SHEAR STRENGTH DEGRADATION DUE TO DUCTILITY DEMAND IN R.C. COLUMNS AND BEAMS

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Abstract

A clear distinction can be made between brittle shear failure, occurring before the flexural strength of the column has been attained, ductile shear failure that occurs after that a flexural plastic hinges has been activated, and plastic rotations increased. The shear strength reduction is due to degrading of several resisting mechanism:- aggregate interlock due to reduction of the roughness of the crack surface by the smoothing action of the cyclic load, and bond slippage; dowel action due to cover rupture, hoops and longitudinal rebar plastic strain, and eventually buckling of the latter; strength of chord and web concrete due to compression softening and development of cracking for cyclic load.

Several studies addressed the shear strength reduction due to ductility demand on the basis of smeared cracking non-linear models, such as the Modified Compression Field Theory (MCFT). However, despite their success in modeling several structural type behaviors, they do not appear suitable to handy provide relationships required for designers or to be implemented in software for seismic analysis of whole structure. In the past, many equation for modelling the reduction of shear strength due to ductility demand were proposed and included in design code, all of them based on the evaluation of the shear as the sum of the tensile concrete, truss mechanism and external axial force contribution.

However, these models are not consistent with the model included in the present Eurocode 2 and Eurocode 8 for shear strength for static action, that is derived neglecting the contribute of tensile concrete by using the truss mechanism (or the equivalent stress fields approach) with variable concrete strut inclination.

In this context a proposal is formulated that allows to evaluate the residual shear strength of reinforced concrete columns for an assigned ductility demand by limiting the range of the deviation angle between the inclinations of the yield θ and the crack line $\theta_{I,}$, taking also into account the reduction of the compressive concrete strength due to cyclic action. To this aim, a previous model derived on the basis of the stress-field approach proposed by Bach et al. is updated. Firstly, aiming at stressing how such limit on the inclination of the web compressed concrete stress field modifies the strength domains of RC members, the effects of the progressive reductions of the deviation angle δ on *N-M-V* domains are shown. The results prove that the progressive reduction in the yield surface inclination (angle θ) causes a major reduction in the maximum shear strength; by contrast, it does not have any influence on the ultimate bending moment. Then the expression of the limitation of the angle of inclination of the web concrete stress fields is derived on the basis of indication contained in Eurocode 8 for evaluating the shear strength reduction in transversally under-reinforced existing concrete structure.

Keywords: shear strength, ductility, degradation, cyclic action

1. Introduction

Inelastic failure of reinforced concrete (RC) structures under seismic loadings can be due either to loss of flexural, shear, or bond capacity. According to evidence from past strong earthquakes, the seismic capacity of existing structures is often reduced by shear failure. A clear distinction can be made between brittle shear failure, occurring before the flexural strength of the column has been attained, ductile shear failure that occurs after that a flexural plastic hinges has been activated, and plastic rotations increased. Inelastic rotation of the hinge region does not affect significantly the flexural strength. By contrast, they reduce the shear strength due to the degrading of several resisting mechanism:- aggregate interlock due to widening of the cracks, reduction of the roughness of the crack surface by the smoothing action of the cyclic load, and bond slippage; dowel action due to cover rupture, hoops and longitudinal rebar plastic strain, and eventually buckling of the latter; strength of chord and web concrete due to compression softening and development of cracking for cyclic load.

Experimental studies by Ang et al. [1], Aschheim and Moehle [2], Wong et al. [3], Moretti and Tassios [4], Ho and Pam [5], and Lee and Watanabe [6], Priestley et al. [7] showed that columns subjected to cyclic lateral loading may fail early, in shear, after flexural yielding.

Many finite element models (FEMs) have been developed for modelling the non linear seismic behavior of shear critical RC columns. In many microscopic FEMs the behavior is reproduced on the basis of smeared cracking non-linear approach, such as the Modified Compression Field Theory (MCFT). Mullapudi and Ayoub [8] formulated an inelastic nonlinear beam element with axial, bending, and shear force interaction. The element considers shear deformation and is based on the section discretization into fibers with hysteretic models. The concrete is modelled according a smeared approach for cracked continuous orthotropic concrete with the inclusion of Poisson effect. It accounts for the biaxial state of stress in the directions of orthotropy in accordance with the Softened Membrane Model, in addition to degradation under reversed cyclic loading. Correlation studies with specimens tested under quasi-static and shake table excitations showed that the model is able to reasonably capture the response of shear-critical RC elements and predict the proper failure mode.

However, despite FEMs's success in modeling several structural type behaviors, they do not appear suitable to handy provide relationships required for designers. An alternative approach is the use of macroscopic model [9,10,11], but these are analysis oriented model, rather than designed oriented one.

Many design equations has been proposed for evaluation of shear strength degradation in RC columns due to ductility demand. Most of them are based on the additive approach that evaluate the strength as the addition of tensile concrete, truss mechanism and axial force contributions. By contrast, the attempts to cast the problem of cyclic shear strength degradation in plastic hinge of RC members within the framework of European design code, that neglects the contribute of the tensile concrete V_c, and are based upon the strut variable inclination of the truss model, often failed [12].

In this context a new proposal is formulated that, in the framework of existing European codes, allows to evaluate the residual shear strength of reinforced concrete columns and bridge piers for an assigned ductility demand by limiting the range of the deviation angle between the inclinations of the yield θ and the crack line θ_I and reduction of chord and web compressive concrete strength.

2. Ductility demand dependent shear-strength degradation models for RC columns

Ang et al. [1] and Wong at al. [3] proposed a shear strength model for circular column based on the additive approach, in which the shear strength is considered to be the sum of the concrete shear strength resisting mechanism V_c and the strength of the truss mechanisms, V_s. The degrading shear strength vs. displacement ductility relationship was obtained by evaluating the residual shear strength by reducing the concrete contribution, but increasing the truss mechanism contribution corresponding to a steeper inclination θ of the diagonal compression strut. A limit value of $ctg\theta_{max}=[0.2\ f'_c/(\rho_{sl}\ f_{sy})\ -1]^{05.}$ was assumed, being f'_c and f_{sy} the nominal concrete compression strength and steel yielding strength respectively, and ρ_{sl} the geometrical ratio of the tension longitudinal reinforcement, taken as 0.5 of the value evaluated considering the total longitudinal reinforcement area.

Watanabe and Ichinose [13] proposed a model for shear strength evaluation for rectangular sections, neglecting the tensile concrete contribution, and based on superimposition of arch (strut in his terminology) action and truss

action, using a lower bound plasticity approach. They limited diagonal compression stress resulting from combined arch and truss action in concrete hinge regions due to densely intersecting large flexural shear cracks by variation of the compressive strength effectiveness factor $\nu = f_{c2}/f'_c$, where f_{c2} is the reduced strength due to compression softening. Outside the hinge region $\nu = \nu_0 = 0.7 - \sigma_0/200$ $\sigma_0 = N/A_g$ (in Mpa), where σ_0 is the stress due to axial force N actiong on the gross section A_g, while in the plastic hinge region $\nu = \nu_0\,(1\text{-}15\,R_p) \geq 0.25\,\nu_0$, R_p being the plastic hinge rotation. They also reduced aggregate interlocking due to widening of flexural shear cracks by limiting the inclination θ of compression strut in the truss action as follows: $ctg\theta_{max} = [\,\nu_0 f'_c\, b\, s\, /\, (A_{sw}\, f_{sy})\, \text{-}1]^{0.5}$ being b the cross section width, s the stirrup spacing and A_{sw} the stirrup cross section area.
Priestley et al [7] provided an improved prediction and reduced scatter compared to the previous shear strength models, detecting that Ang et al model works well for low ductility, but scatter increases at moderate to high ductility levels, because of the residual shear strength being assumed independent of the axial load level and the aspect ratio. The Author evaluated the shear strength by superimposition of tensile concrete V_c, truss mechanism V_s and axial load V_N components. The tensile concrete component is evaluated as $V_c = 0.8\, A_g\, k\, (f'_c)^{0.5}$, where k is the degradation coefficients of concrete shear strength with ductility depicted in Fig.1. The axial load component is not degraded with ductility, and was evaluated as $V_N = N\,(h\text{-}x_c)/(2\,a)$, where h, x_c and a are respectively the cross section depth, compression zone depth, and shear length. Lastly the truss mechanism component was evaluated assuming a 30° angle of the web concrete strut and column axis, as $V_s = A_{sw}\, f_{sy}\, h'/s$ ctg30°, where h' is the distance between centers of the peripheral hoop. In a following paper [14] regarding circular columns only, the degrading coefficient was substituted with an updated expression $\alpha\,\beta\,k$, where $1 \leq \alpha = 3 - a/D \leq 1.5$ (D=diameter of circular section) take into account the element aspect ratio, and $\beta = 0.50 + 20\,\rho_l \leq 1$ the amount of longitudinal steel ratio. All these models refer only to diagonal tension failure, recognized in the degrading of the tensile concrete contribution only. Moreover, they considered the axial force contribution at the shear resistance as an independent additional terms V_N. Moehle and al. [15] accounted for the contributed of axial force within the term V_c and attributed the reduction due to ductility demand to both concrete and truss mechanism terms. Biskinis et al. [8] used two alternative models for the degradation of shear resistance of elements failing for diagonal tension, as controlled by cyclic displacement ductility demand, able to assess the behavior of sections of any shape. Both of them evaluate the shear strength by summation of the contributions of web reinforcement on the classical 45-degrees truss analogy and tensile concrete where the total longitudinal reinforcement ratio $\rho_{l,tot}$ replaced the classical one related to the tensile longitudinal reinforcement ρ_l, and taking into account the effect of axial compression. Only the plastic part μ_p of ductility μ ($\mu_p = \mu$ -1) was assumed as the parameters that control the degradation coefficient, evaluated as ratio of plastic part of the chord rotation at failure to the value of the yielding rotation. The inclination of the concrete strut in the truss model was set equal to 45° and the terms due to axial load was evaluates as in the previous model. Thus, by fitting the results of a large database, they proposed the two following predictive equations for degrading shear strength:

$$V_{Rs1} = \frac{h - x_c}{2a}\min[N_s, 0.55 A_g f'_c] + 0.16\left(1 - 0.095\min[4.5, \mu_\Delta^{pl}]\right)\max[100\rho_{l,tot}, 0.5]\left(1 - 0.16\min[5, \frac{a}{h}]\right)\sqrt{f'_c}\,A_g + \frac{A_{sw}}{s} f_{sy}\, z \quad (1)$$

$$V_{Rs2} = \frac{h - x_c}{2a}\min[N_s, 0.55 A_g f'_c] + 0.16\left(1 - 0.05\min[5, \mu_\Delta^{pl}]\right)\left\{0.16\max[100\rho_{l,tot}, 0.5]\left(1 - 0.16\min[5, \frac{a}{h}]\right)\sqrt{f'_c}\,A_g + \frac{A_{sw}}{s} f_{sy}\, z\right\} \quad (2)$$

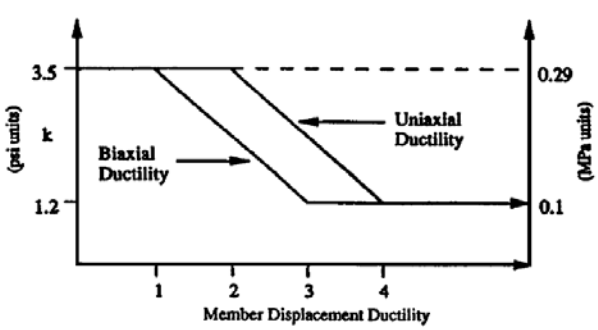


Fig. 1 – Priestly at al. model for degradation of concrete shear strength with ductility [7]

The authors stressed that the second model that considers shear strength degradation in both the web transversal reinforcement and tensile concrete contribution due to ductility demand was able to fit the experimental data with larger accuracy .The Authors proposed also the following predictive equation for the degradation of shear resistance of concrete columns failing in diagonal compression:

$$V_{Rc} = \frac{4_1}{7}\left(1-0.02\min[5,\mu_{\Delta}^{pl}]\right)\left(1+1.35\,\frac{N}{A_g\,f_c^{'}}\right)\left(1+45\,\rho_{l,tot}\right)\sqrt{\min[f_c^{'},40]}\,b_w\,z\;\sin 2\alpha_0 \tag{3}$$

where α_0 is the angle between the diagonal and the axis of the column ($\tan\alpha_0$=h/(2a)). Equations (2) and (3) were included in the Eurocode 8 (EN 1998-3: 2005) for evaluation of column shear capacity in existing building at the Near Collapse Limit State. Sezen and Mohele [12] stressed that the increment of the shear stress corresponding to the onset of diagonal tension cracking is related to the axial load, and that degradation of the concrete lead to reduction in bond capacity of longitudinal and transversal reinforcement, thus reducing the contribute of the truss mechanism also.

Attempts to cast the problem of cyclic shear strength degradation in plastic hinge of RC members within the framework of European design code fails [8]. The model neglects the contribute of the tensile concrete V_c, and are based upon the strut variable inclination of the truss model. Thus, important parameters, such as the shear length *a/h* and the longitudinal reinforcement ratio ρ_l were missing, and large scatter of the data was found.

In this context, a new proposal is formulated based on generalization of an existing model for internal forces N-M-V interaction domain evaluation based on Bach et al's stress field approach to the presence of seismic forces. The model allows to evaluate the residual shear strength of reinforced concrete columns and bridge piers for an assigned ductility demand by limiting the range of the deviation angle between the inclinations of the yield θ and the crack line θ_I and modelling the compression softening of the concrete.

2. Internal force interaction domain by stress field model under static actions

When RC elements are simultaneously loaded by axial force N, bending moment M and shear force V, the stress distribution in the cross-section is complex; thus an analytical model based on plastic theory and able to predict the stress distribution cannot easily be derived. Based on the stress-field approach proposed by Bach et al. [16], Recupero et al. [17-19] proposed a model in which at the Ultimate Limit State the resistant mechanism is composed by (Fig2):- two chords, the tensile cord made by the longitudinal reinforcement and the compressed chord made by the compressed concrete and its reinforcement, both chords modeled by element with finite length y_1 and y_2, respectively, having geometrical shape depending on the cross section shape; - the web concrete portion of height $y_3=h-(y_1+y_2)$, *h* being the height of the transversal cross sections, is subjected to an uniaxial stress field, inclined by an θ angle (yield surface) with respect to the longitudinal axis of the element. The shear action is carried by the last stress field. Adjunctive resisting contribute can be provided by the skin reinforcement, if it exists.

In order to derive an analytical model able to provide the *N-M-V* interaction strength domain of RC elements, according to Recupero *et al.*[17], the following assumptions are made: - (i) the chord and web concrete are only subjected to uniform compressive stress-fields; - (ii) both the stirrups and the longitudinal web reinforcement (if any) are subjected to a purely axial force (i.e. dowel action is considered elsewhere, as explained in the following); (iii) compared to the size of the structural members, the spacing of the stirrups and of the web longitudinal bars is so small that their actions can be modeled via different uniform stress fields on the basis of the theory of plasticity; (iv) the concrete stress field in the web is inclined by the angle θ to the longitudinal axis, which may differ from β that is the alignment of the first cracks in a structural member subjected to axial force, bending and shear; the maximum shear capacity is achieved for $ctg\theta$ varying in the range $0.4 \leq \mathrm{ctg}\theta \leq (\mathrm{ctg}\theta)_{\max}$; (v) the constitutive laws of the materials are consistent with the theory of plasticity; (vi) the contributions to the shear capacity of dowel action, aggregate interlock are indirectly taken care of by introducing (through the angle θ) different orientations for the principal directions of the stress fields and the cracks; (vii) the contribution due to the tensile strength of concrete is neglected; (viii) the arch action, which plays a remarkable role in the D (Disturbed) regions, is neglected; hence, the validity of the model is limited to B (Bernoulli) regions. It has to be

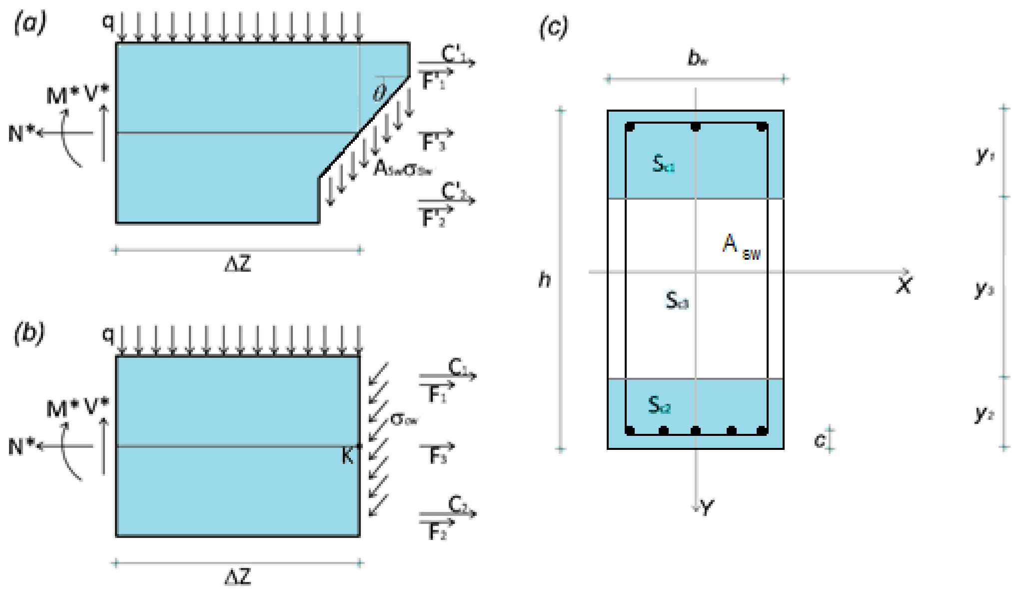


Fig. 2 a) Layered structural element, b) external and internal forces scheme assumed; c) geometrical model of the beam.

pointed out that assumption (iii) may be used for beam with a transverse minimum shear reinforcement mechanical ratio of 0.16/fc 0.5 being fc the concrete strength in compression.The model has been used for evaluation of M-N-V internal force interaction strength domain, according the numerical procedure utilized in the case of RC [17,19] or PC [18] structural elements. The actual values of the internal stresses at the collapse condition are evaluated considering that the stresses have to ensure equilibrium with the internal actions *N*, *M* and *V*. The equilibrium conditions are obtained by imposing the condition that the response is governed by the weakest stress-field failure.

With reference to the RC element in Figs 2, and obtained by a parallel cut to the web concrete stress field at the abscissa $z+\Delta z$ (Fig 2a), the following equilibrium equation in the y direction can be written:

$$V^{*}-q\cdot z=V=\left(b_{w}y_{3}\right)\rho_{sw}\sigma_{sw}\cot\vartheta \tag{4}$$

where V^* is the shear external action at the abscissa z; q is a distributed vertical load to demonstrate how the formulation is as general as possible; σ_{sw} is the stress of steel stirrups; $\rho_{sw} = A_{sw}/(b_w s_w)$ is the geometrical ratio of steel stirrup. The compressive concrete stress field inclination (θ) is not pre-established, and it has to be determined.

Next, a new segment is considered, obtained by cutting the element with two section planes with slope $\theta = 90°$ to the beam axis at the abscissa z and $z+\Delta z$ (Fig. 2b); thus the equilibrium equation of the segment in the y direction reads:

$$V^{*}-q\cdot z=V=\left(b_{w}y_{3}\right)\sigma_{cw}\cos\vartheta\sin\vartheta \tag{5}$$

where σ_{cw} is the strut concrete compressive stress in the central layer having area $S_{c3} = b_w\, y_3$. Furthermore, the expressions of the internal forces in the tension chord and in the compression chord must satisfy the following relations:

$$\sigma_{c1}\int_{S_{c1}} dS_c + \sigma_{c2}\int_{S_{c2}} dS_c + \sigma_{c3}\, ctg\theta \int_{S_{c3}} dS_c + \sigma_{s1}\int_{S_{s1}} dS_s + \sigma_{s2}\int_{S_{s2}} dS_s + \sigma_{s3}\int_{S_{s3}} dS_s = C_1 + C_2 + F_1 + F_2 + F_3 = N \quad (6)$$

$$\sigma_{c1}\int_{S_{c1}} y_c\, dS_c + \sigma_{c2}\int_{S_{c2}} y_c\, dS_c + \sigma_{c3}\, ctg\theta \int_{S_{c3}} y_c\, dS_c + \sigma_{s1}\int_{S_{s1}} y_s\, dS_s + \sigma_{s2}\int_{S_{s2}} y_s\, dS_s + \sigma_{s3}\int_{S_{s3}} y_s\, dS_s = M \quad (7)$$

where: σ_{ci} and σ_{si} are the axial stress of the concrete and steel related to the *i-th* layer, respectively; C_i and F_i are the resultant forces in the concrete and the steel related to the *i-th* layer, respectively; y_c and y_s are the lever arms (algebraic values) calculated starting from the centroid of the cross-section.

The structural element collapse may occur either by concrete crushing or by transversal reinforcements yielding. Using the nominal values for the steel yielding f_{ym} and for the concrete compression strength f_{cm} and $f_{c2} = \nu_0 f_{cm},$ the terms related to the areas S_{c1}, S_{c2}, and S_{c3} depend on the depth of the web layers y_1, y_2, and y_3, which may vary according to the following geometrical and static conditions:

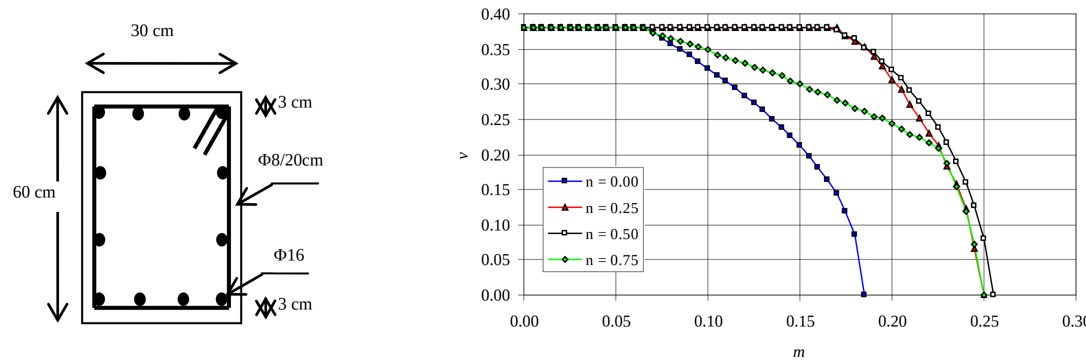
$$\sum_{i=1}^{3} y_i = h\,; \quad -f_{cm} \le \sigma_{ci} \le 0 \quad (i=1,2)\,; \quad -f_{c2} \le \sigma_{cw} \le 0\,; \quad -f_{ym} \le \sigma_{si} \le f_{ym} \quad (i=1,2) \quad (8a,b,c,d)$$

It is noteworthy that, for concrete stress field, a concrete effective compressive strength f_{c2}, reduced by the effect of transverse tension and by the need to transmit stress across the cracks by interlock effect, is adopted. Lastly, it has to be emphasized that the model is general, and able to provide the strength interaction domain when all the three internal axial force M, N, and V are present (for vanishing axial force, some simplification arises). The procedure for evaluation of strength domain requires the knolewdge of the mechanical and geometrical parameters defining the the materials and the geometry of each element segment and the values of the slope of the first cracking β and of the web compressed concrete field θ. The value of β and can be approsimated with the first cracking slope at the Seviciability Limit State (SLS), that can be assumed equal to 45° for beams, while for columns it depends on the ration betwenn axial and shear force at the SLS. When static loads are considered, the variation range of the slope of web concrete stress field is assumed $22° \le \theta \le 68°$, i.e. $0.4 \le ctg\theta \le (ctg\theta)_{max} = 2.5$ as reported in Eurocode 2, part.1. When beam are considered, and $\beta = 45°$ i assumed, the deviation δ between the β and θ laying ($\delta = \beta - \theta$) is comprised in the range $-23° \le \delta \le 23°$; however, since the largest shear strength is obtained when value of ctgθ no smaller than one is retained, values of $\beta \ge 45°$ are never considered since would lead to smaller shear strength. In Fig. 3a a rectangular cross section of a column made of concrete with characteristic concrete cylindrical strength f_{ck}=25 MPa and steel yielding strength f_{yk}=450 MPa is shown. In order to draw the interaction domain, the non dimensional internal resisting axial force n, bending moment m and shear ν are defined by the following relations:

$$n = \frac{N_{rd}}{b_w \cdot h \cdot f_{c1}} \qquad m = \frac{M_{rd}}{b_w \cdot h^2 \cdot f_{c1}} \qquad \nu = \frac{V_{rd}}{b_w \cdot (h-4c) \cdot f_{c2}} \quad (9\ a,b,c)$$

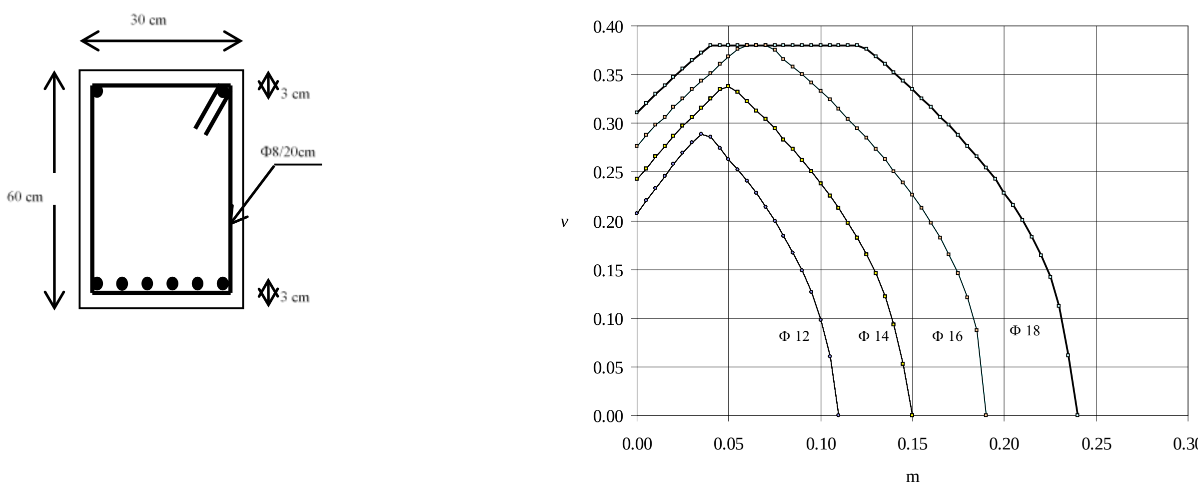
where b_w and c are the column cross section base and cover respectively, and the design strength of the compressed concrete under uniaxial stress f_{cd1} and the reduced value for the contemporary presence of shear stress f_{cd2} are evaluated according to the Model Code as follows:

$$f_{c1} = 0.85 \cdot \left(1 - \frac{f_{ck}}{250}\right) \cdot f_{ck} \qquad\qquad f_{c2} = 0.60 \cdot \left(1 - \frac{f_{ck}}{250}\right) \cdot f_{ck} \quad (10a,b)$$



a) b)

Fig. 3 a) column cross section; b) dimensionless shear ν vs. bending moment *m* interaction strength domain for different values of specific axial force *n*



a) b)

Fig. 4 a) beam cross section; b) dimensionless shear ν vs. bending moment *m* interaction strength domain for different values of diameter of longitudinal reinforcement

In Fig.3b the nondimensional shear ν bending moment *m* interaction domains for the cross section in Fig.3a are shown for four different value of the nondimensional axial force, namely *n*=0; 0.25; 0.5; 0.75. Larger values of axial force are not consistent with the column seismic design. In evaluating the interation domain, the longitudinal reinforcement are assumed continued along the element axis, whitout any interruption. Therefore, the upper limit of the variation range of the θ angle can be considered without any particular caution. the interation domain show that the presence of bending moment close to the maximum one abruptly reduce the shear strength of the column.

Noticeable reduction of the shear strength can be recognized in beams, where the axial force vanishes. As an example, in Fig. 4a the cross section of a beam is depicted, and the corresponding shear bending moment interaction domain for different values of the diameter of the longitudinal reinforcement and the same value of transversal reinforcement are shown in Figure 2b. In this case also, the same range of variation of the θ angle as the previous one were assumed. The domains show that for small diameter of the longitudinal reinforcing bars, i.e. for small amount of tensile longitudinal reinforcement, the shear strength is strongly reduced by the bending moment. Only when over reinforced beam are considered, the shear strength can be assumed almost independent

of the bending moment. More detailed indications on the presented procedure and more deep discussion of the results can be found in [17-19]
The two examples reported herein show that the interaction among the internal force cannot be neglected even when static load are considered. The following section will show that a suitable reduction of the variation range of the compressed concrete stress field θ is able to reproduce the results of the classical additive model for shear strength degradation.

3. Effects of cyclic action on internal force interaction domain

In this section, a tools for the evaluation of the internal force interaction domain depending on the flexural ductility demand is provided. The model is based on a procedure able to limit the range of deviation angle δ between the inclinations of the yield line θ and the initial cracking line β as a function of the geometrical and mechanical properties of the concrete element and steel reinforcement, and of the plastic deformation undergone under the effect of the seismic action. The procedure aims at transposing the suggestion provided by Biskinis et al for the reduction of the shear strength of reinforced concrete elements loaded by cyclic action, to the model based on the stress field approach. In the Biskini's model and its generalization, the contribution due to the tensile concrete in the strut model is reduced depending on the value of the curvature ductility demand. However, several mechanisms may be cited to explain the degradation of shear strength: i) the reduction of aggregate interlock along diagonal cracks, as their interface are ground and became smoother with cyclic loading or because the diagonal cracks gradually open up due to bond slippage and accumulation of inelastic strains in the stirrup tying the cracks; ii) the degradation of dowel action with cycling of the shear force and with accumulation of inelastic strains in the longitudinal reinforcement; iii) the development of flexural cracks throughout the depth of the member causing reduction of the contribution of the compression zone to shear resistance and the softening of concrete in diagonal compression due to accumulation of transverse tensile strains.

In the stress field model, the difference of the slope of the yield surface in comparison to that of the cracking surface is partly generated by the effects of aggregate interlock, which avoid slips along cracks, that is a function of the roughness of the crack sides in contact, and dowel action. When the maximum deformations and/or the accumulated damage due to small amplitude of cyclic actions increase, the roughness of the sliding surfaces is reduced. Thus, the range of the deviation angle δ is limited. The proposed model assumes a limit value of the angle δ that should depends on a measure of the damage generated by the combined effects of amplitude of maximum flexural ductility demand and cumulated effect of cyclic action, i.e. on a damage index that should include both the two aforementioned contributes. As an example, the Park and Ang index appears to be a suitable damage index for governing the limitation of the deviation angle δ.

Firstly, aiming at stressing how such an assumption modifies the strength domains of RC members, the effects of the progressive reductions of the deviation angle δ on *N-M-V* domains are shown In Fig 5a-d and in Fig. 6 for the column section shown in Fig.3a and beam section in Fig.4a respectively, by setting the values of $\theta=\beta-\delta$. The normalized strength domains are shown in Fig. 5a-d for four limit values: $ctg\ \theta = 2.5$ ($\theta \approx 22°$), $ctg\ \theta = 2.0$ ($\theta \approx 26$), $ctg\ \theta = 1.5$ ($\theta \approx 34$) and $ctg\ \theta = 1.0$ ($\theta = 45$) and for four normalized axial force values $n = N_{sd}/(f_{ck}\ A_c) = 0$, 0.25, 0.50 and 0.75. Figs. 5 show that the progressive reduction in the yield surface inclination (angle θ) causes a major reduction in the maximum shear strength; by contrast, it does not have any influence on the ultimate bending moment. The domains show also that for large values of the axial force, the reduction of the shear strength is large for small values of the bending moment and any value of the stress field inclination also. Fig.6 shows that the reduction of the shear strength in beams due to the reduction of the deviation of the web concrete stress field with respect to the initial cracking inclination can overcome the 50%. In order to characterize the relation between angular deviation δ and flexural ductility demand on the basis of the indications provided by Eurocode 8 (or Biskinis et al[12.]), it is observed that both for columns and beams the limit of the web concrete stress field influences the horizontal line of the strength domain corresponding to small values of the bending moment, for which the failure of the structural element is reached by the attainment of the limit stress in the transversal web reinforcement, and the shear strength is evaluated as follows:

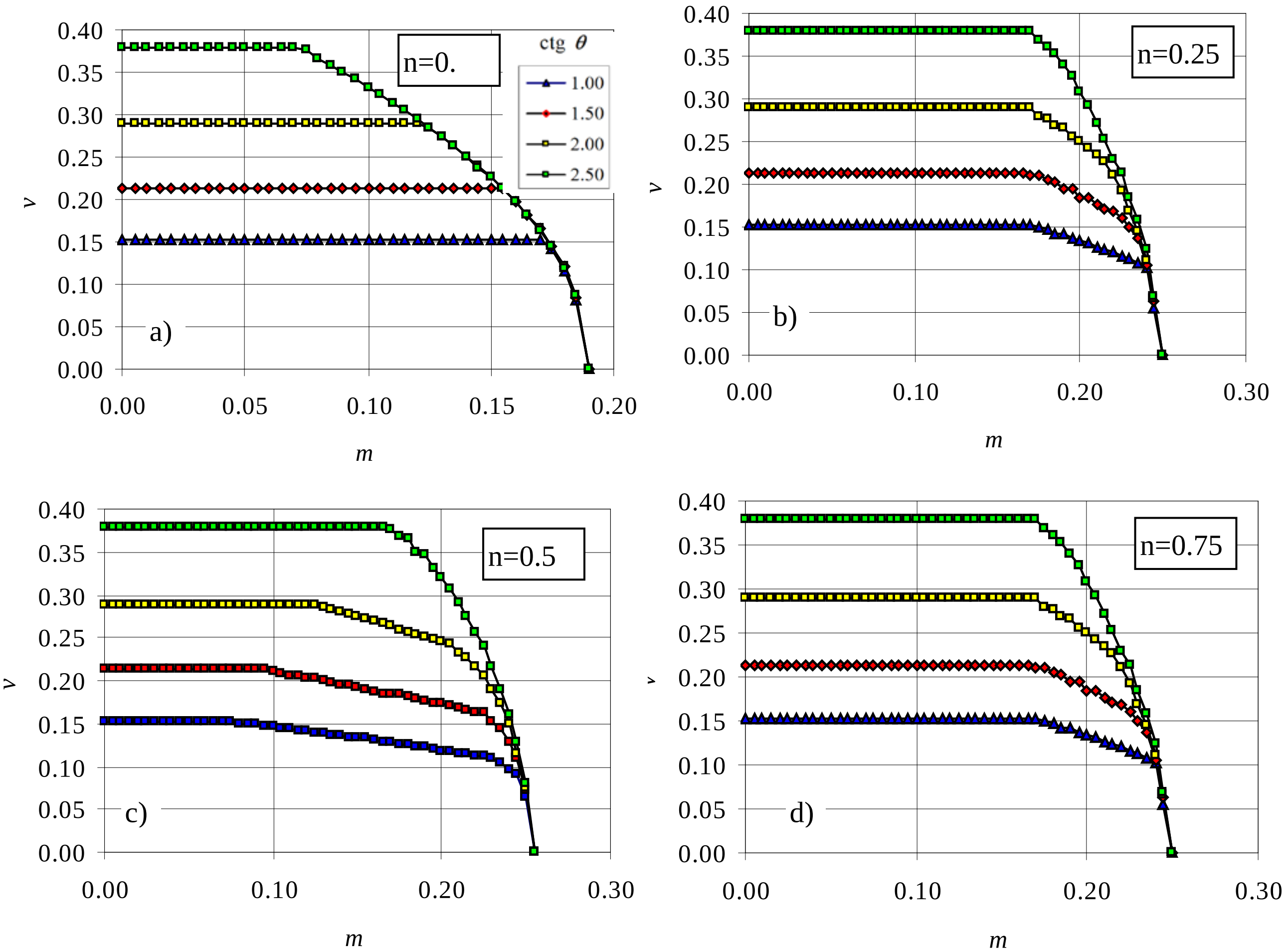


Fig.5-Interation strength domains as function of the compressed concrete stress field slope for different axial force: a) n=0; b) n=0.25; c) n=0.50; d) n=0.75

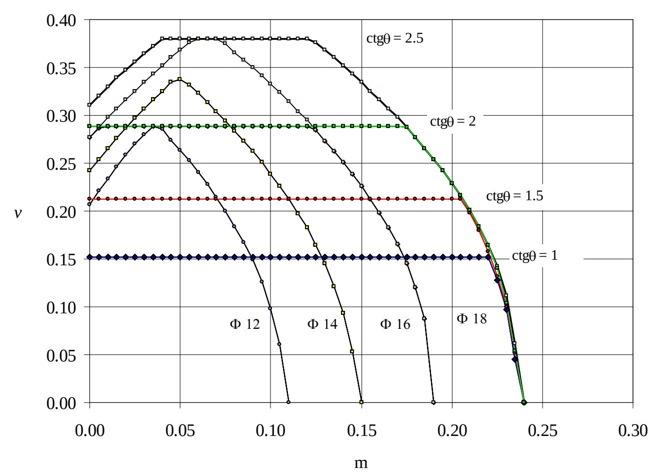


Fig. 6 – Shear-bending moment interaction strength domain for the beam in Fig.2a as function of the compressed concrete stress field slope for different longitudinal bar diametr Φ

$$V_{Rsd} = \frac{A_w}{s} \cdot y_3 \cdot f_{yd} \cdot ctg\theta \qquad \theta = \theta_I \pm \delta \tag{11}$$

Eq. (11) represent the shear strength that the element can withstand when the collapse is determined by the attainement of the strength of the transversal reinforcement. Moreover, in order to take into account the effect of axial load in the D region, the direction of the stress field in the compressed chord is rotated in the direction α of the compressed concrete strut according [7]. Lastly, in order to reproduce the reduction of the concrete strength due to large crack and accumulation of transversal tensile strain, the strength of the concrete in the compression chord and in the web stress field is reduced by a coeffcient $k_{f'c}$, Since the degradation of the concrete is reduced when effect of confinement is increased, the softening coefficient $k_{f'c}$ is assumed to be dependent not only on plastic part on ductility demand μ_Δ^{pl}, but longitudinal ρ_{st} and tranversal ρ_{sw} reinforcement ratios also. Thus, Eq (8) is modified in the form $-k_{fc,\mu_\Delta^p,\rho_{sl},\rho_{sw}} f_{cm} \le \sigma_{ci} \le 0 \quad (i = 1,2,w)$, and in the right han side ef Eqs (4), and (5) is added the term

$$b\, y_1 \sigma_{c1} \frac{(h_N - 0.5 y_1)}{a} \tag{12}$$

where h_N is the distance of the axial load application point from the axis of the more external stirrup.

In order to reproduce the reduction of the shear strength due to the flexural ductility demand predicted by the Biskinis et al's model in Eq.(2) modeling the decay of the resistance by a limitation of the angle of inclination of the web concrete stress fields θ and reduction of concrete compressive strength, element with lack of adequate transversal shear reinforcement have to be considered, i.e. element for which the prosed model predict the shear collapse by the modified form of Eq.(4), in which the slope of the stress field is the maximum. Under the above hypothesis, the following relation can be imposed:

$$\begin{aligned}[V_{Rd}]_{\max} &= \frac{A_{sw}}{s_w} y_3 \frac{f_{yk}}{\gamma_s} [ctg\theta]_{\max,\mu_\Delta^p,\rho_{sl},\rho_{sw}} + k_{fc,\mu_\Delta^p,\rho_{sl},\rho_{sw}}\, b\, y_1 f_c' \frac{(h_N - 0.5y_1)}{a} = \frac{h - y_1}{2a} \min[N_s, 0.55 A_g f_c'] \\ &+ 0.16\left(1 - 0.05\min[5,\mu_\Delta^{pl}]\right)\left\{0.16\max[100\rho_{l,tot}, 0.5]\left(1 - 0.16\min[5,\frac{a}{h}]\right)\sqrt{f_c'} A_g + \frac{A_{sw}}{s} f_{sy}\, z\right\}\end{aligned} \tag{13}$$

where the subscript $[\bullet]_{\mu_\Delta^p,\rho_{sl},\rho_{sw}}$ stress the circumstance that·the minimum angle of inclination of the web concrete stress fields corresponding to $ctg\theta_{max}$ and the compressive concrete strength reduction factor $k_{f'c}$ are dependent on plastic part on ductility demand μ_Δ^p, longitudinal ρ_{sl} and transversal ρ_{sw} reinforcement ratios. The two parameters should be estimated by fitting of test results available in literature. However, a roughly estimate of $[ctg\theta]_{\max,\mu_\Delta^p,\rho_{sl},\rho_{sw}}$ can be done by assuming a simplify expression of the compressive efficiency factor similar at that proposed in [13] $k_{fc,\mu_\Delta^p,\rho_{sl},\rho_{sw}}$ $=\nu_0$ (1-0.0535 $\gamma\, \mu_\Delta^p$)≥ 0.25 ν_0, where ν_0=0.7- f_c' / 200 and γ=2 is assumed. Thus by means of Eq.(13) the expression of $[ctg\theta]_{\max,\mu_\Delta^p,\rho_{sl},\rho_{sw}}$ can be derived:

$$\begin{aligned}[ctg\theta]_{\max,\mu_\Delta^p,\rho_{sl},\rho_{sw}} &= \Bigg\{\frac{h - x_c}{2a}\min[N_s, 0.55A_g f_c'] + 0.16\left(1 - 0.05\min[5,\mu_\Delta^{pl}]\right) \\ &\left\{0.16\max[100\rho_{l,tot}, 0.5]\left(1 - 0.16\min[5,\frac{a}{h}]\right)\sqrt{f_c'} A_g + \frac{A_{sw}}{s} f_{sy}\, z\right\} - k_{f_c',\mu_\Delta^p,\rho_{sl},\rho_{sw}}\, b\, y_1 f_c' \frac{(h_p - 0.5y_1)}{a}\Bigg\} \Bigg/ \left(\frac{A_{sw}}{s_w} y_3 \frac{f_{yk}}{\gamma_s}\right)\end{aligned} \tag{14}$$

4. Model corroboration

In order to corroborate the proposed model, experimental results reported in Nagasaka [20] are reproduced. They had the following the geometrical and mechanical characteristics: b=200 mm, h=200 mm, a=300 mm, cover c=12 mm, f'_c= 21.6 Mpa, 4 longitudinal reinforcing bars having diameter ϕ=12.7 mm, number, and steel yielding and ultimate strength in the f_{yl}=371 Mpa and f_{ul}=541 Mpa respectively, hoops diameter and specing ϕ_w=6 mm and s_w=20 mm having yielding and ultimate strength in the f_{yw}=344 Mpa and f_{uw}=434 Mpa, and are subjected to axial load of values N=147 kN and 294 kN. In Fig. 7 the experimental shear strength is compared against the prpoposed model, where the value of the inclination of the web concrete stress field $[ctg\theta]_{\max,\mu_{\Delta}^{p},\rho_{sl},\rho_{sw}}$ is evaluated by Eq. (14). Only the two specimens that exhibit a failure for combined action of shear and bending moment were considered. Moreover, since the model in its present formulation is not able to take into account the further shear strength degradation due to repeated load cycles of the same amplitude, only results pertaining to the first attinement of a value of displacement ductility demand is considered. A mean value of the model and test shear strength ratio of 0.98 whit a Coefficient of Variation of 0.21 are found. The large values of the scattering proves that a more accurate calibration of the effectiveness coefficient $k_{fc,\mu_{\Delta}^{p},\rho_{sl},\rho_{sw}}$ and $[ctg\theta]_{\max,\mu_{\Delta}^{p},\rho_{sl},\rho_{sw}}$ on the basis of large test database is required.

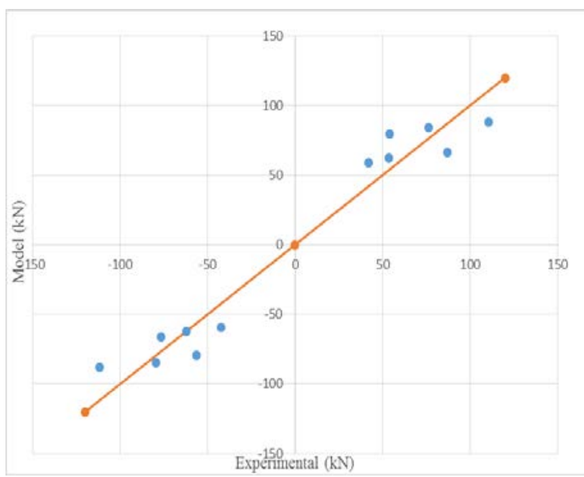


Fig. 7 – Model to experimental shear strength for specimens tested by Nagasaka [11].

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5. References

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