ESTIMATION OF HORIZONTAL SEISMIC BEDROCK MOTION FROM VERTICAL SURFACE MOTION BASED ON HORIZONTAL-TO-VERTICAL SPECTRAL RATIOS OF EARTHQUAKE MOTIONS

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Abstract

Estimating bedrock motion is a popular method to understand the amplification characteristics of the observation site and investigate strong ground motion characteristics without any site effects. We calculated average Horizontal-to-Vertical spectral ratios of weak motions and identify the subsurface structure as a ratio of transfer functions of horizontal and vertical components based on the diffuse field theory. Then, we attempted to estimate the horizontal seismic bedrock motions of weak motions and strong motions using the proposed method and compared with results from the equivalent linear analysis for non-linear soil response. For the weak motions, the bedrock spectra and the bedrock motions estimated by both analyses were consistent. The estimated bedrock spectra of strong motions were similar, but the bedrock waveform estimated by the equivalent linear analysis at MYG006 was larger than the one estimated by the proposed method. The cause of this larger amplitude was the excessive amplitude reduction of the transfer function that is used to divide the surface spectrum in order to get the bedrock spectrum. Except for that case, the results of the proposed method basically corresponds to the previous methods, and so we have shown here a new possibility of the application of the diffuse field theory.

Keywords: Horizontal-to-Vertical spectral ratio, seismic bedrock motion, diffuse field theory
1. Introduction

Estimating a seismic bedrock motion, which is supposedly coming directly from the source to the interface of the bedrock, is a popular method to understand the amplification characteristics of the site and strong ground motion characteristics without any site effects [1]. To obtain the bedrock motion, researchers need to first identify the subsurface structure by means of various survey methods [2][3], and then to remove the site amplification factor taking non-linearity into consideration by means of the equivalent linear analysis (ELA) [4] or the real non-linear response analysis. However, it is difficult to manage the parameters appropriately for the non-linearity response analysis, and it is also difficult to show the evidence that those non-linear parameters are optimal for the sites. If we use improper parameters for non-linear site amplification estimate, the resultant bedrock motion could be totally unrealistic. In this study, in order to avoid the use of non-linear response analysis to obtain the bedrock motion, we attempt to estimate the horizontal seismic bedrock motion from the vertical motion at the surface based on the Horizontal-to-Vertical spectral ratios (HVRs) derived by the diffuse field theory for earthquakes [5].

According to the theoretical relations on the transfer functions that can be used to obtain the HVR at the surface for earthquake motions, if the vertical transfer function does not change between weak motions and strong motions of S-wave portion [6], the horizontal seismic bedrock motion of S-wave portion is equal to the vertical motion at the surface divided by the linear transfer function of the vertical component, with the coefficient of the square root of the ratio of S-wave velocity and P-wave velocity of the seismic bedrock. Since we can observe much larger numbers of ground motions with linear amplification levels compared to the ground motions with non-linear amplification levels, we can evaluate the linear transfer function with much higher accuracy. First, we make a new P-wave velocity conversion curve from PS loggin g data at K-NET and KiK-net [7] strong motion observation stations in Japan. Next, we calculate average HVR of weak motions and identify the subsurface structure as a ratio of horizontal and vertical transfer functions based on the diffuse field theory. Finally, we attempt to estimate the horizontal seismic bedrock motions using the proposed method at several observation sites and compare with results from conventional methods such as ELA for non-linear soil response.

2. Relationship between S-wave velocity and P-wave velocity

Based on the diffuse field theory for earthquakes [4], the HVR of earthquake is equal to the ratio of transfer function of horizontal and vertical components multiplied by the square root of P-wave velocity over S-wave velocity at seismic bedrock, as shown in Eq. (1). Here $\alpha$ and $\beta$ are P- and S-wave velocities of the seismic bedrock, respectively. TF is the transfer function between the seismic bedrock and the surface.

$$ \text{HVR}_{\text{theory}} = \left( \frac{\alpha}{\beta} \right)^{1/2} \ast \left( \frac{\text{TF}_{\text{horizontal}}}{\text{TF}_{\text{vertical}}} \right)^{1/2} $$ (1)

The parameters to construct the subsurface structure are S-wave velocity, P-wave velocity, thickness, density and damping. A lot of previous studies on site structures such as [2][3] investigated S-wave velocity and thickness, because those parameters are closely linked to the horizontal amplification and hence damage of buildings. Researchers used the observed density and P-wave velocity directly or converted them from S-wave velocity. Ludwig et al. [8] is the most-frequently used conversion relationship to calculate P-wave velocity from S-wave velocity, but this relationship was obtained for S-wave velocity over 600m/s. When we interpret HVR based on the diffuse field theory, not only S-wave velocity but also P-wave velocity are important parameters in spite of the lower resolving power of P-wave velocity than S-wave velocity. Therefore we made a new P-wave velocity conversion formula from PS logging data obtained by borings at K-NET and KiK-net [7] stations in Japan.

Fig.1 shows 7298 pairs of S-wave and P-wave velocities from the PS logging data observed at K-NET and KiK-net stations in Japan which we could download from the webpage of National Research Institute for Earth Science and Disaster Prevention (NIED). We show the relationship of Ludwig et al. in Fig.1, and it is corresponding to the average of PS logging data over 600m/s of S-wave velocity. We divided the PS logging data into bins for each 100m/s of S-wave velocity and calculated averaged P-wave velocities for each bin. The averaged P-wave velocities, their standard deviations and the approximated curve of the averaged P-wave
velocities are also shown in Fig.1. The averaged P-wave velocities are increasing as S-wave velocities are increasing, but the average values start to fluctuate from about 2500m/s of S-wave velocity probably because of insufficient numbers of observed data. The standard deviations, approximately 500m/s in low to high S-wave velocity ranges, seem not so large, that the averaged P-wave velocities can be considered to be representative of the relationship between S-wave velocity and P-wave velocity among whole sites in Japan. Eq. (2) is the approximate curve of averaged P-wave velocities, where $V_p$ and $V_s$ are P-wave velocity and S-wave velocity, respectively. The units of $V_p$ and $V_s$ are in m/s.

$$V_p = 1.89 \times 10^{-4} \times V_s^2 + 2.15 \times V_s + 619$$  \hspace{1cm} (2)

This curve is consistent with the relationships of Ludwig et al. in the range over 600m/s of S-wave velocity. We obtained the P-wave velocity conversion formula covered from low to high S-wave velocity ranges, which means the proposed formula covers the subsurface structure from the surface to the seismic bedrock.

3. Identification of subsurface structure

We calculated HVRs of earthquake at K-NET station MYG006 and IBR013 to identify subsurface structures based on the diffuse field theory. The PS logging data and boring data are shown in Table 1, and we calculated the averaged HVRs of weak motions for each horizontal direction excluding high acceleration records, as shown in Fig. 2, to get the linear characteristics. These HVRs were calculated from 40.96 seconds earthquake records of

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the S-wave and early coda portions. The observed HVR at MYG006 has the first peak at 0.2Hz and then a wider peak with several small fluctuations from 0.6Hz to 10Hz. The observed HVR at IBR013 has the first peak at 0.35Hz and the second peak at 5Hz. These first peaks were also observed at temporary observation sites which we deployed around each K-NET station for about a half year to one year, so we considered that these peaks reflect the common and deep subsurface structure of each area. The observed HVRs at both sites do not show any azimuth dependence, and so the subsurface structures of these areas can be considered as one-dimensional (flat-layered) structures.

Fig. 2 – Observed HVRs of weak motions at MYG006 and IBR013. The averaged HVRs of NS and EW components (blue and red solid lines), and the one standard error of NS and EW components (blue and red dotted lines).

We identified the one-dimensional subsurface structures beneath the observation sites by reproducing the observed HVRs based on the diffuse field theory for earthquakes. We used Hybrid Heuristic Searching method [9], which is a combined method of real type genetic algorism and annealing simulation, to search the optimal $V_s$ and thickness. $V_p$ and density $\rho$ were converted from $V_s$ by Eq. (2) and Eq. (3) [10], respectively, and the damping was assumed to be 1.1% as a hysteresis damping type for all the layers.

$$\rho = 1.4 + 0.67 \times (\frac{V_s}{1000})^{1/2} \quad (3)$$

We did not allow any inverted $V_s$ layers, thus $V_s$ increases gradually with depth. We set the variable of generation, population, crossing ratio, and mutation ratio to be 300, 400, 0.7 and 0.1, respectively. We used the PS logging model and the deep subsurface structure model down to seismic bedrock [11] as the initial models for inversion. We used the root mean square value of observed HVRs of two horizontal directions. We identified the subsurface structures 10 times changing random number seeds to minimize the misfit calculated by Eq. (4), and we regarded the minimum misfit model in those 10 results as the optimal model.

$$\text{misfit} = \sum (HVR_{\text{observed}} - HVR_{\text{theory}})^2 / f \quad (4)$$

We weighted the misfit by the inverse of frequency $f$ to evaluate the misfit on logarithmic frequency axis. We identified the structures in the frequency range from 0.1Hz to 20 Hz.

Fig. 3 shows the identification results at MYG006 and IBR013. We show the observed HVRs, the theoretical HVRs and the S-wave velocity structures of the PS logging model, 10 times identification and the optimum model among them. We also show the P-wave velocity structures converted from the S-wave velocity.
At MYG006, the PS logging model cannot reproduce the observed HVR well, but the identified models explain the observed HVR very well in a wide frequency range from 0.1Hz to 10Hz. The identified S-wave structures

(a) MYG006

Fig. 3 – Identification results at (a) MYG006 and (b) IBR013. The observed HVR (green line), the theoretical HVR and S and P-wave velocity structure of PS logging model (orange lines), the results of 10 times identification (gray lines) and the results of optimum model (red lines).
are slower than the PS logging model, and the P-wave velocity structures are faster in the first layer and slower in the second to fourth layers than the PS logging model. At IBR013, the peak frequency of the theoretical HVR calculated from the PS logging model at 5Hz corresponds to the second peak of the observed HVR. The identified S-wave velocity structures reproduce the observed HVR well, and they are corresponding to the PS logging model. But the converted P-wave velocity structures are faster in the first layer and slower in the second to fourth layers than the PS logging model. The variability of the identified models at both sites is small except for the deep structures at MYG006 and the depth of seismic bedrock at IBR013, respectively. However, all of identified structures reproduce the observed HVRs well in a wide frequency range. So the resolving power of the first peaks, which are related to the deep structures, was not enough to determine the deep structure definitively, but any of those identified structures can explain the observed ground motion characteristics.

4. Deconvolution of seismic bedrock motion

Based on the diffuse field theory for earthquakes, the HVR is interpreted as Eq. (1). When we rearrange Eq. (1), we get a formula describing the seismic bedrock motion as Eq. (5). The left term of Eq. (5) is equal to the horizontal incident wave at seismic bedrock. This equation indicates that the horizontal incident spectrum is equal to the vertical incident spectrum with a coefficient of $\alpha$ and $\beta$.

$$S_{\text{horizontal}}^{\text{BED}} = S_{\text{horizontal}} / TF_{\text{horizontal}} = (\alpha / \beta)^{1/2} * S_{\text{vertical}} / TF_{\text{vertical}}$$  \hspace{1cm} (5)

In the previous study [6], the vertical transfer function can be considered to be unaffected by the input level of strong motion. So we can apply the transfer function of vertical component obtained by the identification of weak earthquakes to Eq. (5) even for strong motions. If we use the transfer function of vertical component of weak motions, we can calculate the right term of Eq. (5) under linear condition ignoring nonlinear characteristics and nonlinear solution. In this section, we calculate the horizontal seismic bedrock waves by Eq. (5) and compared them with the results from ELA [4], and compare them to see the validity of Eq. (5).

We use the subsurface structures identified in section 3 and the nonlinear characteristics shown in Fig. 4 [12]. To conduct ELA correctly, the selection and assignment of nonlinear characteristics are very important and delicate. We assigned the clay nonlinear characteristics to the Fill soil, Silt and Volcanic ash clay layers, the sand characteristics to Sand and the gravel characteristics to Gravel soil according to the soil profile shown in Table 1. First, we applied ELA to the horizontal component of the weak motion and applied our proposed method to the vertical component. Fig. 5 shows the bedrock spectra estimated by these analyses, the horizontal and vertical transfer functions and the estimated bedrock waveforms. In ELA, the strain was too small to cause non-linearity, therefore the analysis was performed linearly. The spectra of bedrock motions estimated by Eq. (5) are consistent with the spectra estimated by ELA. The bedrock waveforms estimated by Eq. (5) also shows good agreement with the bedrock waveforms estimated by ELA at both MYG006 and IBR013.

![Fig. 4 – Nonlinear characteristics of Clay (black), Sand (gray) and Gravel (light gray) [12], decreasing ratio of share modulus (G/G0) (solid lines) and increasing ratio of damping (dashed lines) to Effective strain.](image)
Next, we applied these methods to the strong motion which causes the subsurface structures to become nonlinear. Fig. 6 shows the results of the analyses for the strong motions. The results of ELA converged and the maximum effective strains were in the applicable range up to 1%. The G/G₀ and the damping were decreased and increased according to the nonlinear characteristics, respectively, thus the horizontal transfer functions in the high frequency range, which are related to the shallow structures, shifted to lower frequency and the amplitude got smaller than the transfer functions calculated linearly. On the other hand, based on our proposed method, we just divide the spectra observed at the surface by the linearly-calculated vertical transfer functions to obtain the bedrock wave. The bedrock spectra obtained by Eq. (5) are generally corresponding to the spectra obtained by ELA. The bedrock waveforms at IBR013 by both of analyses are consistent, but the bedrock waveform estimated by ELA is larger than the one estimated by Eq. (5) at MYG006. The peak frequency of the horizontal transfer function is shifted to lower frequency and the amplitude became less than 1 in a frequency range higher than 7 Hz at MYG006 because of the strong nonlinear behavior. In ELA the observed spectrum was divided by this nonlinear transfer function, so the spectrum in a high frequency range got larger than the original one. This increase of the high frequency spectrum is the reason of the large amplitude of bedrock waveform estimated by ELA at MYG006. From the above results, our proposed method can give us the seismic bedrock wave corresponding to the one obtained by the previous method, except for the case where too strong de-amplification is taking place due to nonlinear analysis, so it can be said that Eq. (5) was established for not only the linear behavior but also for the nonlinear behavior.

5. Conclusion

To identify the subsurface structure which explains the observed data well based on the diffuse field theory for earthquakes, we developed a new P-wave velocity conversion formula from the PS logging data obtained at K-
Fig. 6 – Horizontal transfer function calculated by ELA (dark blue line), vertical transfer function calculated from identified structures linearly (dark red line), estimated bedrock spectra and waveforms at (a) MYG006 and (b) IBR013. Blue lines show the results of ELA and red lines show the results of proposed method.

NET and KiK-net stations in Japan. Then we identified the subsurface structures at MYG006 and IBR013 using the developed conversion formula based on the diffuse field theory, and we obtained the structures which reproduced the observed HVRs quite well. We rearranged the fundamental equation of the diffuse field theory for earthquakes given as Eq. (1), and proposed an equation of the horizontal and vertical incident spectra at the seismic bedrock as Eq. (5). Eq. (5) indicates that the horizontal incident spectrum is equal to the vertical incident spectrum with a coefficient of P-wave velocity and S-wave velocity of seismic bedrock. If we assume that the vertical transfer function behaves linearly during strong shaking, then we can estimate the seismic bedrock wave by the linear deconvolution on the vertical component. We compared our proposed method with the equivalent linear analysis, and we found good agreement not only in weak motions but also in strong motions except for the high frequency range of MYG006. This difference was caused by the excessive reduction of the amplitude of transfer function because of nonlinear behavior, so we should choose and assign the nonlinear characteristics according to a priori information to avoid a peculiar solution. From these results, it can be said that Eq. (5) was established even for strong motions. Now we are able to show a new possibility of the diffuse field theory application. If the horizontal and vertical spectra are known because they are observed, and if the vertical transfer function is also known because we can estimate it from many weak motions, then we can evaluate the horizontal transfer function during strong shaking directly without performing nonlinear analysis. This result is very interesting to investigate the nonlinear behavior and the strong motion prediction. So we will study about the horizontal transfer function during strong shaking, and we will also try to identify a P-wave velocity structure simultaneously with S-wave velocity and thickness, because the vertical transfer function calculated from P-wave velocity structure becomes more important than ever in order to estimate the seismic bedrock wave and the nonlinear horizontal transfer function based on the proposed method. In this study, we assumed one-dimensional model to express the velocity structure. In most cases, the HVRs in the lower frequency range does not show clear azimuth dependence, meaning that the flat-layer assumption is valid. However, in some cases, the velocity
structure is not flat-layered and the HVRs are affected by the lateral heterogeneity, mostly in the higher frequency range. Strictly speaking, the diffuse field theory for earthquake is constructed on the assumption of one-dimensional model, so we need to carefully analyze the observed HVRs with azimuth dependence to clarify how it is affected by the complex velocity structure. On the other hand, for sites with azimuth dependence of HVRs, we can estimate the equivalent one-dimensional structure which reproduce the HVR of each direction and by comparing the difference of the ground motion characteristics between the identified equivalent one-dimensional structure and the one of the actual irregular structure to check in what extent the equivalent one-dimensional structure will be valid. And if the vertical transfer function of the identified structure is corresponding to the one of the irregular structure, our proposed method to estimate the bedrock motion and the horizontal transfer function become more convenient.

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References


