



## DAMAGE DETECTION IN A STRUCTURAL SYSTEM VIA BLIND SOURCE SEPARATION

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### **Abstract**

A technique for vibration based system identification for detection of earthquake induced damage in structural systems is discussed. A numerical case study of detection of damage in a building frame subjected to a strong earthquake is presented for the application of a blind source separation algorithm to the problem of output only modal identification. The problem of base-excitation, as in earthquake loading, presents a situation wherein the governing equation of motion is usually formulated in terms of the deformations relative to the base motion whereas the response recordings are of absolute acceleration. The conditions under which the blind source separation formulation of representing observed system response as a mixture of independent sources can be adapted for the case of base excited systems are discussed. The modal parameters (mode shapes, natural frequencies,) are extracted from the vibration response and the changes in the identified modal parameters are tracked in time through a moving time-window analysis. The implications of the assumptions in the formulation and their validity in practical problems is also discussed.

*Keywords: Structural Health Monitoring (SHM); Blind Source Separation (BSS); Second Order Blind Identification (SOBI).*



## 1. Introduction

The state of health of civil infrastructure after an extreme loading event is a major concern for planning and scheduling maintenance. Several methods for structural health monitoring (SHM) have been proposed in the last few decades to detect and identify damage in a structural system. The objective is to provide an early indication of damage for facilitating preventive maintenance of the structural system. In recent years, vibration based monitoring techniques [1-2] are receiving attention because of their potential for online monitoring of the health of structural systems. These techniques are based on the premise that any change in physical properties (mass, damping and stiffness) of the system will cause predictable changes in modal properties of the system (natural frequencies, mode shapes and modal damping). These modal parameters can be ascertained from the vibration signatures of the structural system and the changes in identified modal parameters can be correlated with damage/deterioration in the structural system.

Recently, a powerful signal processing technique called blind source separation (BSS) [3-6] has been used for system identification in structural dynamics problems and appears suitable for monitoring the health of large-scale civil engineering structures [7-11]. In the BSS setting, a set of independent sources are separated from their mixture without any prior information about the sources and the mixing process. The BSS technique was first used by Zang *et al.* [7] for damage identification by using the independent component analysis (ICA) procedure in combination with artificial neural network (ANN). In 2005, Antoni demonstrated the difficulties associated with BSS techniques while dealing with dynamical systems involving convolutive type mixture pattern [8]. This problem was subsequently resolved by using the concept of virtual source, where the response of a mechanical system is interpreted as a static mixture of source [9-10] and the dynamic parameters such as natural frequencies and modal damping are estimated from the source vectors through some post processing and the mixing matrix is interpreted as the modal matrix. All previous BSS studies on structural dynamics problems are for the systems with direct loading [9-15] with the measured response directly corresponding to the variables in mathematical models. However for structures excited by earthquakes, the measured total acceleration response differs from the variables in the governing equations in terms of the motion relative to the ground. We present a formulation of the BSS problem tailored to the case of structures subjected to base-excitation. The modal parameters are estimated from the identified independent sources. Subsequently, damage identification techniques [16-17] are applied to detect damage in the structural system by comparing the modal parameters for the healthy state of the structure with those for the damaged state.

## 2. Blind Source Separation: Background

The BSS techniques were initially introduced in the context of neural network and have been shown to have applications in various fields including image processing, biomedical and telecommunications. In the past few years an increasing number of applications of the BSS to structural dynamics have been found. Since only output data are used for the analysis, this algorithm falls under the category of output only system identification. A simple BSS problem can be written as Eq. (1), where  $\mathbf{x}(t)$  is the vector of observed output,  $\mathbf{s}(t)$  is the vector of source signals, and  $\mathbf{A}$  is the mixing matrix.

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) \quad (1)$$

In the presence of measurement noise, this can be expressed as:

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

where,  $\mathbf{n}$  represents the measurement noise. In literature two main BSS techniques have been identified: Independent Component Analysis (ICA) [3-4, 6], and Second Order Blind Identification (SOBI) technique [5].

### 2.1 Independent Component Analysis (ICA)

ICA method is based on the assumption that the sources are statistically independent at each time instant. It determines the demixing matrix  $\mathbf{W}$ , such that the source signal  $\hat{\mathbf{s}}(t)$  is estimated by:



$$\hat{\mathbf{s}}(t) = \mathbf{W}\mathbf{x}(t) \quad (3)$$

where,  $\mathbf{W}$  is an approximation for the inverse of the mixing matrix  $\mathbf{A}$ , i.e.,  $\mathbf{W} \approx \mathbf{A}^{-1}$ . ICA is based on the principle of central limit theorem which states that the sum of independent random variables tends toward the more Gaussian distribution, i.e., a mixture of independent variables is more Gaussian than any one of the original variables. Using this theorem, independent source components are determined by maximizing the non-Gaussianity of  $\mathbf{W}\mathbf{x}(t)$  by using kurtosis or negentropy.

## 2.2 Second Order Blind Identification (SOBI)

The SOBI method is based on second order statistics and its objective is to take advantage of the temporal structure of the sources for facilitating their separation. Implementing SOBI is a three step process: first is whitening, which involves the linear transformation of the observed data such that the whitened data are uncorrelated with unit variance, followed by the orthogonalization, applied to diagonalize the time lagged covariance matrix of  $\mathbf{R}_x^w(\tau)$  whitened data. Third step is the unitary transformation in which whitened covariance matrix  $\mathbf{R}_x^w(\tau)$  is diagonalized by the unitary matrix  $\mathbf{U}$  related to the whitening matrix  $\mathbf{W}$  as  $\mathbf{U} = \mathbf{W}\mathbf{A}$ .

$$\begin{aligned} \mathbf{R}_x^w(\tau) &= \mathbf{W}\mathbf{E}[\mathbf{x}(t)\mathbf{x}^T(t+\tau)]\mathbf{W}^H \\ &= \mathbf{W}\mathbf{A}\mathbf{E}[\mathbf{s}(t)\mathbf{s}^T(t+\tau)]\mathbf{A}^T\mathbf{W}^T \\ &= \mathbf{U}\mathbf{R}_s(\tau)\mathbf{U}^T \end{aligned} \quad (4)$$

The requisite unitary matrix can be estimated from the eigenvalue decomposition of time lagged whitened covariance matrix and the mixing matrix can be estimated from the relationship between whitening matrix and the unitary matrix. The independent sources can be subsequently estimated from the measured data and pseudo-inverse of the mixing matrix.

## 2.3 Estimation of modal parameters from BSS:

Consider a multi degree of freedom linear damped mechanical system given by Eq. (5)

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \quad (5)$$

where,  $\mathbf{M}$ ,  $\mathbf{C}$ , and  $\mathbf{K}$  are the mass, damping, and stiffness matrices of the system respectively. The excitations that are applied on the structure are represented by the  $\mathbf{f}(t)$  and system responses is represented by  $\mathbf{x}(t)$ . The system response  $\mathbf{x}(t)$  can be written as a convolution between the impulse response function (IRF),  $\mathbf{h}(t)$  and the external force vector  $\mathbf{f}(t)$  as:

$$\mathbf{x}(t) = \mathbf{h}(t) \otimes \mathbf{f}(t) \quad (6)$$

where,  $\otimes$  denotes the convolution. While applying the BSS technique to dynamic systems, the system response  $\mathbf{x}(t)$  constitute convolutive mixture of independent sources  $\mathbf{f}(t)$  and its separation is non-trivial. Kerschen *et al.* [9] proposed the concept of virtual sources for representing the response of a structural system and is based on the modal expansion of system response:

$$\mathbf{x}(t) = \sum_{i=1}^n \boldsymbol{\varphi}_{(i)} q_{(i)}(t) = \boldsymbol{\Phi}\mathbf{q}(t) \quad (7)$$

where, the coefficients  $q_{(i)}(t)$  represents the normal coordinates,  $\boldsymbol{\varphi}_{(i)}$  measures the normal modes of structural system. Comparison of Eq. (1) with Eq. (7) suggests that the mixing matrix  $\mathbf{A}$  can be interpreted as modal matrix and the normal coordinates as virtual independent sources irrespective of the number and type of the physical excitation. Thus the mode shapes are directly provided by the columns of the mixing matrix but some post



processing is required for the identification of other modal parameters (natural frequencies and modal damping) from the sources.

### 3. Proposed Method: Structure Subjected to Base Excitation

For any structural system subjected to base excitation as shown in Fig.1, the equations of motion of the system are generally given in terms of the relative displacement between ground and structure whereas the accelerometers pickup the the total acceleration response. The equation of motion for the earthquake excitation case is given by:

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = -\mathbf{M}\mathbf{r}\ddot{\mathbf{x}}_g(t) \quad (8)$$

Where  $\ddot{\mathbf{x}}_g(t)$  represents the ground acceleration, and  $\mathbf{r}$  is rigid body coefficient vector.

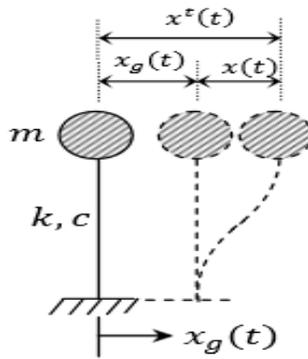


Fig. 1-Single degree freedom system subjected to ground excitation

To obtain the solution in terms of absolute response Eq. (8) can be reduced in the form of

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{M}\mathbf{r}\ddot{\mathbf{x}}_g(t) = -(\mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t)) \quad (9)$$

or,

$$\left(\ddot{\mathbf{x}}(t) + \mathbf{r}\ddot{\mathbf{x}}_g(t)\right) = -\mathbf{M}^{-1}(\mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t)) \quad (10)$$

Substituting structural relative response  $\mathbf{x}(t)$  from Eq. (7) to Eq. (10) we get the total response in terms of modal coordinates as:

$$\ddot{\mathbf{x}}^t(t) = -\mathbf{M}^{-1}(\mathbf{C}\boldsymbol{\phi}\dot{\mathbf{q}}(t) + \mathbf{K}\boldsymbol{\phi}\mathbf{q}(t)) \quad (11)$$

Generally, the damping force is very small in comparison to the restoring force in a structural system and the system response can be approximated as:

$$\ddot{\mathbf{x}}^t(t) \approx -\mathbf{M}^{-1}\mathbf{K}\boldsymbol{\phi}\mathbf{q}(t) \quad (12)$$

where,  $\mathbf{M}^{-1}\mathbf{K}$  can be written as  $\boldsymbol{\phi}\boldsymbol{\Lambda}\boldsymbol{\phi}^{-1}$  from standard eigenvalue problem [18,19]

Hence, Eq. (12) can be written as:

$$\ddot{\mathbf{x}}^t(t) \approx -\boldsymbol{\phi}\boldsymbol{\Lambda}\boldsymbol{\phi}^{-1}\boldsymbol{\phi}\mathbf{q}(t) \quad (13)$$

or,

$$\ddot{\mathbf{x}}^t(t) \approx -\boldsymbol{\phi}\boldsymbol{\Lambda}\mathbf{q}(t) \quad (14)$$



Now, if we compare Eq. (14) with the basic BSS problem i.e Eq. (1), we can easily interpret the modal parameters from the absolute response of the system i.e.  $\ddot{\mathbf{x}}^t(t)$ . The modal coordinates, weighted by the square of respective natural frequencies, constitute the virtual sources and contribute to amplify the higher mode response.

### 3.1 Numerical Study

To validate the above formulation, a numerical study has been carried for UCLA Doris and Louis Factor building (UCLA FB). UCLA FB is a seventeen storey special moment resisting (SMF) steel frame structure. There are 12 SMF bays in both the EW and NS directions of the building. The UCLA FB and its typical floor plan are shown in Figs. 2 and 3 respectively. Due to a slight overhang on the east and west sides of the building, the floor area from 10<sup>th</sup> to 16<sup>th</sup> floor increases by 13.5% approximately. The building is symmetric about the East-West axis (considered as X axis) and slightly asymmetric about the North-South axis (considered as Y axis). Except for some concrete caissons, the structure is supported by concrete spread footing. The building consists of 72 uniaxial forced balanced accelerometers with an onsite recording system. Four accelerometers exist at each floor above grade, oriented to record translational motions near the perimeter of the floor (two in each direction). Building has two basement levels, each of the two basement levels has an accelerometer to record translation in two directions, as well as two accelerometers to record vertical responses, due to this the building is one of the most densely permanently instrumented buildings in North America [20, 21].

In this study, the factor building responses data recorded during Parkfield, CA earthquake ( $M_w = 6.0$ ) of September 28<sup>th</sup>, 2004 are used for the analysis. System identification of UCLA FB has been carried out using second order blind identification technique (SOBI) [5]. Here the recorded response at each floor is the absolute response of the structure, so it has been used directly as output data. Mixing matrix and source vectors have been identified from the SOBI technique. Mixing matrix gives information about the mode shapes while natural frequencies have been identified from the fast fourier transformation (FFT) of sources data. The natural frequencies for first three modes in EW and NS direction obtained from the proposed method are compared with system identification (SI) results given by Skolnik [20] and modified cross correlation (MCC) technique given by Hazra *et al.* [12] for the same UCLA FB and details are given in Table 1 and Table 2. The Proposed method works quite well by considering the absolute response of the system as the output parameter for SOBI.

Table 1- Natural frequencies (in Hz) of the UCLA Factor building in EW direction

Mode No.	Proposed Method	SSI, Skolnik (2005)	MCC, Hazra <i>et al.</i> (2010)
1	0.46	0.47	0.48
2	1.50	1.49	1.55
3	2.68	2.68	2.69



Fig. 2- UCLA Factor Building [21]

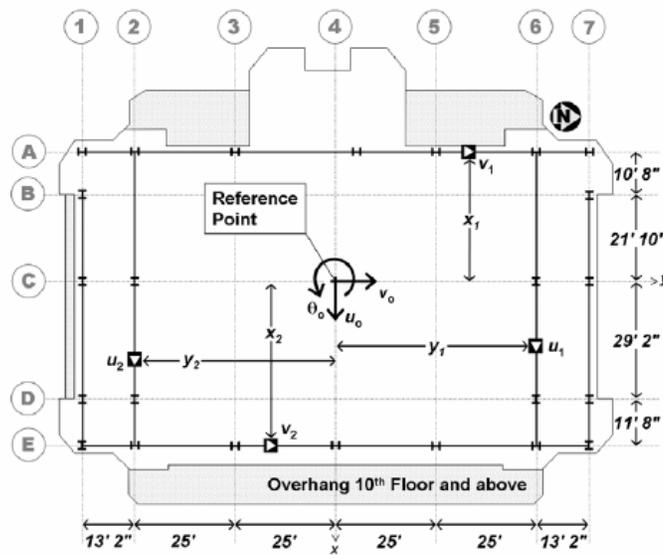


Fig. 3-Typical floor plan of UCLAFB [21]



Table 2-Natural frequencies (in Hz) of the UCLA Factor building in NS direction.

Mode No.	Proposed method	SSI, Skolnik (2005)	MCC, Hazra <i>et al.</i> (2010)
1	0.54	0.51	0.52
2	1.71	1.66	1.57
3	2.85	2.86	2.94

## 4. Damage Identification

After identifying the modal parameters next step is to detect the damage in the structural system due to strong earthquake. In this study, a numerical simulation has been carried out for damage identification by modelling the UCLAFB building in SAP 2000 [22]. A linear and nonlinear direct integration time history analysis have been carried out using Hilber–Hughes–Taylor (HHT- $\alpha$ ) method. The nonlinearity in the structure is modelled in terms of plastic hinges defined by FEMA 356 [23], which is an in-built function in SAP 2000. The ground motion recorded during Parkfield, CA earthquake (Mw = 6.0) is used for base excitation. The building is analysed for both the linear and nonlinear cases and the modal parameters are identified from the total acceleration response of the building using SOBI algorithm. When the building is analysed for nonlinear case, the formation of plastic hinges shows certain damage in the system. The damaged state can be identified by the measuring the changes in the modal parameters. In this study we consider the changes in the natural frequencies in moving time window for damage identification.

### 4.1 Damage detection in moving time window

Moving time window technique is used to detect changes in modal parameters due to damage sustained during the ground shaking. The changes in the natural frequencies for identifying damages are estimated using the short time fourier analysis of the identified sources. The short-time Fourier transform (STFT), is the tool with its ability to simultaneously unveil features of signal in time and frequency plane. STFT is the fourier transform (FT) of a short portion of signal  $\mathbf{s}_h(\tau)$  sampled by a moving time window  $\mathbf{h}(\tau - t)$  and represented by,

$$\mathbf{s}_t(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{s}_h(\tau) e^{-j\omega\tau} d\tau \quad (15)$$

where,  $\mathbf{s}_h(\tau)$  is defined as follows:

$$\mathbf{s}_h(\tau) = \mathbf{s}(\tau)\mathbf{h}(\tau - t) \quad (16)$$

The changes in the natural frequencies of the system can also be represented by obtaining the spectrogram of the signals. Spectrograms are the collection of all signals at different time windows and are expressed as the energy density  $\mathbf{P}(t, \omega)$  of the STFT.

$$\mathbf{P}(t, \omega) = |\mathbf{s}_t(\omega)|^2 \quad (17)$$

or,

$$\mathbf{P}(t, \omega) = \left| \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \mathbf{s}(\tau)\mathbf{h}(\tau - t) e^{-j\omega\tau} d\tau \right|^2 \quad (18)$$

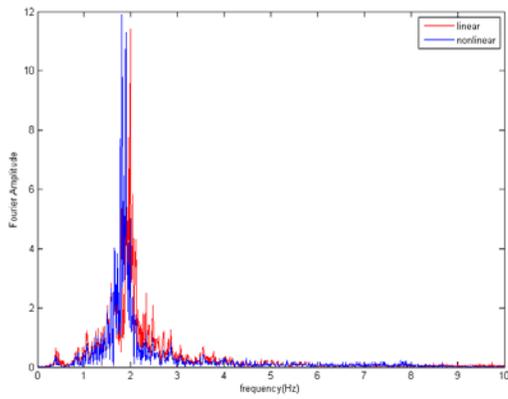
The damage scenario in the building can be seen through the spectrograms plotted for different sources (obtained via SOBI). While plotting spectrogram five second Hamming windows are considered with fifty percent overlapping. When we compare the spectrogram of linear and non linear sources (both in NS and EW



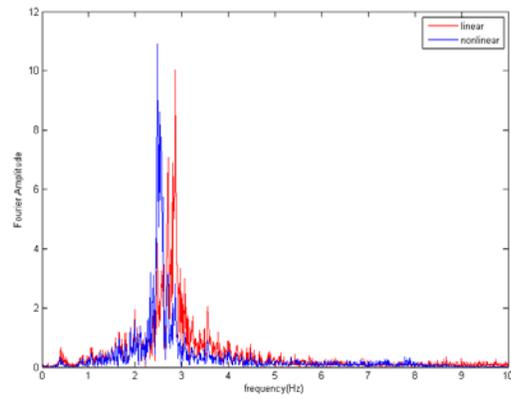
direction) as shown in Fig. 4 and Fig 5, significant frequency shift is observed. This shift is obtained as the system develops plastic hinges, and the shift in energy distribution becomes prominently visible in the spectrogram. The mode of vibration also plays very important role in damage identification. When we compare Fig. 4a with Fig 4b, and similarly Fig. 5a with Fig. 5b, a large frequency shift has been observed in frequency spectrum of fourth source in comparison to third source due to changes in the structural parameters. This shows the ability of the higher modes to capture the local effects like plastic hinges in beams.

## 5. Conclusions

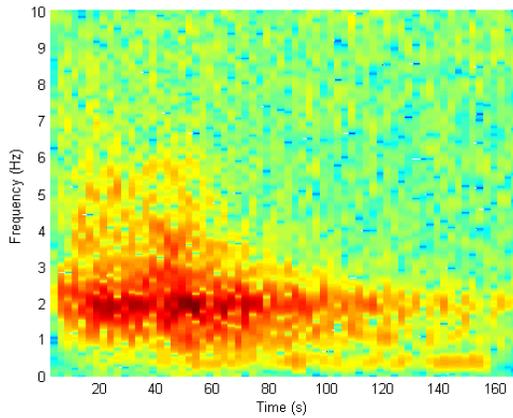
In this paper second order blind identification algorithm (SOBI) is proposed for identification of earthquake induced damage in structural systems. The general BSS problem is formulated for the case when structure is subjected to base excitation in terms of building absolute response rather than using relative response, which was done in earlier studies. This formulation is validated through a numerical simulation of UCLA factor building. The modal parameter (natural frequencies) are obtained from the absolute response of linear and nonlinear analysis of UCLA factor building and used for damage identification. Damage identification is carried out by tracking the changes in the natural frequencies of the building in moving time window analysis. The frequency shift is more prominent in the higher modes and hence it is important to use acceleration response data, which enhances the participation of higher modes, for identification.



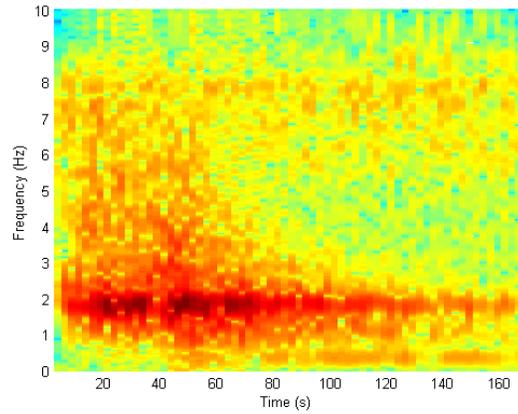
a. Fourier Spectrum of 3<sup>rd</sup> source



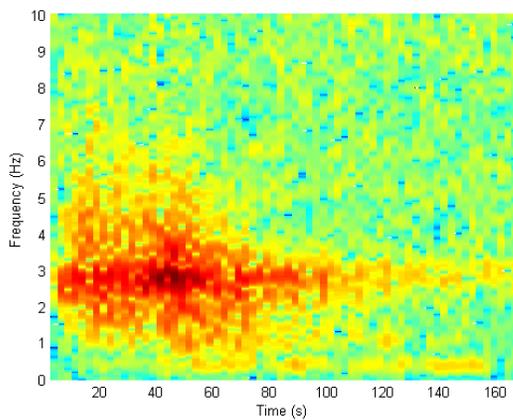
b. Fourier Spectrum of 4<sup>th</sup> source



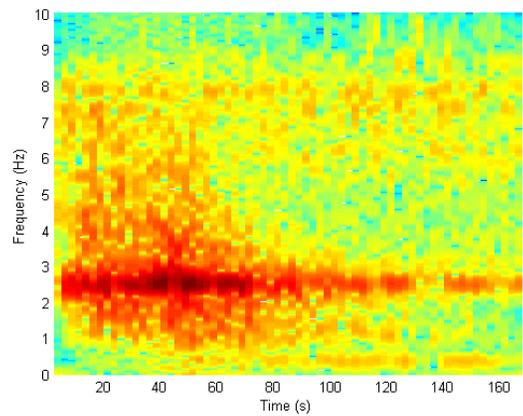
c. Spectrogram of 3<sup>rd</sup> source from linear response of UCLA building model



d. Spectrogram of 3<sup>rd</sup> source from nonlinear response of UCLA building model

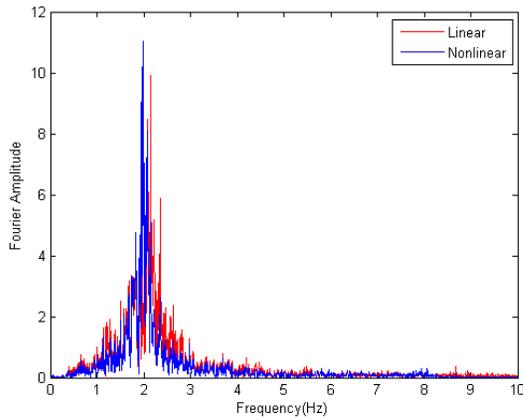


e. Spectrogram of 4<sup>th</sup> source from linear response of UCLA building model

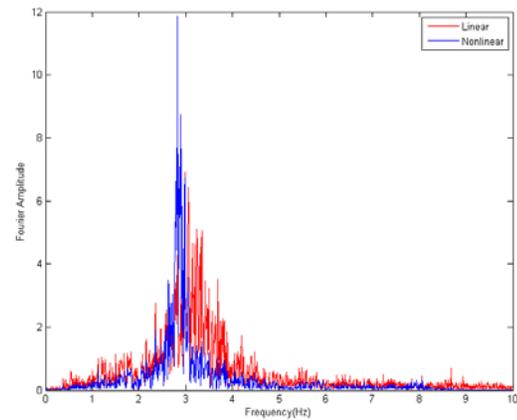


f. Spectrogram of 4<sup>th</sup> source from nonlinear response of UCLA building model

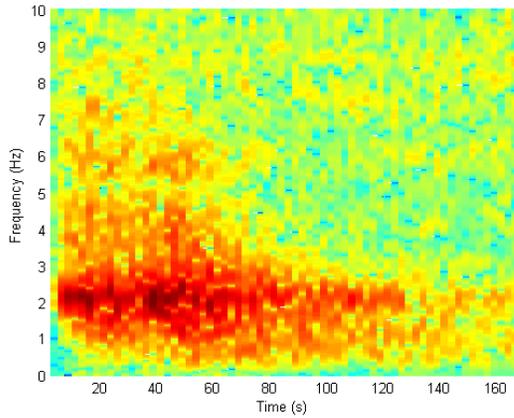
Fig. 4-Fourier spectrum and spectrogram of 3<sup>rd</sup> and 4<sup>th</sup> sources (NS ) direction



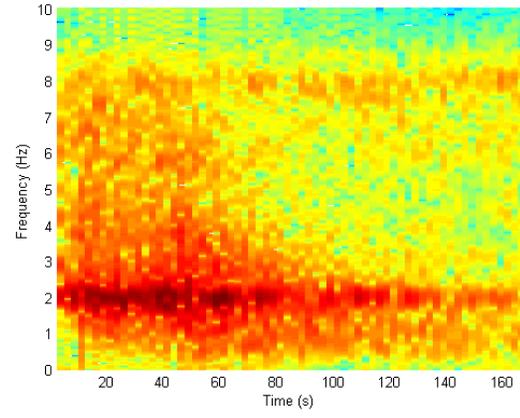
a. Fourier Spectrum of 3<sup>rd</sup> source



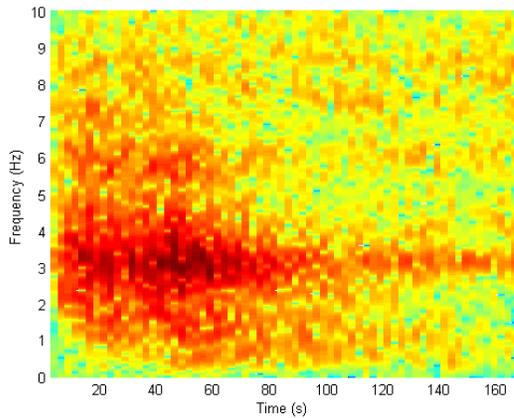
b. Fourier Spectrum of 4<sup>th</sup> source



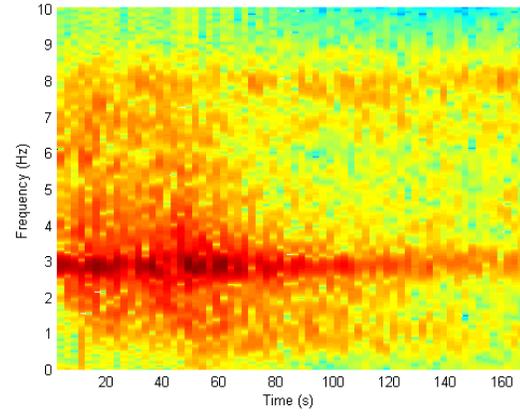
c. Spectrogram of 3<sup>rd</sup> source from linear response of UCLA building model



d. Spectrogram of 3<sup>rd</sup> source from nonlinear response of UCLA building model



e. Spectrogram of 4<sup>th</sup> source from linear response of UCLA building model



f. Spectrogram of 4<sup>th</sup> source from nonlinear response of UCLA building model

Fig. 5- Fourier spectrum and spectrogram of 3<sup>rd</sup> and 4<sup>th</sup> sources (EW direction)



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