer used. It is referred to by its acronym **HDRB**, which stands for High Damping Rubber Bearing.
- **Lead core elastomeric isolator**. This device is an elastomeric isolator into which one or more lead cores are inserted. The lead is very ductile and its plastic deformability increases the damping capacity of the isolator. It is generally stiffer than a simple elastomeric isolator. It is referred to by its acronym **LRB**, which stands for Lead Rubber Bearing.
- **Curved-surface sliding elements**. This device is made up of two metal parts in contact with each other. One is shaped like a large plate and the other like a lens. The interface between the plate and the lens is a spherical sliding surface with a controlled coefficient of friction. During a seismic event, the flexibility of the isolation system is linked to the radius of the spherical surface. It is as though the isolated structure is suspended from a pendulum of the same radius. The damping of the system is obtained by friction between the two parts. It is referred to by its acronym **PS**, which stands for Pendulum System.
- **Flat-surface sliding elements**. This is a pot bearing, for example. It is used in addition to the three types described above. It provides the system with no elasticity and almost no damping, and is used to take up the vertical loads without blocking seismic motion.

Table 4 – Relevance of pseudo acceleration when designing isolation systems, depending on the technology

Isolator type	Damping range	Period range	Relevance of pseudo acceleration (obtained from the accelerograms)	Relevance of pseudo acceleration (normative target spectra)
HDRB	7 to 16%	2 to 3 seconds	Reasonable difference (less than 10%)	Reasonable difference (less than 10%)
LRB	16 to 30%	2 seconds	Significant difference (greater than 20%)	Reasonable difference (less than 10%)
PS	16 to 30%	3 to 5 seconds	Significant difference (greater than 20%)	Significant difference (greater than 20%)

Table 4 sets out the range of use of these devices and the relevance of pseudo acceleration as a measurement of the absolute acceleration of the system fitted with them.

When the difference is significant, a spectral analysis of the isolated system would lead to the underestimation of the acceleration of the superstructure and the under-designing of the devices. In this case, it is appropriate to calculate absolute acceleration directly. The next section gives a simple method for calculating absolute acceleration.

5. Proposed absolute acceleration response spectrum

Given that the difference between absolute acceleration and the target spectrum becomes very significant for high damping rates, where the spectrum is proportional to $1/T^2$, an absolute acceleration spectrum can be defined that considers a smaller exponent than 2 on 1/T. Considering $1/T^{4/3}$ gives satisfactory results at 28% damping. A decline at $1/T^{1.7}$ gives satisfactory results for 16% damping. However, the gradient must be kept at $1/T^2$ for 5% damping. An interpolation function is then constructed, linking the value of the exponent to the damping. The following definition is therefore proposed for periods greater than T_D:

$$T_D \leq T: \quad S_a(\xi,T) = 2.5 a_g S \eta \frac{T_C}{T_D}\left(\frac{T_D}{T}\right)^{-2.9\xi+2.145} \tag{10}$$

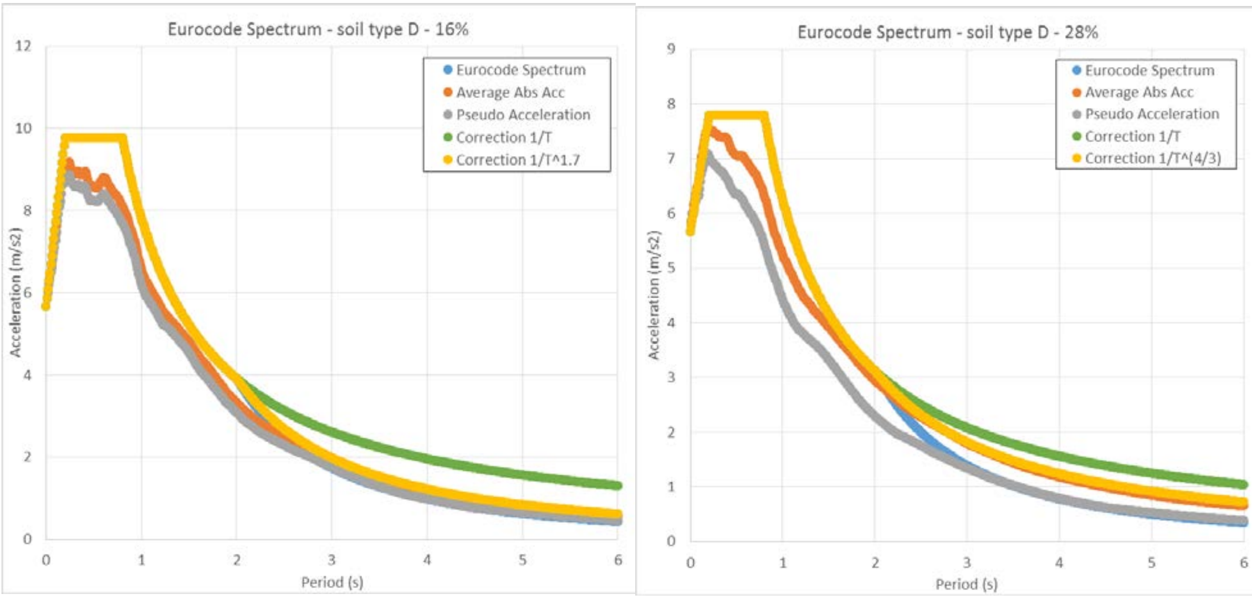

Fig. 5 – Corrected response spectrum at 16% and 28% damping

Fig. 6 shows the mean differences observed on period ranges characteristic of the Eurocode spectra and the corrected spectra. The proposed correction of the spectrum makes it possible to give an envelope value of absolute acceleration whatever the type of Eurocode spectrum.

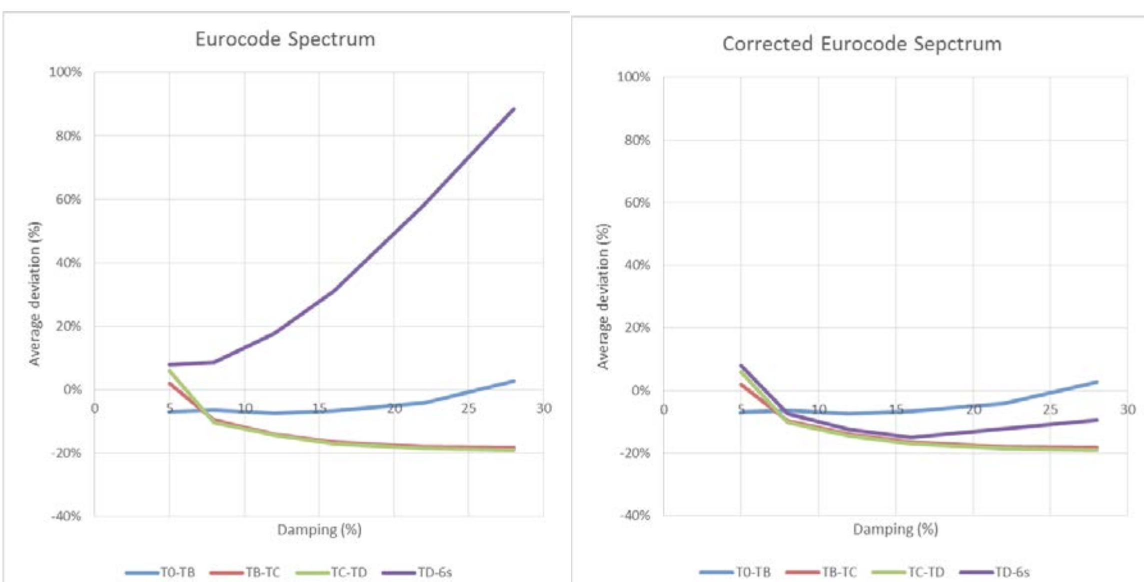

Fig. 6 – Deviation between results obtained with uncorrected and corrected spectral analyses, calculated for different period intervals, with the Eurocode spectra and type D soils.

6. Conclusion

This study shows that pseudo acceleration is not an accurate measurement of absolute acceleration for damped, isolated systems. The difference between these two quantities results in particular in the under-designing of the isolation systems when damping greater than 15% is required.

An absolute acceleration response spectrum calculation is proposed that gives satisfactory results in the range of application of the spectral method.

The use of this absolute acceleration spectrum could possibly lead to the extension of the area of use of the simplified linear analysis of Eurocode 8, which is currently limited to periods of less than 3 s. The calculation of the shear stress at the base of the isolation systems could be obtained by multiplying the mass of the superstructure by absolute acceleration, up to 6 s.

7. References

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