



THE USE OF SENSITIVITY ANALYSIS FOR THE PROBABILISTIC-BASED SEISMIC ASSESSMENT OF EXISTING BUILDINGS

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Abstract

Seismic assessment of existing buildings is treated by international codes and guidelines with semi-probabilistic approaches based on the use of a Confidence Factor (CF) directly related to a target Knowledge Level (KL) of the structure under assessment. Since for existing buildings, the knowledge reached after investigations and deepening could not be complete, the CF is addressed to solve the problem of residual uncertainties still remaining. It has been revealed by many authors the inadequacy of such approach, since it does not cover the probabilistic aspect of the problem in a robust way, and in some cases it is leading to unsafe results. Alternative procedures have been proposed by some authors or in some international and national guidelines. The main idea behind these approaches is to frame the seismic assessment of existing buildings by a fully probabilistic approach capable to take into consideration in an accurate way the propagation of uncertainties in the response of the structure. Although these methods can detect accurately the safety of the structure, they are still computationally demanding and difficult to be integrated in the engineering practice field as standard tool. In this paper, the sensitivity analysis is proposed as a tool to improve the seismic assessment under different points of view: to point out in an explicit way the influence each uncertain parameter has on the structural response; to support the set of an effective investigation plan in order to improve the knowledge of the structure; to compute on basis of a limited number of analyses the essential parameters to pass to a verification based on a probabilistic approach. To the latter aim, results from these analyses are used to determine the median Intensity Measure (IM_{LS}) and, with the help of the Surface Response technique, its dispersion β_{LS} that define the fragility curve representing the capacity in the safety assessment. The proposed methodology is applied on two cases of study where the values of IM_{LS} and dispersions β_{LS} are calculated and compared with reference values obtained from nonlinear static analyses performed on a large number of models generated using Monte Carlo simulations. Results obtained from the procedure based on the sensitivity analysis show a good estimate of the IM_{LS} and β_{LS} parameters, providing, in terms of annual probabilities of occurrence, safety in almost all cases.

Keywords: Sensitivity Analysis; Investigation plan; Probabilistic approach; Response surface technique; Fragility curves

1. Introduction

Many difficulties arise in facing the performance-based seismic assessment (PBA) of existing buildings. They are mainly associated to: 1) overcome the “incomplete” knowledge by effectively acquiring as-built information on material parameters and structural details and by balancing costs, invasiveness and reliability needs; 2) interpret and model their seismic response in the most accurate way despite the huge variety of possibilities that characterize them; 3) properly include in the final assessment the residual uncertainties that in general still remain. As far as the issue 2) concerns, in the assessment of existing buildings, the use of nonlinear analysis approaches (static or dynamic) arises as a very effective tool, especially in case of masonry structures. As far as the issue 3) concerns, it is known that different sources of uncertainties of aleatory and epistemic nature are involved.

Codes at international or national levels usually face the problem within a semi-probabilistic approach based on the use of a Confidence Factor (CF) defined on basis of the Knowledge Level (KL) reached on the structure under examination and applied to a given mechanical parameter assumed a priori to be the most affecting the structural response. As illustrated in §2, the current CF-based approach presents many deficiencies in guarantying results which are always on the safe side. The alternative is to pass to a full probabilistic approach that is emerging not only at research level [1], but also in recommendation documents as SAC-FEMA guidelines [2], firstly illustrated in [3], or the CNR-DT212 [4] recently issued by the Italian National Research Council. Although in principle the full probabilistic approach is the most appropriate in facing all the complex issues involved, many difficulties still endure in its application in engineering practice-oriented procedures due to the huge computational effort and the expertise it usually requires. Indeed, the SAC-FEMA procedure has attempted to address the issue in a convenient way for analysts by converting the probabilistic approach in a semi-probabilistic format by defining conventional factors representative of building typologies. However, the huge variety of existing buildings makes very difficult the identification of well codified values which are enough versatile to cover all their specificities.

Within this context, the objective of this paper is to propose the use of sensitivity analysis for improving the steps 1) and 3) aforementioned of the seismic assessment. After a short description about the current approaches adopted in codes (§2), in §3 it is discussed how the results of a sensitivity analysis “*effectively executed*” may be useful for addressing the investigation plan and evaluating the basic parameters necessary to proceed to the seismic assessment according to a full probabilistic approach. Based on the will of pursuing a practice-oriented approach, the assessment is faced by using nonlinear static analyses instead of the more demanding dynamic ones. Then in §4 the feasibility of the approach proposed is tentatively verified through an application to an URM masonry building. Results achievable by the limited number of analyses performed within the context of sensitivity analyses are then compared with fragility curves built from nonlinear static analyses performed on models generated by a Monte Carlo Simulation. The final safety is checked by mean of the computation of probabilities of occurrence calculated on basis of the closed-form expression presented in [3].

2. Current CF-based and full probabilistic approaches

International standards and guidelines (e.g. [5, 6]) treat the topic of seismic PBA of existing buildings by semi-probabilistic procedures, without explicitly taking into account the probabilistic aspect of the problem. It means that for each Limit State (LS) a single verification is made by defining the *demand* as a proper fractile of the annual probability of occurrence of the seismic event (considered random) in the site under examination, and the *capacity* through proper nominal mean or fractile values. Differently from design that is usually based on linear models and fractiles of the structural parameters (then corrected by proper safety factors), the assessment of existing buildings promote the use of nonlinear models, which usually refer the use of mean values. Then the latter need to be properly “corrected” in order to take into account the residual uncertainty intrinsically involved in the assessment.

The common approach of standards is based on the definition of a Knowledge Level (KL), usually divided into three different levels KL1, KL2 and KL3, with increasing knowledge. The achievement of each KL depends

on the available data together with the knowledge acquired on geometry, structural details or condition assessment in [6], and material properties of the structure under examination. To each KL corresponds a predefined value of a Confidence Factor (CF) (ranging from 1.35 to 1) that must be applied to one specific parameter, assumed *a priori* by the code as being the most critical in affecting the response of the structure. This CF recovers the incomplete knowledge, after investigation, on the parameters used in the final analysis of the structure. In [5] it is suggested to apply the CF to a mechanical parameter, usually related to strength, while in [6] the CF is applied to strength parameters or to deformation capacity, depending on the type of component (if deformation or force controlled).

Many authors studied the procedures based on the use of CF [7, 8, 9, 10, 11] with the aim of investigating the effectiveness and the degree of safety provided by this kind of approach. Some numerical simulations carried on reinforced concrete [7] or masonry structures [9] proved that sometimes the obtained results are not safe. The main critical issues that can be singled out are: i) the parameter, which the CF is applied to, is selected a priori, without being sure that this is the one having the highest uncertainty/sensitivity on the structure; ii) the CF is related to the KL, which is conceptually correct, but its value is conventional and the KL reached is defined as the worst among the different examined aspects (geometry, material properties, constructive details) without considering the various effects they may have on the structural response; iii) in case of use of nonlinear analyses (which is recommended in case of existing buildings), the stability of the result from a continuous variation of the assumed relevant parameter is not sure. With the aim of overcoming some of these drawbacks various proposals have been recently outlined in literature. For example in [10] it has been proposed to apply the CF directly to the value of the capacity, in terms of the Intensity Measure (IM) compatible with the attainment of a given LS. An alternative to face the problem of using conventional values of CF has been proposed in [11] through the use of the sensitivity analysis to calibrate these values and choose the parameter to which apply the CF, as the one mostly affecting the response without any *a priori* selection.

The alternative for including in the assessment the uncertainties treatment in a more robust and rigorous way is to pass to a fully probabilistic approach. This would require the knowledge of the fragility curve of each LS, usually defined by a cumulative lognormal distribution in terms of two parameters: the median value of IMs leading to the attainment of a certain limit state, IM_{LS} , and corresponding dispersion β_{LS} as shown in Eq. (1):

$$p_{LS} = P(d > D_{LS} | im) = P(im_{LS} < im) = \Phi \left(\frac{\log \left(\frac{im}{IM_{LS}} \right)}{\beta_{LS}} \right) \quad (1)$$

where d is a displacement representative of the building seismic behavior, D_{LS} is its LS threshold, IM_{LS} is the median value of the lognormal distribution of the intensity measure im_{LS} that produces the LS threshold and β_{LS} is the dispersion. In [4] different methods based on the execution of nonlinear Incremental Dynamic or Static Analyses have been proposed. However, they require a significant computational effort and expertise, what makes it not yet feasible as the “standard” tool for applications, at least for ordinary existing buildings. On the other hand, an effective analytical closed-form expression for the computation of p_{LS} has been proposed in [2]:

$$p_{LS} = k_0 (IM_{LS})^{-k} e^{\frac{1}{2} \beta_{LS}^2 k^2} \quad (2)$$

This expression is based on the assumptions that the hazard function is approximated by a linear regression on the log-log space (defined by k_0 and k), and the demand and the capacity are independent variables (what makes more simple the computation of dispersion β_{LS}). The linear regression used to assume the hazard in Eq. (2) presents some drawbacks, especially with return periods outside the range [30, 2475] years. To overcome this problem, a second order function has been proposed in [12]. Then in [13] Eq. (2) was also converted in a practical format very effective for engineers, for the safety checking, by proposing also conventional values for the main parameters involved. However, studies in literature are not able to cover the huge variety of features of existing buildings highlighting the conventionality to adopt reference values representative of a whole class in the context of the assessment of a single building. To this aim, in §3 the potential of a limited number of analyses is explored for computing IM_{LS} and β_{LS} , with the main advantage to be “specifically targeted” to the building under examination.

3. Potential of the sensitivity analysis for improving the reliability of seismic assessment of existing buildings

Sensitivity analysis is an innovative technique used in both research and practice areas in order to point out the dependence of an outcome of study on its dependent parameters. In seismic assessment of existing buildings, the usefulness of this analysis was revealed by many authors [7, 11] because of its capability to overcome the problem of uncertainties mentioned in the introduction. In [11] the sensitivity analysis was explored to identify the uncertain parameters that mostly affect the response of the structure, to route the investigation plan and deepen their knowledge only when this is relevant. To this aim, as proposed in [11], it is helpful to switch from a global scale KL (referred to the whole structure) to different KLs associated to each parameter according to its degree of sensitivity. Herein its potential is explored to improve the setting of investigation plan and to determine, on basis of a limited number of analyses, the two parameters that characterize the fragility curve of the buildings (IM_{LS} , β_{LS}). In particular, firstly the partial dispersions are used (§3.1) and, then, they are combined to define the total one (§3.2). In both cases, results are validated with a more rigorous probabilistic procedure in order to verify the effectiveness of performing a limited number of analyses: in the case of partial dispersion the target reference is the complete factorial analysis; in the case of the total one the Monte Carlo sampling.

In general a preliminary phase of knowledge of the building is essential to set up a structural model and to define the aleatory and epistemic uncertainties involved in the assessment of the building under examination. For epistemic variables, usually related to modelling uncertainties, a possible way to proceed is using the logic tree approach, representing evaluations with alternative models, and attributing to each uncertainty a subjective probability quantifying its reliability. In this paper, focus will be only on aleatory uncertainties, mostly related to geometry, material and diaphragms properties, by proposing a simplified sensitivity analysis capable to detect the influence of each one on the response of the structure. The engineer is supposed to define to each one of the aleatory parameters (X_k), a plausible range of variation characterized by median (X_{med}), minimum (X_{low}) and maximum (X_{up}) values, determined using information available in codes, literature or previous studies performed on buildings in the vicinity. Parameters could be considered separately or combined into one group assuming that their variation (in terms of lower and upper values) must be identical in each model.

The full way to proceed with a sensitivity analysis is to create different models of the same structure characterized by the possible combinations between the lower and upper values of all the aleatory parameters in order to assess the variability of the outcome of the PBA. The execution of a complete factorial analysis requires performing 2^N analyses, where N is the number of aleatory variables (or group of variables): thus, the number of analyses will increase rapidly by adding more parameters, what could be extremely exhaustive and time consuming, even more than a full probabilistic assessment (i.e. faced by the Monte Carlo approach). In order to balance the computational effort, it is herein proposed to start with a sensitivity analysis comprising only $2N+1$ models and then, if necessary, as highlighted by the results of such first phase, to add additional targeted analyses. Particularly, each one of the $2N$ models is formed by considering the median values of all the aleatory parameters but one set at one of its extremities. The additional analysis (+1) is performed using a model with all parameters set at their median values. The result of each analysis is summarized by a Structural Performance Indicator (SPI) represented by the maximum value of the IM compatible with the attainment of a given LS (im_{LS}), which is selected by the engineer to be the best representative of the structural response. In general, for masonry structures that are the object of the case study examined in §4, a good assumption of the im_{LS} is the Peak Ground Acceleration (PGA): such approximation (instead of the use of spectral ordinate associated to the fundamental period) is justified by the fact that they are characterized by a period of vibration rather low. This quantity can be calculated using nonlinear static procedures based on the use of overdamped or inelastic spectra.

3.1 Computation of partial dispersions to address the investigation plan

The values of im_{LS} collected from the sensitivity analysis are firstly used to define the partial dispersions β_i that reflect the sensitivity of each aleatory parameter on the structural response, by capturing the variability of the IM when moving from X_{low} to X_{up} of a parameter. They can be considered as the angular coefficient of the

hyperplane that fits the response surface of the variable $\log(im_{LS})$ in the hyperspace of the normalized variables representing the aleatory parameters, and calculated using the Response Surface Technique [14] as in Eq. (3):

$$\beta_i = (Z^T Z)^{-1} Z^T Y \quad (3)$$

where: Z is the matrix of normalized variables -1 and +1 corresponding to X_{low} and X_{up} values of the aleatory parameters respectively and Y is the matrix of the im_{LS} quantities deriving from the analysis performed on the model represented at the same row in matrix Z . In the case of the $2N+1$ analyses, the normalization of the median value is assumed to be equal to zero, and the regression is made in a two dimensional plane where the points are defined by the two analyses performed with the two interval extremities of each variable.

The partial dispersions are useful to direct the investigation plan aiming to deepen the knowledge of the structure. In fact, this step allows the identification of the most relevant investigations, focusing on the parameters that create a great uncertainty in the safety of the assessment, that are those having a high value of β_i . The investigation plan assigned is now formed by target KLs, also graduated in High (KLH), Medium (KLM) and Low (KLL), related to each one of the aleatory parameters. A definite and automatic classification of the KL corresponding to each parameter (mechanical properties of masonry, geometry, construction details, masses, stiffness of horizontal elements, etc.) is problematic, as it should be the result of interaction between different instrumental investigations and visual inspections as well as documented information (available drawings of the project, documented photographs of the phases of construction or later modifications, etc.). However, this phase remains so important since it directs the engineer on what kind of investigation and the number of tests he needs for each part or constituent of the structure. Another important point to take into consideration is the reachability of the target KL. Indeed, it may be required from the sensitivity analysis to assign to a specific parameter a KLH since it represents a high sensitivity, but in reality, reaching this KL for this parameter is not possible. In addition to that, care about costs and invasiveness should be highlighted since it also affects the definition of the target KL, especially in the case of cultural heritage structures where the level of intervention is rather low. Usually, it is known from the beginning if a certain parameter could be deepened through experimental tests or not, but even though, it is useful to integrate it in the sensitivity analysis in order to find out its degree of effectiveness on the response of the structure. In some cases, the variation between im_{LS} of the two analyses performed with the limits of a certain aleatory parameter could be not monotonic, so it is better to run more analyses by varying the values of the other parameters (previously set at the median value) in order to form a more clear idea about its importance and then calculate β_i . The results of the new investigations can lead to confirm or update the median values X_{med} , of the intervals of variation of the aleatory variables (assuming implicitly, without the need of a direct estimation, that due to the investigations executed, the initial interval will be reduced to some extent). As for the epistemic uncertainties, the additional investigations help to acquire information useful in choosing the most reliable model among the alternatives originally assumed, or at least assign to each model a subjective weight, representative of the reliability of each choice. A practical way to update the median values and the ranges of variation of the aleatory variables could be the Bayesian approach as demonstrated in [8] and [10].

3.2 Computation of the total dispersion to pass to a full probabilistic assessment

Since it is impossible to reach a complete knowledge of the whole structure (KLH for all parameters) even after investigations, the im_{LS} value representing the capacity of the structure will remain different from the one obtained from the analysis performed with all the parameters set at their median values. The results of a new sensitivity analysis could provide a good estimate of IM_{LS} and β_{LS} . After executing the investigation plan and updating the median values and the intervals of variation of the parameters with high sensitivity, two possible alternatives of proceeding may arise: i) in case the median value of any parameter is significantly modified, the sensitivity analysis should be rerun by adopting the modified values for the new models; ii) in case the rational intervals already assumed are significantly modified, it is necessary to rerun only the analyses where it was used X_{min} or X_{up} of the updated parameter. It may seem that the computational efforts in the second case are less than the first one, but in fact, the worst case is to rerun $2N+1$ analyses again, resulting at the end $4N+2$ nonlinear static analyses, which is still considered a low number compared to a full probabilistic procedure.

The new results of sensitivity analysis are used to define the median value of all the im_{LS} calculated at each analysis performed with the updated variables. On the other hand, by reapplying the response surface

technique on the logarithm of the new im_{LS} quantities, it is possible to define the new Partial Dispersions β_i and the total one β_{LS} of the fragility curve representative of each LS as shown in Eq. (4).

$$\beta_{LS} = \sqrt{\beta_i^T \beta_i} \quad (4)$$

4 Application of the procedure

4.1 Description of the cases of study

For testing the effectiveness of sensitivity analysis, a first case study (referred to as *case-noRC*) consisting of a three-story residential masonry building is selected. It is an existing building made of brick masonry and lime mortar located in San Felice sul Panaro, Italy and hit by the seismic event of 2012 (Fig.1 a). The diaphragms are made of concrete beams with flooring blocks, while the roof is a timber structure constructed with trusses and strut layers. The response of the structure is examined in the following through the equivalent frame modeling approach, using Tremuri program [15] and by performing nonlinear static analysis. The nonlinear response is concentrated at walls divided into piers and spandrels (Fig.1 b), for which it was adopted a model with nonlinear beams characterized by a piecewise linear constitutive law (Fig.1 c) based on a phenomenological approach that allows to describe the nonlinear monotonous response associated with increasing levels of damage (ending at collapse), by assigning progressive strength drops β_{Ei} at predetermined drift levels δ_{Ei} [16]. The reliability of such modeling approach in analyzing the actual response of the buildings was already proven in [4].

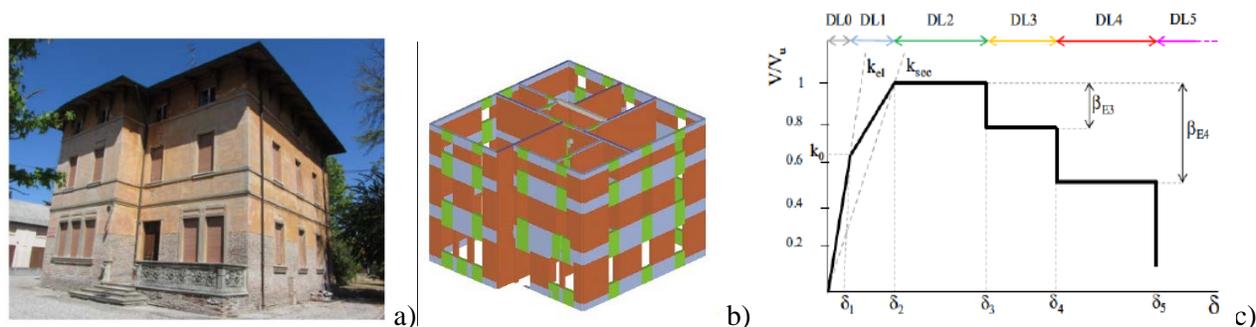


Fig. 1 – a) Exterior view of the case study; b) 3D view of the macro-element model of the structure; c) Piecewise linear constitutive law adopted to simulate the response of panels

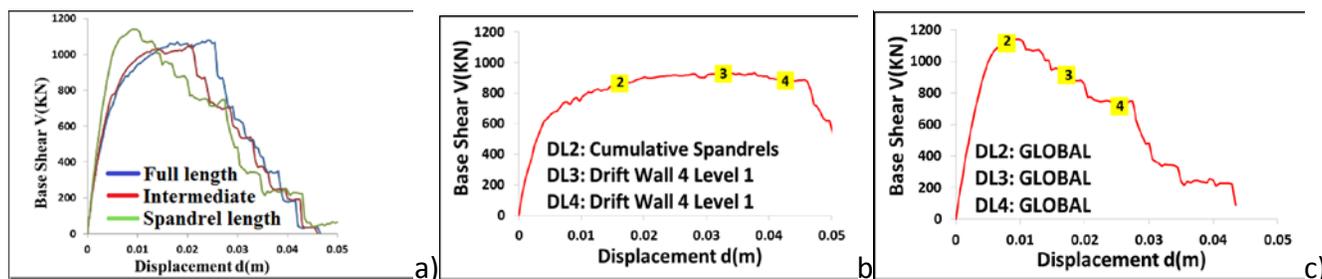


Fig.2 – a) Pushover curves to detect the effect of varying the length of the ring beams coupled with spandrels in *case-RC*; Positions of DLs on Pushover Curves in the X direction for b) *case-noRC* and c) *case-RC*.

Starting from the same geometrical configuration and assuming the same materials mechanical properties, a second case study (referred to as *case-RC*) has been analyzed with the presence of reinforced concrete ring beams coupled with spandrels on each floor. With this modification in structural details, the structure tends to move from a failure mode with damage concentrated at the level of spandrels (*case-noRC*), to a soft story behavior (*case-RC*), what will probably affect also the sensitivity of the mechanical parameters. Primary, some parametric analyses were performed with different effective length of the ring beams; it was assumed equal to the width of the openings, the distance between two adjacent nodes, or an intermediate length between the two. Although in reality the ring beam is continuous at the floor level, these possibilities correspond to different

hypotheses of the bond made between the wall and the reinforced concrete ring beam (in particular at the opening levels). After checking the structural response for the three cases (Fig.2 a), it was decided to deepen the first case, in order to examine two configurations with a stronger variation in the response.

4.2 Selection of the aleatory uncertainties

For both cases of study, ten aleatory variables (or group of variables) are defined:

- X_1 – mechanical properties of masonry: a group comprising the modulus of elasticity E , shear modulus G , average shear strength f_{vm0} , coefficient of equivalent friction μ , and compressive strength f_m . Further details on the strength criteria assumed to interpret the failure modes of panels are described in [4].
- X_2 – parameters that regulate the degradation of the initial elastic stiffness: the parameters k_0 and k_{in} . As indicated in Fig.1c, k_0 defines the value of the shear for which starts the degradation of stiffness, normalized to the ultimate shear while k_{in} the ratio between the initial and the secant stiffness.
- $X_3/X_4/X_5$ – stiffness of the intermediate floors, roof, and stairs, respectively, represented by the equivalent shear moduli G_{floor} , G_{roof} and G_{stairs} (using a slab of conventional thickness equal to 4cm). In fact, diaphragms are modeled as orthotropic membrane in Tremuri program.
- X_6 – piers: a group of parameters comprising the drift corresponding to the different levels of damage (Θ_{M3} , Θ_{M4} , Θ_{M5}) and the percentage of residual strength after collapse (β_{M3} , β_{M4}), different for the two possible failure modes considered: shear and rocking.
- X_7 – spandrels: it is a group of parameters comprising the drift corresponding to the different levels of damage (Θ_{F3} , Θ_{F4} , Θ_{F5}) and the percentage of the residual strength after collapse (β_{F3} , β_{F4}).
- $X_8/X_9/X_{10}$ – masses of intermediate floors, roof, and stairs, respectively: permanent and accidental loads (factorized) p_{floor} , p_{roof} , p_{stairs} .

As shown in Table 1, it is required to define a plausible range of variation for each parameter. The significant change associated with the floor stiffness reflects the uncertainty of mechanical properties and the quality of connection with the perimeter walls that may have an important influence on the structural response. Those of the mechanical parameters of masonry are defined from the proposed values in [17]. The ranges of variation of the parameters that regulate the stiffness degradation and the drift limit of the piers and spandrels, are calibrated using the data available from reference literature. The uncertainties of loads on diaphragms reflect those of the finishing and, for example, the thickness of the slab in case of intermediate floors and stairs.

Table 1 – Plausible ranges of variation for all the uncertain parameters assumed in both cases of study, represented with lower, upper, and median values

Aleatory parameters		X_{low}	X_{up}	X_{med}
X1	E [MPa]	600	1350	900
	G [MPa]	200	450	300
	f_{mvo} [MPa]	0.1	0.1875	0.137
	μ	0.333	0.5625	0.433
	f_m [MPa]	2.4	6	3.795
X2	$k_0 - k_{in}$	0.5 – 1.75	0.8 – 1.25	0.65 -1.5
X3/4/5	$G_{floor/roof/stairs}$ [MPa]	1250–100-1250	12500 – 1000 - 12500	3953 – 316 - 3953
X6	$\Theta_{M,T3}/\Theta_{M,T4}/\Theta_{M,T5}$	0.0023/0.0039/0.0056	0.0037/0.0061/0.0084	0.0029/0.0049/0.0069
	$\Theta_{M,PF3}/\Theta_{M,PF4}/\Theta_{M,PF5}$	0.0046/0.0078/0.01204	0.0074/0.0122/0.01796	0.0058/0.0098/0.0147
	$\beta_{M,T3}/\beta_{M,T4}/\beta_{M,PF4}$	0.6/0.25/0.8	0.8/0.55/0.9	0.7/0.4/0.85
X7	$\Theta_{F,T3}/\Theta_{F,T4}/\Theta_{F,T5}$	0.0015/0.0045/0.0151	0.0025/0.0075/0.0249	0.0019/0.0058/0.0194
	$\Theta_{F,PF3}/\Theta_{F,PF4}/\Theta_{F,PF5}$	0.0015/0.0045/0.0151	0.0025/0.0075/0.0249	0.0019/0.0058/0.0194
	$\beta_{F,T3}/\beta_{F,T4}/\beta_{F,PF4}$	0.3/0.3/0.3	0.7/0.7/0.7	0.5/0.5/0.5
X8/9/10	$P_{floor/roof/stairs}$ [kN/m ²]	0.805/0.8/0.805	1.196/1.2/1.196	0.981/0.98/0.981

4.3 Execution of the sensitivity analysis

For the execution of the sensitivity analyses, nonlinear static analyses are performed in X and Y directions, in the two senses, positive and negative, for both cases of study, with load patten distributed proportionally to masses. This distribution derives from the evidences collected from previous numerical simulations performed on this structure [4], even with a nonlinear dynamic approach, where it proved to be the most reliable in simulating the actual seismic response of the structure. Studying the two cases in both directions X and Y is beneficial, since analyzing a structure in two different directions is similar to analyze two different structures. For our cases of study, reference is made to the attainment of damage levels (DL) 2,3 and 4 assumed to correspond respectively to the Damage limitation, Life Safety, and Collapse limit states as provided in [5]. The position of the DL on the pushover curve is defined using a multiscale approach proposed in [18, 19], that combines controlling operators at three different scales: structural element, macroelement (walls), and global. Particularly, the controlling variables and thresholds assumed for each scale are: at element scale, the cumulative damage of the elements that have reached a predetermined DL (through the achievement of some drift limits), at macroelement scale, the interstory drift; and at global scale, the attainment of proper thresholds of the maximum base shear as defined by the pushover curve. The value of the PGA is calculated using the Capacity Spectrum Method, and an overdamped spectrum. For the conversion of the pushover into equivalent oscillators, it is referred to the principle proposed in [5] and [17], using the participation factor Γ and equivalent mass m^* .

From the analyses performed, we can notice that for case-*noRC*, it is obvious that the position of the first DL is controlled by the element scale and in particular the spandrels. While in the case-RC, and apart from the differences observed in terms of global strength and ductility, the dominant scale defining the LS is the global one (Fig. 2b and 2c). The purpose behind performing both the complete and the simplified sensitivity analyses is to investigate if the latter is capable to capture accurately the parameters that affect the most the response of the structure. So for both cases of study, it is performed $2^N = 1024$ and $2N+1 = 21$ nonlinear static analyses. Collected im_{LS} are integrated in Eq. (3) to generate the β_i values. Comparison of some results is shown in Fig.3.

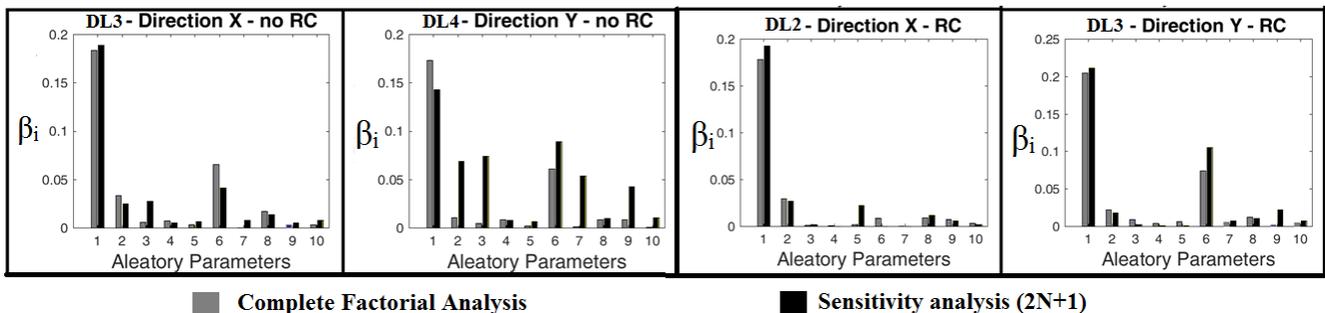


Fig.3 –Partial Dispersions obtained for the 10 aleatory variables considered

Taking the complete factorial analysis as reference, it is obvious that in most of cases the $2N+1$ analyses are capable to capture the parameters with highest sensitivities among the 10 aleatory variables. In some cases, the simplified analysis tends to overestimate (e.g. parameter 2 in DL4 – Direction Y – noRC) or to underestimate the uncertainty of certain parameters (e.g. parameter 6 in DL3 – Direction X – noRC). The latter is more serious since in this case, the simplified sensitivity is not capable to catch the degree of vulnerability of the structural response to this specific parameter. This means that no more investigations will be carried on this uncertainty, and the corresponding value adopted in the model will not be updated and it will remain affecting the response of the structure to a great degree. For the two examined cases of study, the majority of errors recorded by the $2N+1$ analyses are related to overestimation. High irregularities appear in case-*noRC* in the Y direction, where it is clear from the illustrated example in Fig.3, that the sensitivity analysis shows great differences with the factorial one, in terms of partial dispersions for some variables. It is important to notice that the parameters that are not captured in a good way are all overestimated, what means that the engineer will perform further investigations on these parameters while this is not needed. In terms of safety, these additional investigations are still useful, since they will increase the general knowledge of the structure, leading to more precise IM_{LS} but with a drawback presented in the additional cost and efforts needed in testing process that could be avoided.

4.4 Setting the investigation plan and rerunning the sensitivity analysis

Table 2 summarizes the investigation plan outlined on basis of results achieved by the sensitivity analyses. The high sensitivity of the mechanical properties of masonry (X_1) is common for both directions, so a high level of knowledge is asked for this parameter. This is assumed to be achieved through investigation techniques that provide direct mechanical parameters results (not necessarily invasive). The masonry characterizing this building is, in fact, homogeneous in all its areas, and for the brick and lime mortar used, it exists a lot of studies and data available in literature useful to limit the number of tests. The data obtained from the double flat jack is useful also to acquire some information about the propensity of the stiffness degradation of materials (useful for X_2), even if with a less degree of reliability (since the response at the wall scale affects also other factors, for example the prevailing collapse mechanism). In some cases (X_8), even if the sensitivity class was not so high, it is decided to reach a high level of knowledge because doing this is a bit onerous (in terms of cost and invasiveness). As for the response of the piers and spandrels (X_6 , X_7), that showed a high to medium sensitivity in both configurations, it is assumed to not be able to improve the knowledge because this needs the execution of highly invasive investigations. In the two cases of study, it was assumed that the investigation allows us to minimize the dispersion of the intervals of variation without changing the median values, so it is asked to run again only the analyses related to the parameters included in the investigation plan.

Table 2 – Investigation plan and KLs of the ten aleatory variables, proposed for the two cases of study

X	KL	Investigation technique
X_1 – materials	KLH	Double flat jack and characterization tests for individual constituents
X_2 – stiffness degradation	KLM	Double flat jack
$X_3/X_4/X_5$ –floor/roof/stairs stiffness	KLM	Visual inspections and local cores to check the quality of connections
X_6/X_7 – piers/spandrels	KLL	
$X_8/X_9/X_{10}$ –mass floor/roof/stair	KLH	Local cores to define the thickness of the slab and check the finishing with additional tests for roof to determine the section of resisting elements

4.5 Generation of reference IM_{LS} and β_{LS} using Monte Carlo simulations

The approach used for the generation of reference median IM_{LS} and dispersion β_{LS} is the construction of fragility curves through the application of a fully probabilistic procedure based on the use of Monte Carlo simulation. In particular, for each case, the approach adopted requires the attribution to each aleatory variable an appropriate distribution of probability and a relative parameter that characterize it with dispersion of the variables selected in a way to have a median value equal to the one of the intervals updated after the investigation and fractile limits of 16% and 84% corresponding to the lower and upper values, respectively. Lognormal distributions are assumed for the parameters that have values ranging between 0 and infinity ($X_{1,3,4,5,8,9,10}$) and beta distribution for those varying in [0 1] ($X_{6,7}$) or having, from a physical point of view, a limited range of variation equal to one (X_2). The generation of a sufficient number of models, from a statistical point of view (in this case 100 models in each case), is done using Monte Carlo technique, based on the above distributions of the 10 variables (or set of variables). It has been verified that the selected number of samples is sufficient to reach a good convergence in the estimation of parameters that define the fragility curves. The execution of the nonlinear static analysis leads to the construction of the fragility curves corresponding to each direction and each LS; in particular, they are obtained by taking the lower PGA between the two directions (positive and negative) and putting in an ascending order the values obtained in the 3 LSs. Out of these numerical fragilities, it is calculated a median value IM_{LS} and a dispersion β_{LS} , used to fit these fragility curves by lognormal ones. The results of the probabilistic approach, based on the use of Monte Carlo simulations, and those from the simplified sensitivity analysis are presented in Fig.4 and Fig.5, where it could be detected the reliability of this approach.

From Fig.5, it is clear how the sensitivity analysis based on 2N+1 analyses is able to result, in almost all cases, a value of IM_{LS} close enough to the reference one and at the same time, slightly lower, guarantying a safer state when adopting it. The bars related to the analysis performed using the median values of all the aleatory parameters (the +1 analysis), illustrate the fact that relying on a single analysis is not enough. It is true that in this analysis, according to the assumption made about the update of the aleatory variables after investigations, the median values remain the same, but at the same time, the dispersions of the intervals of variation of the updated parameters are lower in the probabilistic approach and diverting to the same median.

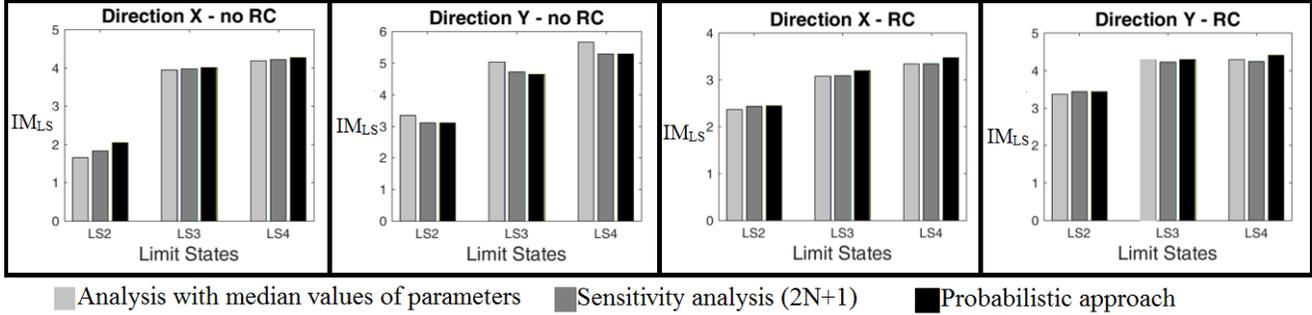


Fig.4 – Comparison of the IM_{LS} values of the fragility curves obtained from the different approaches examined.

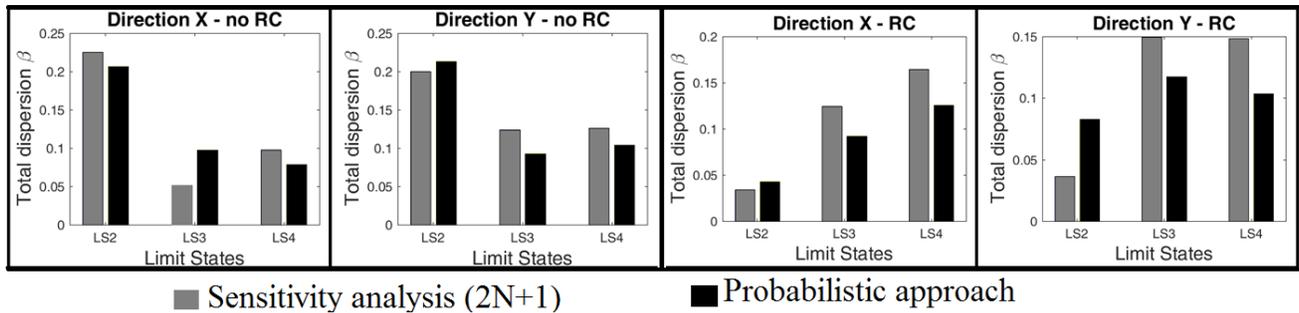


Fig.5 - Comparison between the dispersions β_{LS} of the fragility curves obtained by the probabilistic approach with those obtained by the Sensitivity analysis.

The results of β_{LS} are similar to those of IM_{LS} but with less accuracy. In some cases, the quantities collected by the simplified sensitivity analysis are close to the real values, but in others, the compatibility with the probabilistic approach is not so relevant. In the majority of the analyses and for all the LSs, the response of spandrels and piers showed high sensitivity for case-*noRC* and case-*RC* respectively, and since it was assumed that improving knowledge about these two parameters is impossible, the uncertainty they impose in the response of the structure remains the same. This affects the value of the total dispersion since it is calculated based on the partial dispersions (Eq. (4)) related directly to the diversity of the structural response captured by 2 analyses only (one with X_{low} and one with X_{up}). When following a full probabilistic approach, this problem is overcome.

The degree of safety provided by the two derived parameters, IM_{LS} and β_{LS} , is checked by the annual probability of occurrence p_{LS} of each LS calculated using Eq. (2) with k_0 and k assumed 0.05 and 2.5, respectively. Thus Fig.6 shows the degree of safety provided by following the procedure based on the sensitivity analysis. For the majority of cases the calculated IM_{LS} and β_{LS} result a value of annual probability of occurrence in a favor of safety compared to the one obtained by the probabilistic approach (ratios less than 1). Even though in the three cases the ratio is larger than 1, the maximum value (1.03) is close enough to 1 to be considered reliable. In general, those results are considered promising for future deepening and studies since the proposed method is capable to derive a good assumption of the fragility curve parameters. To make the concept of such probabilistic-based seismic assessment simpler to be used in engineering field, it is convenient, by rearranging Eq. (2), to express the results in terms of the value IM^* that assures the same probability of occurrence p_{LS} . This aim may be pursued by applying to IM_{LS} a confidence factor CF_1 as shown in Eq. (5), computed by β_{LS} :

$$IM^* = IM_{LS} e^{-\frac{1}{2}\beta_{LS}^2 k} = \frac{IM_{LS}}{CF_1} \quad (5)$$

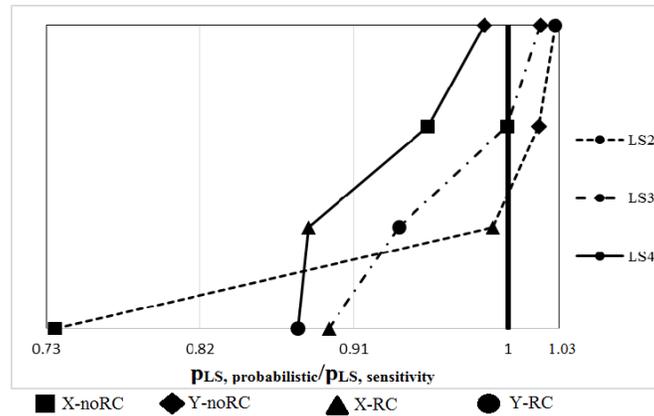


Fig.6 –Ratio between the failure probabilities calculated as a function of the different configurations examined on basis of the full probabilistic approach (based on the Monte Carlo sampling) and the sensitivity analysis

5 Conclusion

International codes propose for the seismic assessment of existing buildings, semi-probabilistic procedures for defining the capacity, using a CF determined based on a target KL assigned to the structure under assessment. This kind of procedures proved to be inaccurate and sometimes leading to unsafe results, what raises the need for procedures able to improve our capability in including the effect of uncertainty in the assessment.

Recognizing the highest versatility and accuracy of passing to a full probabilistic approach, in the paper the potential of the use of a limited number of analyses, firstly performed for the aim of a sensitivity analysis, is explored in order to compute the two basic parameters (IM_{LS} and β_{LS}) necessary to compute the fragility curve and the probability of occurrence of a given limit state. As introduced in §4.5, the computation of such parameter could be converted in a convenient format also for practice-oriented procedures as already proposed in a similar way in [3]. Moreover sensitivity analysis is the tool used to set an accurate and beneficial investigation plan, aiming to decrease the cost and invasiveness of the tests performed but at the same time increase the knowledge on parameters presenting high uncertainty in the response of the structure.

In the paper, a first application on two URM case studies is presented by executing such sensitivity analysis, with a limited number ($2N+1$) of nonlinear static analyses (N : number of uncertainties). A comparison with values obtained from fragility curves build with the execution of a large number of nonlinear static analyses on models generated using Monte Carlo simulations, proved that the use of such targeted but limited number of analyses is quite effective. Of course, further studies need to be carried out in order to increase the reliability of the such analysis results concerning the setting of investigation plan (defined by the partial dispersions β_i), and propose, at the same time, a clear procedure that the engineer can follow in assessing existing buildings.

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