16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 3682 (Abstract ID)

Registration Code: S- A1460627484

MODAL DECOMPOSITION AND BEHAVIOR OF FREE VIBRATION RESPONSE WITH GROUNDING AND UPLIFTING

Yoshihiro Mogi[(1)], Shinnosuke Okayama[(2)]

[(1)] *Engineer, Design Dept. of Taisei Corporation, e-mail: mg-ysh00@pub.taisei.co.jp*
[(2)] *Engineer, Design Dept. of Taisei Corporation, e-mail: okysnn00@pub.taisei.co.jp*
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Abstract

The effect of uplift on earthquake response of buildings had been investigated by nonlinear earthquake response analyses and shaking table tests of small-scale building models. As a result of these studies, it is found that uplift reduces story shear force that causes structural damage. If it is possible to level off earthquake loading acting on a building by allowing uplift, an epoch-making design system that is independent of input earthquake size or its periodic characteristics may be able to be established. However, it has been found that an uplifting building is vibrating randomly due to higher mode vibration. Therefore it is difficult to predict external load distribution and its size acting on the building. This study presents a modal analysis approach applied to vibrating structures in grounding and uplifting phases. By assuming the non-linear term can be treated as an external force, the modal analysis problem can be simplified and it becomes possible to carry out modal decomposition. Based on this assumption, modal decomposition on response of the uplift building could be carried out as "spring pendulum under gravity". The behavior of SDOF spring pendulum which represents a higher mode is very simple and clarified that vibration was simply caused by movement of an equilibrium position accompanying change of gravity value and direction.

Keywords: Uplift, Modal decomposition, Higher-mode vibration, Spring-pendulum, Impact

1. Introduction

Recent investigations showed that one of the significant problems to establish the aseismic design method with allowing uplift was influence of higher-mode vibration. In terms of physical static equilibrium of external load and structural internal stress, external load may be limited when a base overturning moment due to external load exceeds the resistance of self-weight overturning moment. However, according to dynamic analyses and shaking table tests of small-scale building models, it has been found that an uplifting building was vibrating randomly due to higher mode vibration. Moreover, some internal stresses were larger than ones derived from static analyses. Therefore, it is difficult to predict external static design load distribution and its size. One of the aims in this research is to find physical principle about mechanism of amplifying of higher-mode vibration.

Past related researches showed that uplift of foundation may have an advantageous effect on seismic performance of structures. This advantage was initially recognized by Housner(1963) [1]. He had investigated the damaged structures on 1960 Chile Earthquake and reported structures showed good performance against earthquake and also used a rectangular, rigid and free standing block to investigate the rocking behavior of structures. Then he expressed the uplift behavior as “Inverted Pendulum”. Later, shaking table tests and numerical analysis approaches on uplift behavior were studied in all over the world.

Authors think that the original theoretical approach about mechanism of higher-mode vibration was researched by Meek(1975) [2] using a simple single-mass model that has two degree of freedom. As other research, a shaking table test of small-scale 9-story steel frame took place in University of California(1978) [3]. Then Huckelbrige reported an interesting feature of response time histories include “spikes” caused by collision between the footing and the rigid base.

Based on the above knowledge, higher-mode vibration caused by uplifting and landing has been recognized. However, the general characteristics have not been explained yet. So it was one of the problems to be solved.
An early theoretical approach for uplifting structure was proposed by Ishihara et al (2008) [4].They researched higher-mode vibration from a different viewpoint. Within a limit of piecewise-linear response(from grounding condition to uplift and just before regrounding), Meek's method was applied to multi-story model based on modal decomposition and examined the dynamic characteristics. Then it was concluded that mode vectors changed by shifting to dynamic uplifting state from dynamic fix-supported state and then higher-modes are excited due to gravity.
Considering the above background, what is reported in this paper presents a new modal decomposition approach to apply a free vibration system of elastic structure with nonlinear rocking spring. Moreover, new physical interpretation about the mechanism of amplifying of higher-mode vibration was found.
As a fundamental problem, modal decomposition approach can be applyed to linear system only. However, an equation of motion system to continuously simulate grounding state and uplifting state inherently has nonlinear component. In order to deal with the above problem, by assuming that the non-linear component can be treated as an external force, internal force becomes linear system. Using this approximate vibration model, the modal analysis problem can be simplified and it becomes possible to carry out modal decomposition.

2. Simple model of uplifting structure and equation of motion

2.1 Rocking model

It is assumed that a building has a rigid foundation with a stiff girder and has tension-free supports at both edges of the foundation like a building on a hard ground or on a pile foundation which intentionally detaches from the foundation under tension condition (Fig.1). In order to simulate uplift behavior of this simple building, lumped-mass rocking model that has an elastic linear superstructure and an elastic bilinear rocking spring is considered. As shown in Fig.2, horizontal displacement u_i is defined not to include contribution of rocking displacement, θH_i.
M_{UL} is the limit of uplift resistance force and θ_{UL} is the rocking angle when the structure starts uplifting.

M_{UL} due to gravity is defined by the following equation:

$$M_{\mathrm{UL}} = m_{\mathrm{t}} \times g \times B / 2 \tag{2.1}$$

where m_{t} is the total mass of the system. g is gravitational acceleration and B is the width of foundation parallel to the excited direction. (Fig.1)

K_{R} is initial stiffness of rocking spring that represents soil stiffness.

A special feature of this simple model is that the rocking spring which has bilinear-elastic restoring hysteresis represents the resistance of continuous gravity effect during grounding and uplifting state.

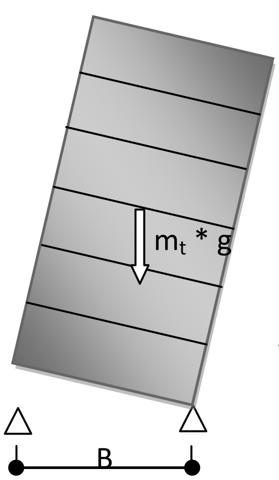


Fig. 1 – Uplift building

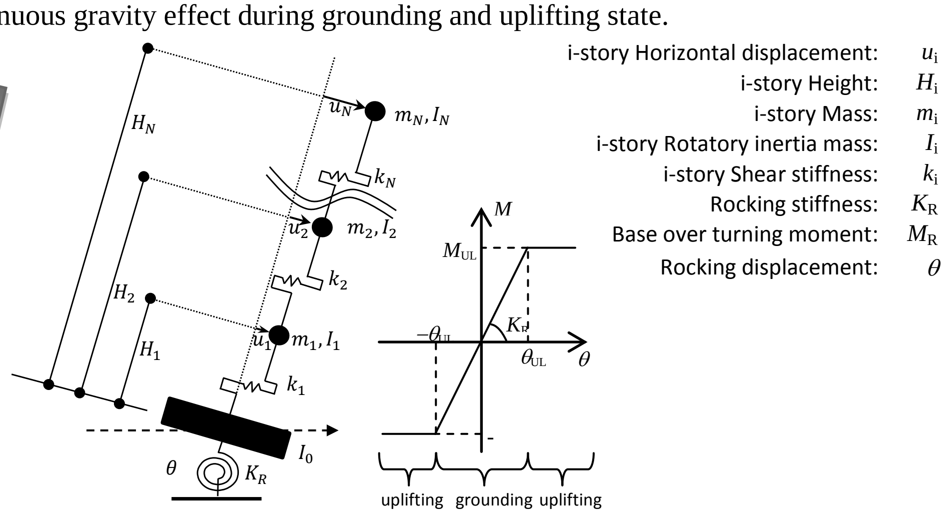


Fig. 2 – Rocking model(Lumped Mass Type)

2.2 Transposition of the nonlinear component and linearization of internal force

An equation of motion represeting the rocking model is:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} + \mathbf{f_{ex}} = \mathbf{0} \tag{2.2}$$

$$x = \begin{Bmatrix} u_N \\ \vdots \\ u_1 \\ \theta \end{Bmatrix} \text{ and } f_{ex} = \begin{Bmatrix} 0 \\ \vdots \\ 0 \\ M_R \end{Bmatrix} \tag{2.3}$$

$$M_{\mathrm{R}} = \begin{cases} K_{\mathrm{R}}\theta & |\theta| \le \theta_{\mathrm{UL}} \\ M_{\mathrm{UL}} & |\theta| \ge \theta_{\mathrm{UL}} \end{cases} \tag{2.4}$$

$$\mathbf{M} = \left[\begin{array}{ccc|c} m_N & & & m_N H_N \\ & \ddots & & \vdots \\ & & m_1 & m_1 H_1 \\ \hline sym & & & \sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^{\,2} \end{array}\right] \tag{2.5}$$

$$\mathbf{K} = \left[\begin{array}{ccc|c} k_N & -k_N & & 0 \\ -k_N & \ddots & -k_2 & \vdots \\ & -k_2 & k_2 + k_1 & 0 \\ \hline sym & & & 0 \end{array}\right] \tag{2.6}$$

For the sake of simplicity, damping is defined as stiffness-proportional type and no damping is applied to rocking component.

Nonlinear component f_{ex} is transposed from the left-hand side (stiffness matrix) to the right-hand side (external load):

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = -\mathbf{f_{ex}} \tag{2.7}$$

f_{ex} is defined as rocking reaction force, which is a nonlinear stiffness component and also can be expressed as a multiplication of mass matrix and gravity acceleration vector.

$$-\mathbf{f_{ex}} = \left\{\begin{array}{c} 0 \\ \vdots \\ 0 \\ \hline -M_{\mathrm{R}} \end{array}\right\} = \mathbf{M}\left\{\begin{array}{c} H_N \\ \vdots \\ H_1 \\ \hline -1 \end{array}\right\}\ddot{\theta}_{\mathrm{ex}} = \mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}} \qquad \ddot{\mathbf{x}}_{\mathbf{ex}} : \text{gravity acceleration vector} \tag{2.8}$$

$$\ddot{\theta}_{\mathrm{ex}} = M_{\mathrm{R}} \Big/ \sum_{i=0}^{N} I_i \ : \text{rocking gravity acceleration}$$

By substituting equation(2.8) for equation(2.7), the motion equation (2.9) which regards rocking external force term as pseudo-gravity can be obtained.

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}} \tag{2.9}$$

Equation(2.9) expresses both uplifting behavior and grounding behavior. The most impotant feature is that mass, stiffness and damping matrices are constant so that the left-hand side is a linear system.

Therefore it is possible to grasp the complex behavior of free vibration system with grounding and uplifting as the superposition of uplifting mode, whose rocking stiffness is zero, coherently during not only uplifting state but also grounding state. However, because there exists some rocking stiffness during ground state in reality, it is difficult to figure out the accurate behavior of grounding state if the rocking stiffness is small. The uplifting structure on the rigid ground is employed as a first approach so that grounding duration is almost zero.

3. Modal decomposition of uplift structure and equilibrium position

3.1 Modal decomposition of linear system which excludes nonlinear component

Response of non-damping or proportional damping system $\mathbf{x}$ is treated as the sum of responses of independent modal SDOF systems.

$$\mathbf{x} = \mathbf{\Phi y} \tag{2.10}$$

$\mathbf{\Phi}$:Mode matrix $\mathbf{y}$:Generalized mode displacement vector

By substituting equation(2.10) for equation(2.9),

$$\mathbf{M\Phi\ddot{y}} + \mathbf{C\Phi\dot{y}} + \mathbf{K\Phi y} = \mathbf{M\ddot{x}_{ex}} \tag{2.11}$$

By multiplying ${}^{\mathbf{t}}\mathbf{\Phi}$ and each term of the equation is shown below,

$$\mathbf{M}^*\mathbf{\ddot{y}} + \mathbf{C}^*\mathbf{\dot{y}} + \mathbf{K}^*\mathbf{y} = \mathbf{M}^*\boldsymbol{\eta}$$

$\mathbf{M}^* = {}^{\mathbf{t}}\mathbf{\Phi M \Phi}$: Generalized mass

$\mathbf{C}^* = {}^{\mathbf{t}}\mathbf{\Phi C \Phi}$: Generalized damping (2.12)

$\mathbf{K}^* = {}^{\mathbf{t}}\mathbf{\Phi K \Phi}$: Generalized stiffness

$\mathbf{M}^*\boldsymbol{\eta} = {}^{\mathbf{t}}\mathbf{\Phi M \ddot{x}_{ex}}$: Generalized gravity

Therefore, the motion equation of s^{th} mode can be determined using the following equations:

$${}_sM_s\ddot{y} + {}_sC_s\dot{y} + {}_sK_s y = {}_sM_s\eta \tag{2.13}$$

$${}_s\eta = \frac{{}_s^{\mathbf{t}}\boldsymbol{\varphi}\mathbf{M}\mathbf{\ddot{x}_{ex}}}{{}_sM} = \frac{{}_s^{\mathbf{t}}\boldsymbol{\varphi}\mathbf{M}\mathbf{\ddot{x}_{ex}}}{{}_s^{\mathbf{t}}\boldsymbol{\varphi}\mathbf{M}{}_{\mathbf{s}}\boldsymbol{\varphi}} \quad \text{: Generalized gravity acceleration} \tag{2.13a}$$

From the above consideration, each modal response is treated as SDOF spring pendulum system under pseudo gravity. (Fig.3)

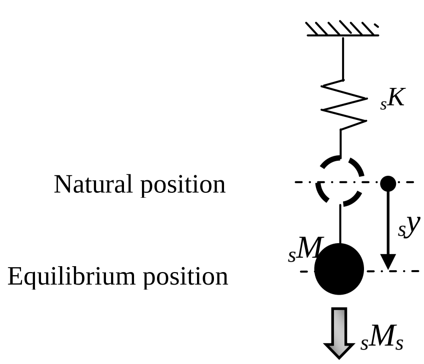


Fig. 3 – Spring pendulum system under pseudo gravity

3.2 Rigid rotation 1st mode

According to past related researches, 1st mode of uplifting structure is known as rigid rotation mode. Its natural frequency and lateral displacements u_i (Fig.2) are zero, but rocking displacement only is non-zero.

Eigenvalue problem of equation(2.11) is the following equation:

$$^{t}\mathbf{\Omega M \Phi} + \mathbf{K\Phi} = \mathbf{0} \tag{2.14}$$

$$\begin{gathered} {}_s\Omega \mathbf{M}_s\boldsymbol{\varphi} + \mathbf{K}_s\boldsymbol{\varphi} = \mathbf{0} \qquad s = 1,2,\cdots N+1 \\ \mathbf{\Omega} = {}^t\{{}_1\Omega \quad {}_2\Omega \quad \cdots \quad {}_{N+1}\Omega\} \quad : \text{eigenvalue vector} \end{gathered} \tag{2.15}$$

Obviously, rigid rotation mode vector of the rocking model is expressed as the following equation(2.16). By substituting equation(2.16) for equation(2.15), it is found that this assumption is correct.

$$\begin{aligned} &{}_1\Omega = 0 \\ &{}_1\varphi = {}^t\{0 \quad \cdots \quad 0 \mid {}_1\lambda\} \end{aligned} \qquad {}_1\lambda \text{ is arbitrary} \tag{2.16}$$

Generalized mass, stiffness, damping, gravity and its acceleration are expressed as the following equations.

$${}_1M = {}_1^t\boldsymbol{\varphi}\mathbf{M}_1\boldsymbol{\varphi} = \left(\sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^{\,2}\right){}_1\lambda^2 \tag{2.17}$$

$${}_1K = {}_1^t\boldsymbol{\varphi}\mathbf{K}_1\boldsymbol{\varphi} = 0 \tag{2.18}$$

$${}_1C = 2\,{}_1h\sqrt{{}_1M\,{}_1K} = 0 \tag{2.19}$$

$${}_1M_1\eta = {}_1^{\mathbf{t}}\boldsymbol{\varphi}\mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}} = -M_R \cdot {}_1\lambda \tag{2.20}$$

$${}_1\eta = \frac{{}_1^{\mathbf{t}}\boldsymbol{\varphi}\mathbf{M}\ddot{\mathbf{x}}_{\mathbf{ex}}}{{}_1M} = -\frac{M_R}{\left(\sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^{\,2}\right){}_1\lambda} \tag{2.21}$$

From the above equations (2.17) (2.18) (2.19) (2.20) (2.21), 1st mode does not have natural period and spring stiffenss.

Therefore it is constantly affected by generalized gravity only as shown in equation(2.20).

By substituting equation(2.18) (2.19) for equation(2.13), the motion equation of rigid rotaion 1st mode is expressed by the following equation:

$${}_1M_1\ddot{y} = {}_1M_1\eta \quad \Rightarrow \quad {}_1\ddot{y} = {}_1\eta = -\frac{M_R}{\left(\sum_{i=0}^{N} I_i + \sum_{i=1}^{N} m_i H_i^{\,2}\right){}_1\lambda} \tag{2.22}$$

During uplifting, M_R is constant value equal to M_{UL}, therefore 1st mode behavior is under the condtitions of uniform acceleration.

3.3 Equilibrium position and limited equilibrium position of higher-mode SDOF

In terms of higher-mode SDOF vibration, if each higher-mode is stationary or not vibrating at equilibrium position ${}_sy = {}_sy_g$ under generalized gravity ${}_sM\,{}_s\eta$, ${}_s\ddot{y} = {}_s\dot{y} = 0$is substituted for equation(2.13).

$${}_sK\,{}_sy_g = {}_sM\,{}_s\eta \quad \Rightarrow \quad {}_sy_g = \frac{{}_sM\,{}_s\eta}{{}_sK} \tag{2.23}$$

From equation(2.23), it is found that equilibrium position ${}_sy$ is proportional to generalized gravity value ${}_sM\,{}_s\eta$.

In contrast to higher mode, 1st mode does not have equilibrium position because ${}_1K$ is zero.

During uplifting, generalized gravity ${}_sM\ {}_s\eta$ becomes constant, therefore equilibrium position ${}_sy$ also becomes constant. These constant values are defined as limited generalized gravity ${}_sM\ {}_s\eta_{\mathrm{UL}}$ and limited equilibrium position ${}_sy_{\mathrm{UL}}$, respectively.

$$ {}_sK\ {}_sy_{\mathrm{gUL}} = {}_sM\ {}_s\eta_{\mathrm{UL}} \quad \Rightarrow \quad {}_sy_{\mathrm{gUL}} = \frac{{}_sM\ {}_s\eta_{\mathrm{UL}}}{{}_sK} \tag{2.24} $$

4. Mechanism of higher-mode excitation

As shown in equation(2.23), equilibrium position is proportional to generalized gravity value. Considering the character of equilibrium position as shown in chapter.3 and rocking restoring force characteristic as shown in Fig.2, the following facts can be understood.

4.1 higher-mode excitation in a detach phenomenon (1^{st} uplifting)

Uplift behavior diagram of a higher-mode spring pendulum system on the rigid ground is shown in Fig.4. First uplifting is a change of the state from grounding to uplifting. When the structure stands on the ground vertically, rocking restoring force by gravity is zero and equilibrium position of the higher-mode spring pendulum is at the natural position. By contrast, when the structure is uplifted to the right direction, generalized gravity ${}_sM\ {}_s\eta_{\mathrm{UL}}$ works vertically down on the higher-mode pendulums (in case of Fig.4). Then equilibrium position is also moved to the same direction. Equilibrium position is central axis of free vibration for spring pendulum system. Therefore, if it is assumed that the structure is on the rigid ground and equilibrium position is moved instantly to the limited equilibrium position, the spring pendulum behaviors as though it has amplitude of ${}_sy_{\mathrm{gUL}}$ at that moment. After then, the spring pendulum goes to the same direction, and reaches the point which has maximum distance from natural position, ${}_sy_{\mathrm{gUL}}+{}_sy_{\mathrm{gUL}}=2{}_sy_{\mathrm{gUL}}$.

4.2 higher-mode excitation in a landing-detach phenomenon (2^{nd} or more times of uplifting)

When the vibration which has occured with uplifting is sufficiently converged by damping, the spring pendulum is stationary at the limited equilibrium position ${}_sy_{\mathrm{gUL}}$. When the uplifting structure reaches to ground and starts to the next uplifting, generalized gravity ${}_sM\ {}_s\eta_{\mathrm{UL}}$ works vertically up (in case of Fig.4). Then equilibrium position also moves to the same direction. Now, the spring pendulum behaviors as though it has amplitude of $2{}_sy_{\mathrm{gUL}}$. After then, the pendulum goes to the same direction and reaches the point which has maximum distance from natural position, ${}_sy_{\mathrm{gUL}}+2{}_sy_{\mathrm{gUL}} =3{}_sy_{\mathrm{gUL}}$.

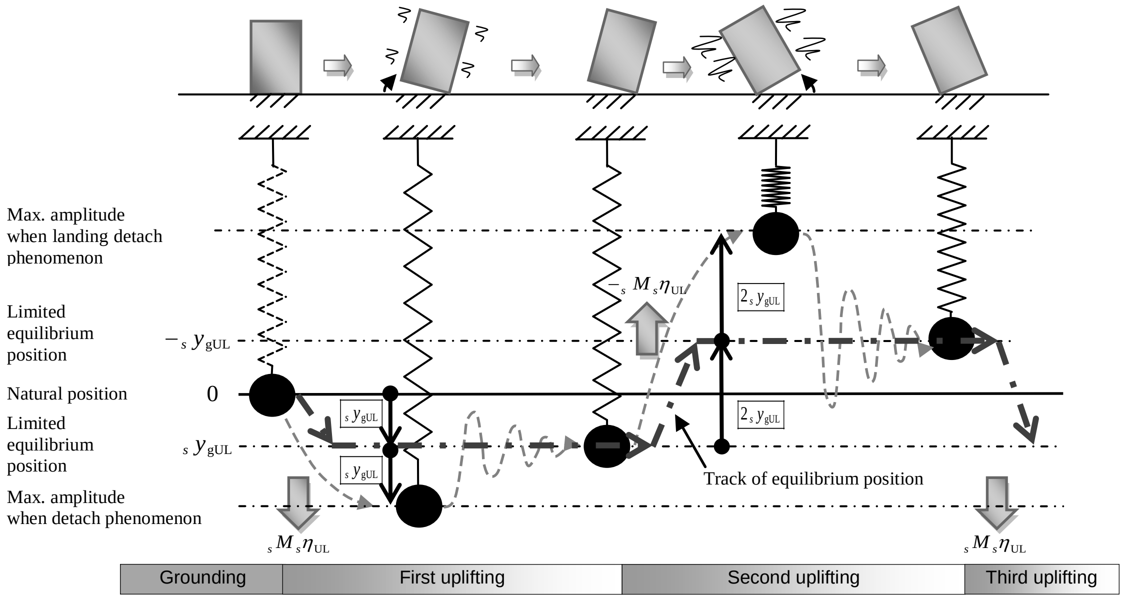
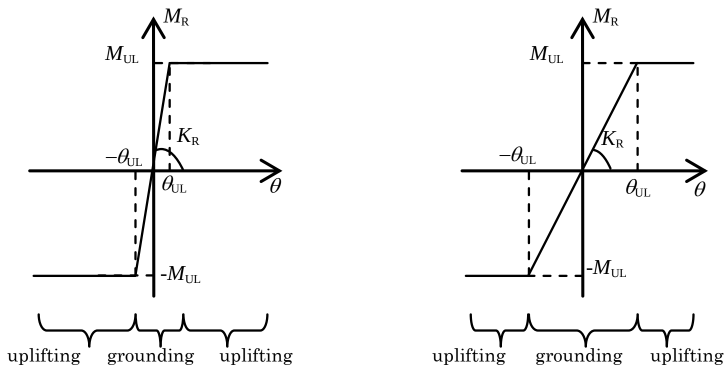


Fig. 4 – Mechanism of generation of higher mode vibration

4.3 determinant of vibration amplitude of spring pendulum

From the above consideration, in case of on the rigid ground as shown in Fig.5(a), equilibrium position ${}_{s}y_{g}$ instantly moves to another oppsite equilibrium position.On the other hand, in case of on the softer ground as shown in Fig.5(b), it takes time for equilibrium position ${}_{s}y_{g}$ to move to another oppsite equilibrium position. Therefore, vibration amplitude is reduced.

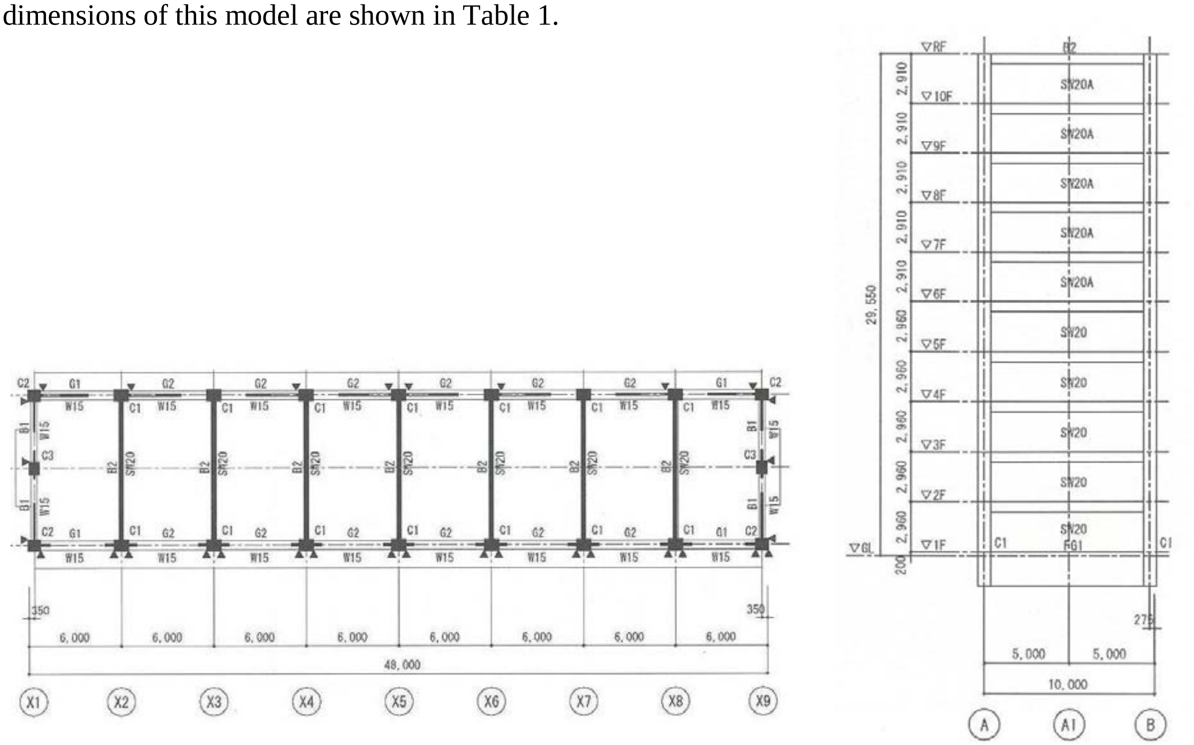


(a)Large K_R (b)Small K_R

Fig. 5 – Effect with difference of K_R

5. Numerical verification

To verify the validity of this theory, eigenvalue analysis and free vibration analysis with initial velocity using 10-storey lumped-mass model with rocking spring are carried out. The plan and elevation are shown in Fig.6 and the dimensions of this model are shown in Table 1.



(a) Plan (b) Elevation

Fig. 6 – Model Plan and Elevation

Table 1 – 10-stoey lumped-mass model

i	ΣH_i (m)	m_i (t)	I_i (t・m^2)	k_i (kN/m)	K_R(kNm/rad)
10	29.35	800	18930	2,504,005	―
9	26.44	789	18690	3,836,492	―
8	23.53	789	18690	4,979,611	―
7	20.62	789	18690	6,051,187	―
6	17.71	789	18690	7,189,547	―
5	14.8	797	18890	8,309,677	―
4	11.84	799	18940	9,758,189	―
3	8.88	799	18940	11,892,545	―
2	5.92	801	19000	14,337,205	―
1	2.96	804	19070	17,891,002	―
0	0	1240	29410	―	486,400,000

Table 2 – Specifications of each mode

	1th	2th	3th	4th
Period sT	–	0.1549	0.1088	0.0928
Generalized mass sM	2885857	1166	377	417
Generalized stiffness sK	0	1918271	1257861	1912446
Natural circular frequency $s\omega$	0	40.6	57.8	67.7
Participation factor $s\beta$	0.045	−0.754	0.546	1.292
Equivalent mass $s\overline{M}$	5844	663	112	696
Equivalent mass ratio	73.0%	8.3%	1.4%	8.7%
Generalized gravity acceleration $s\eta_{UL}$	−0.16	3.79	39.32	24.68
Generalized gravity force $sMs\eta_{UL}$	−450900	4417	14827	10292
Equilibrium position sy	–	0.0023	0.0118	0.0054
Static potential energy sWes_blc	–	5.1	87.4	27.7
Static potential energy ratio	–	4.2%	72.4%	23.0%

5.1 Eigenvalue Analysis

The results of eigenvalue analysis are shown in Table2. sM,sK,$s\eta_{UL}$ are calculated by mode vectors which are generalized as the maximum value of the component is 1. sWes_g$_{UL}$ is static potential energy on the limited equilibrium position. If this value is large, that mode generates large vibration when uplifting. In case of this model, 3rd mode has the largest part of total sWes_g$_{UL}$.

The graphs of eigenvalue analysis are shown in Fig.7,8,9 and 10. Each of (a) to (d) are from 1st to 4th mode and (e) is the total vector of all modes. In Fig.8,9 and 10, the rigid line shows the total displacement, which is the sum of base rocking vector and deformation of structure. On the other hand, the dashed line shows base rocking vector only. So the difference vector of these two lines means the deformation of structure.

Participation vector, which is modal decomposition of ground acceleration vector, is shown in Fig.7. On the other hand, Modal gravity acceleration vector, which is modal decomposition of gravity acceleration vector during uplifting, or equation(2.8), is shown in Fig.8. In Fig.7, 1st mode has the largest contribution. But in Fig.8, higher mode (3rd mode in this case) has the largest contribution. Participation of each mode varies according to assumed force vector. This fact is also pointed out in Ref [4] and is important to understand behavior of the uplifting structure. When the structure is not uplifting, participation vector (Fig.7) and effective mass calculated by the vector are the important indexes of modal analysis. On the other hand, when the structure is uplifting, modal gravity acceleration vector (Fig.8) is also an important index of modal analysis.

Modal displacement vector when the mode is on the limited equilibrium position is shown in Fig.9. This is the situation when the structure is in equilibrium with Generalized Mass sM (equation (2.24)). When the structure is in this situation, the modal spring pendulum is on the “Limited equilibrium position” in Fig.4. By contrast, because 1st mode has no stiffness, it is not in this situation or on the position.

Shear coefficient of each mode when the mode has the displacement in Fig.9, is shown in Fig.10. This is the important basic value when assuming shear response by higher mode vibration during uplifting. More explanation about this is given in the next section, 5.2.

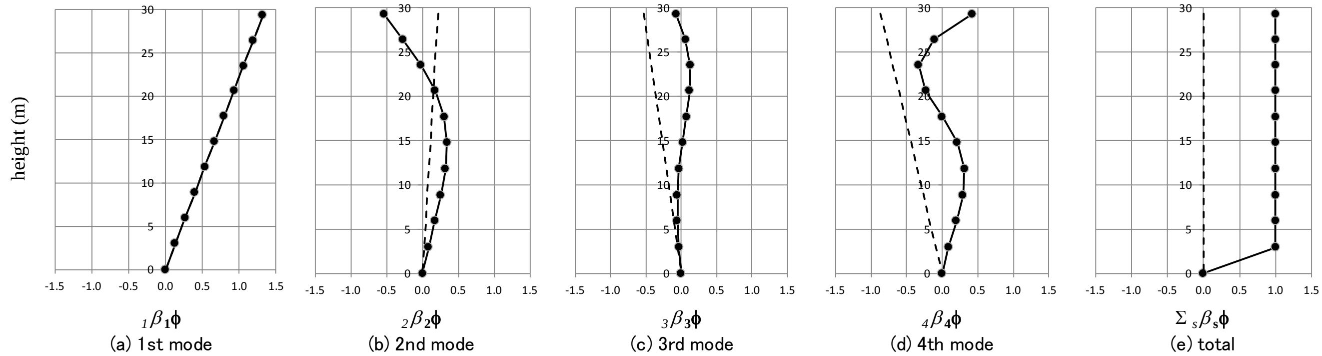


Fig. 7 – Participation vector (uniform horizontal unit acceleration at every story)

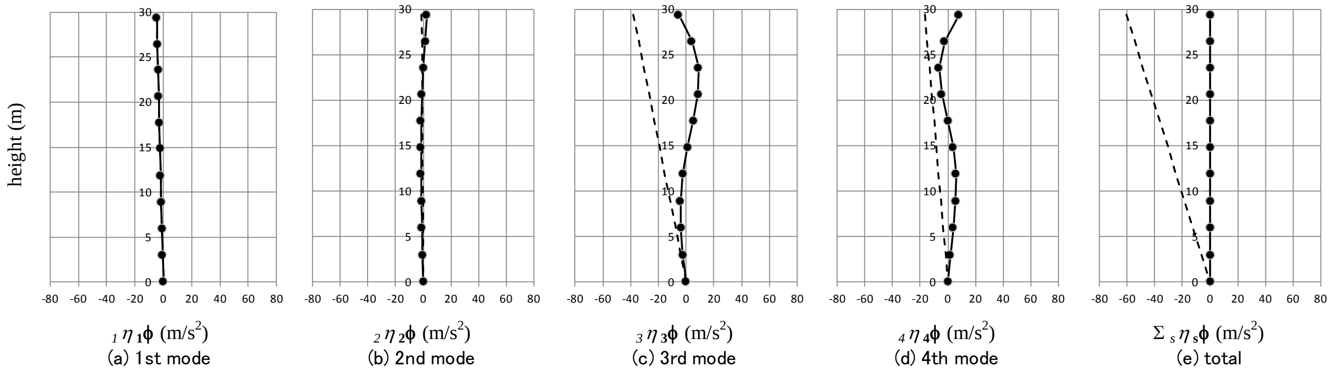


Fig. 8 – Modal gravity acceleration vector

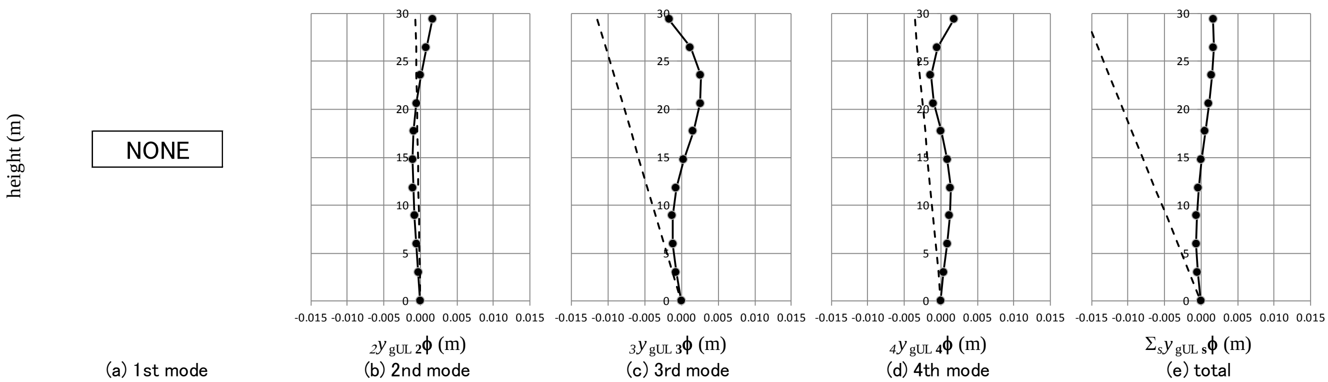


Fig. 9 – Modal displacement vector (at limited equilibrium position)

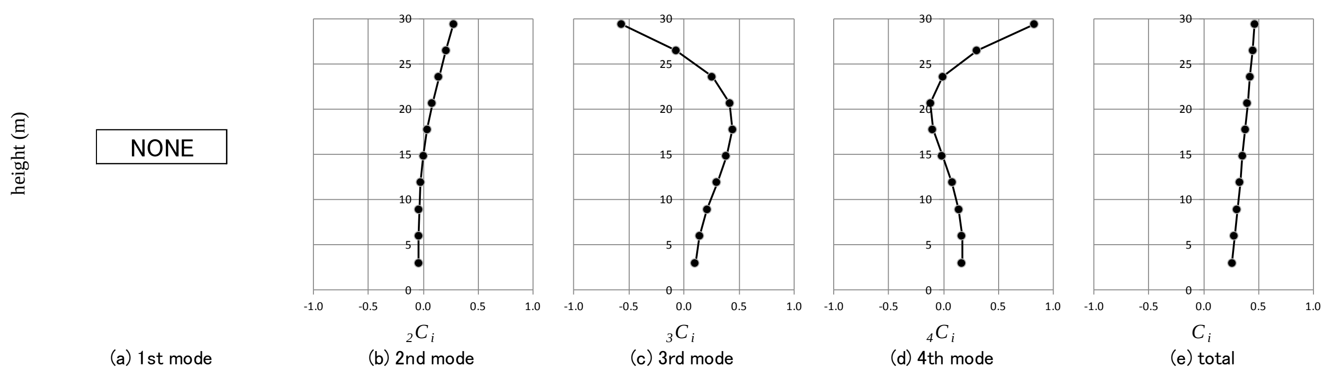


Fig. 10 – Modal shear coefficient (at limited equilibrium position)

5.2 Free Vibration Analysis

Free vibration analyses with initial velocity are carried out. The conditions of analyses are shown below.

i) Newmark-β method (β = 1/4), dt=1/1000 sec

ii) The initial velocities of superstructure except rocking $\dot{u}_i = 0$

iii) The initial rocking displacement $\theta_0 = 0$ and initial rocking velocity $\dot{\theta}_0 = 14094\ \mathrm{kN \cdot m}$

iv) The rocking stiffness of ground is 100 times K_R in Table1 in the case of hard ground, Fig.11(a).

v) The rocking stiffness of ground is 0.5 times K_R in Table1 in the case of soft ground, Fig.11(b).

The results of these analyses and the modal decompositions of those are shown in Fig.11.

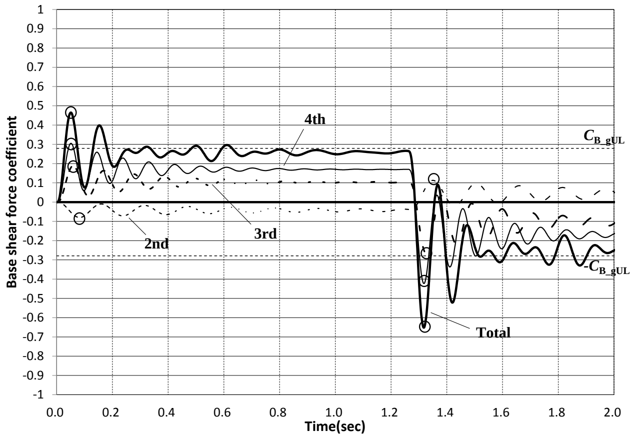
The dashed line, $C_{\mathrm{B\_gUL}} = 0.28$ means total base shear coefficient when all modes are at limited equilibrium position. This value is the sum of all modal shear coefficients ${}_sC_{\mathrm{B\_gUL}}$, at limited equilibrium position. Reffering to Fig.10, the coefficients of 2nd, 3rd and 4th mode ${}_2C_{\mathrm{B\_gUL}}$, ${}_3C_{\mathrm{B\_gUL}}$ and ${}_4C_{\mathrm{B\_gUL}}$ are -0.04, 0.10 and 0.17, respectively. In the both cases of Fig.11(a) and (b), the each center of vibration of each higher mode and the sum of all modes is identical (-0.04, 0.10, 0.17 and 0.28, respectively). As just shown, the center of higher modal and total vibrations are always each limited equilibrium positions respectively regardless of ground stiffness. On the other hand, 1st mode is rigid rocking mode which has no stiffness and it does not support any shear force.

Firstly, focus on the first uplifting - around 0 sec - of Fig.11(a), in the case of rigid ground. The maximum response of base shear coefficient in 2nd, 3rd and 4th modes are -0.08, 0.20 and 0.34, respectively. These values are almost twice as much as ${}_sC_{\mathrm{B\_gUL}}$ as described in section 4.1. But the maximum response of total base shear coefficient is not twice as much as $C_{\mathrm{B\_gUL}}$ because each mode does not reach the maximum response at the same time.

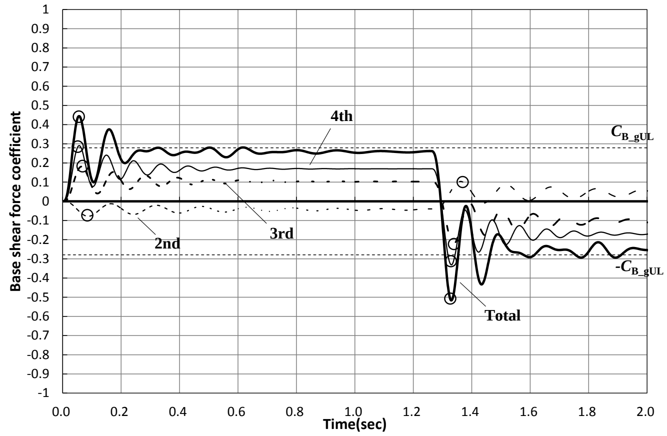
Then, focus on the second uplifting - around 1.3 sec – in Fig.11(a). The responses of 2nd, 3rd and 4th mode are 0.11, -0.28 and -0.41, respectively. These are approximately three times as much as ${}_sC_{\mathrm{B\_gUL}}$ as described in section 4.2. However, it is a little smaller than exact three times because of the damping of superstructure.

On the other hand, the case of soft ground is shown in Fig.11(b). If the ground stiffness is smaller, the maximum response of higher mode is smaller. If the ground is hard, the time from landing to next uplifting is short and equilibrium position moves instantly. By contrast, if the ground is soft, the time from landing to next uplifting is relatively longer and equilibrium position moves slowly. This is the reason why the two sets of base shear coefficients vary according to ground stiffness. It is easy to imagine that the vibration of the spring pendulum becomes small if the gravity force works slowly.

As explained in above, under the ideal condition, the maximum response of higher mode vibration is twice as much as limited equilibrium position at first uplifting and is three times as much as limited equilibrium position at second uplifting. But in Fig.11(a), the response of total modes is a little smaller than that because of the damping of superstructure. Furthermore, in Fig.11(b), the response is more smaller because of the ground softness.



(a) Hard Ground (h=1%)



(b) Soft Ground (h=1%)

Fig. 11 – Time history of modal base shear coefficient

6. Conclusions

In this paper, in order to investigate the dynamic behavior with grounding and uplifting, the approximated modal analysis method, which assumes that the non-linear component is treated as an external force, is applied to nonlinear problem. From the results, the conclusions are summarized as follows:

(1) Higher-mode vibration of a structure with uplift is amplified by movement of equilibrium position due to change of pseudo gravity value or direction in the spring pendulum system of each mode.

(2) The traveling distance between two limited equilibrium positions, during Landing-Detach Phenomenon, is twice the distance during Detach Phenomenon. Hence, the amplitude of each mode is also doubled if it is initially stationary at limited equilibrium position with no damping on perfectly rigid ground.

(3) As for the response of each mode during uplift under the same conditions of (2), the response during Detach Phenomenon is twice as large as the absolute value of limited equilibrium position, and the response during Landing-Detach Phenomenon is three times the absolute value of limited equilibrium position.

(4) Movement of equilibrium position will change gently and the occurrence of vibration is mitigated if soil stiffness becomes soft or the structure has more damping.

7. References

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8. The definitions of some technical words

In this paper, some technical words are used as below definitions.

Uplifting: the state that one side of foundation corner are untouched with ground.

Grounding: the state that both sides of foundation corner are touched with ground.

Landing: the moment when the structure changes from uplifting state to grounding state

Detachment: the moment when the structure changes from grounding state to uplifting state

Detach phenomenon: the phenomenon from grounding to uplifting over the detachment

Landing-detach phenomenon: the phenomenon from uplifting to opposite side of uplifting over the landing, grounding and detachment

Grounding mode: the mode of grounding structure whose rocking stiffness is positive value

Uplifting mode: the mode of uplifting structure whose rocking stiffness is zero

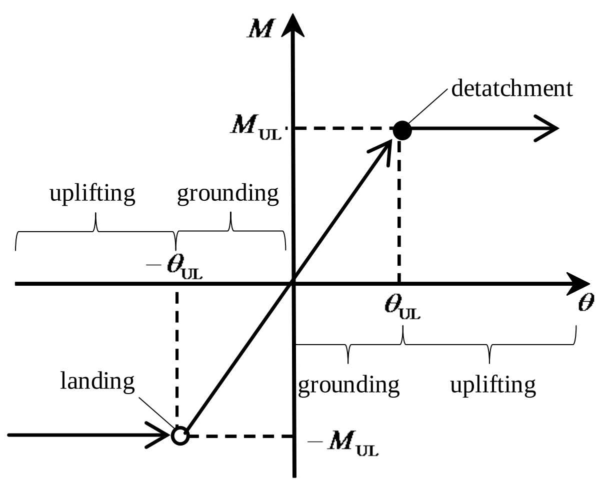

Fig. 12 – The definitions of uplifting, grounding, landing and detachment