16th World Conference on Earthquake, 16WCEE 2017
Santiago Chile, January 9th to 13th 2017
Paper N° 378
Registration Code: S-I1461822751

Experimental Investigation into Vibration Control Effect of Passive Track Nonlinear Energy Sink

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Abstract

Nonlinear energy sinks (NES) are efficient vibration control devices. Its application has been studied in mechanical, automobile and aerospace engineering and made some achievements. However, there are little applications in civil engineering. To validate its vibration control effects on building structures, a series of shaking table tests were conducted on a five-story steel frame which is five meters high and six tons weight. Track of the NES were installed on the roof of the frame with rigid connections and mass of the NES was constrained to slide along the track through use of wheels which were made of either steel or rubber. Five earthquake waves with different frequency spectrums were selected to excite the frame coupled with NES under minor, moderate and major levels. Accelerations and displacements on each story of the frame were measured, recorded and evaluated. The experimental results demonstrate that with small mass ratio (2%) of main structure, NES has good performance in reducing the dynamic responses of the frame under seismic excitations. In the time domain analysis, it is shown that NES can attenuate the displacements, accelerations and RMS of the frame. NES exhibits wide-band frequency vibration controlling attributes that responses of the frame are reduced in multiple vibration modes after the acceleration response data were processed in the frequency domain. Comparisons of vibration reduction capability are made between NES with steel wheels and rubber wheels, and it is verified that different damping of NES makes a difference to the effect of vibration control.

Keywords: nonlinear energy sink, shake table test, dynamic response, structural vibration control

1. Introduction

Since the first passive vibration absorb device was invented by Frahmin in 1911 [1], a variety of such devices have been developed and widely used in civil, mechanical and aerospace engineering area. Most of them are linear vibration absorber which consist of the linear mass-spring system with damper. The nature frequency of the linear system are designed close to the mode frequency of the primary structure being controlled. Energy on the primary structure can be partly transferred to the linear oscillator by resonance mechanism and be dissipated by internal damping of the absorber. However, vibration controlling bandwidth of the linear absorber is relative narrow due to the fixed natural frequency. With the development of the nonlinear dynamic theory, nonlinear oscillators has been studied as passive vibration absorber. Different from the linear absorbers, the nonlinear absorber is able to mitigate vibration in a broader range of frequency for the nonlinearity of restoring force on the oscillator.

In the last decade, nonlinear energy sinks (NESs) which is a new kind of passive vibration absorber exhibited effective capacity in suppressing the vibration of structures. Similar to the linear absorbers, the NES consists of a secondary mass attached to the structure. But the restoring force on the mass is essentially nonlinear which means the links between displacements and restoring force of the secondary mass has no any part of linear component. Extensive research works on NES has been done by analytical, numerical and experimental methods [2-6]. Various types of NESs emerged in these research works, and the main differences of them are the way to implement the nonlinear restoring force. A typical NES has smooth nonlinear restoring force is called Type I NES [7]. In the early stages of the research on this kind of NES, metal wires or springs are employed and arranged perpendicular to motion direction of the mass to realize geometric nonlinearity [8]. The nonlinearity relations between the displacement and restoring force can be approximately expressed by a cubic nonlinear spring [9]. Experimental research results of Type I NESs proved that it can absorb the energy from the structure and dissipate the energy by its own damping. But, most of the structure used in these experiments are small or medium scale structures [7]. Considering the applications of the NES in civil engineering and mechanism of the NES, it is necessary to find alternative ways for the realization of the nonlinear restoring force. Research effort have been made toward this goal and some achievements appeared. In the applications of the NES for vibration mitigation on large-scale structure, elastomeric bumpers are employed to produce nonlinear restoring force, and it is proved that the bumper NESs are able to reduce the response of the structure under impulsive ground motions [10]. Furthermore, an NES system composed by several bumper NESs has been tested on a largescale nine-story frame under impulsive and blast excitations [7]. In addition, tracks with specific shapes are used to realize the nonlinearity of restoring force and this type of NES is defined as TYPE II NES [11, 12]. A track NES system composing of four identical tracks are studied through numerical and experimental methods, the numerical analysis results showed the shape of the tracks has an effect on the performance of vibration attenuation. The experiment of the track NES system are conducted on a small-scale steel frame with two layers, and experimental results demonstrates that the track NES is capable of reducing the responses of the structure in both two mode frequency.

This paper presents large-scale experimental evaluations of a steel frame attached with a track NES systems for mitigation of dynamic response excited by seismic waves. The main objectives of the experiments are to explore the effectiveness and the robustness of the NES in reducing the response of the structure. During the testing, three historic earthquake ground motions are scaled to different levels and implemented as the excitations of the shake table. A series of comparisons and discussions of the experimental results are performed by evaluating the responses of the structure between with and without the track NES. The comparison results of time and frequency domain shows the track NES is capable of reducing the response of the structure in a broad range of frequency.

2. System of track NES

A track NES system is compose by a set of track with identical shape and an auxiliary mass which is free to slide along the tracks. As shown in Fig.1, the NES track is fixed on the primary structure via rigid connections. When the primary structure suffers external excitations, its vibration will result in relative motion between the auxiliary mass and the structure. Meanwhile, the mass is constrained to move along the tracks which provide the mass with nonlinear restoring force. The vibration energy of primary structure is partly transferred to the mass through interaction force on the tracks.

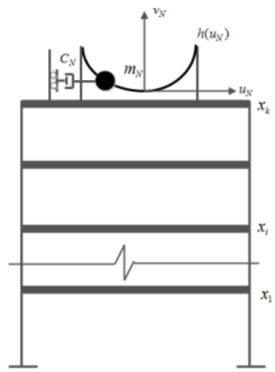


Fig. 1 – Multi-story frame with NES

The mass of the NES is m_N , the vertical and horizontal displacement of the NES relative to the roof of the primary structure are v_N and u_N respectively. The mathematical expression of the track's shape is $h(u_N)$. To simplify the dynamic model, a horizontal viscous damping c_N is assumed to represent all possible damping in the track NES systems, such as rolling friction damping between the wheels and the track. $m_1, m_2, \cdots, m_5$ are the mass of the each storey. $k_1, k_2, \cdots, k_5$ are the total stiffness of the column on each story and $c_1, c_2, \cdots, c_5$ are the viscous damping coefficient of each floor respectively. $x_1, x_2, \cdots, x_5$ represent the displacement of each floor relative to the ground. Supposing the mass dose not rotating and keep contact with the frictionless track as it moves. Using Lagrange's method, the dynamic equation of each floor and the NES are:

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_1 x_1 + k_2(x_1 - x_2) = -m_1\ddot{x}_g \tag{1}$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + c_3(\dot{x}_2 - \dot{x}_3) + k_2(x_2 - x_1) + k_3(x_2 - x_3) = -m_2\ddot{x}_g \tag{2}$$

$$m_3\ddot{x}_3 + c_3(\dot{x}_3 - \dot{x}_2) + c_4(\dot{x}_3 - \dot{x}_4) + k_3(x_3 - x_2) + k_4(x_3 - x_4) = -m_3\ddot{x}_g \tag{3}$$

$$m_4\ddot{x}_4 + c_4(\dot{x}_4 - \dot{x}_3) + c_5(\dot{x}_4 - \dot{x}_5) + k_4(x_4 - x_3) + k_5(x_4 - x_5) = -m_4\ddot{x}_g \tag{4}$$

$$m_5\ddot{x}_5 + c_5(\dot{x}_5 - \dot{x}_4) + k_5(x_5 - x_4) - c_N\dot{u}_N - F_{NES} = -m_5\ddot{x}_g \tag{5}$$

$$m_N\ddot{u}_N + c_N\dot{u}_N + F_{NES} = -m_N\ddot{x}_g \tag{6}$$

And the expression of the force F_{NES} is:

$$F_{NES} = ([h'(u_N)]^2 \cdot \ddot{u}_N + h'(u_N)g + h'(u_N)h''(u_N) \times \dot{u}_N^2)m_N \tag{7}$$

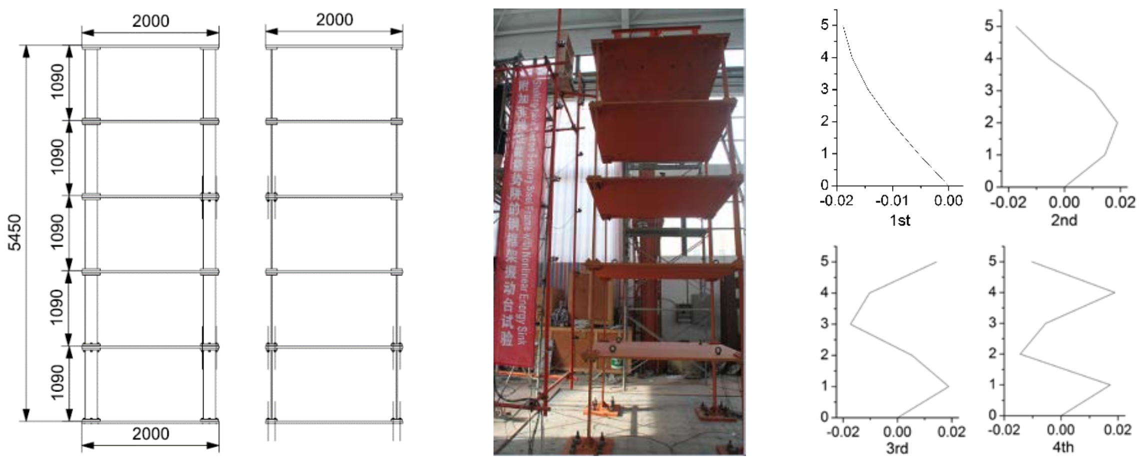
The restoring force of NES defined by Eq. (7) only contains the horizontal component of interaction force between NES and tracks because the force in the vertical direction will have little effect on reducing the lateral vibration of the test frame during the experiment. Eq. (7) shows that the shape of the track is critical to the nonlinearity of the restoring force which is a function of the shape's expression. The parameters in Eq. (7), such as the mass of the NES and the shape of the track should be designed by optimal calculation due to the complexity of the nonlinear dynamics. Wang proposed that $h(u_N) = a \cdot u_N^4$ was an appropriate choice for the shape of the track [12].

3. Experiment Setup

To verify the capability of the NES in vibration attenuating, a series experiments are conducted with a large-scale model structure and a track NES system. In this section, detailed information of the experimental structure, realization of the NES and the earthquake waves employed in the testing are discussed.

3.1 Experimental Structure

The test structure of the experiments is a five-story steel frame which has a height of 5.48m and a mass about 5600kg. Each floor is made of Q345 grade steel plates and its outline dimension is $2\text{m}\times2\text{m}\times0.03\text{m}$. Height of each floor is 1.09m and the mass is 1120kg. The direction with lower stiffness is weak axis of the frame. During the test, the excitations are loaded along this direction by the shake table to get larger vibration amplitudes. Fig. 2(a) shows the configuration of the frame. The fundamental frequency of the frame is about 1Hz. The frame had a damping ratio of 0.002. The natural frequencies of the frame at weak axis are 1Hz，2.9Hz，4.8Hz，6.1Hz and 7Hz.



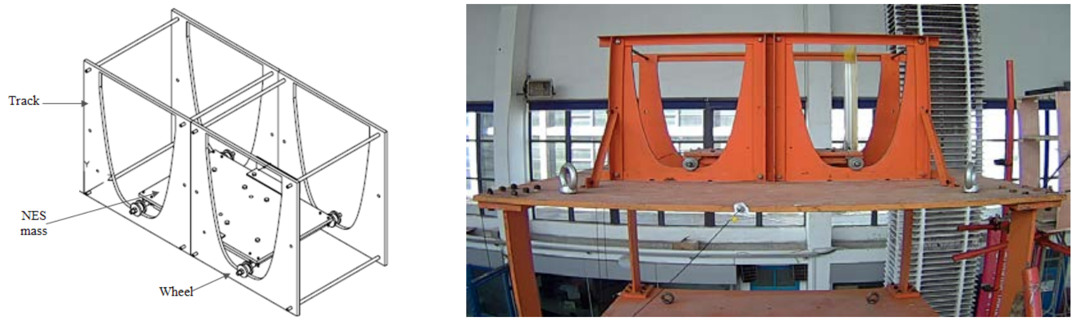
(a)Configuration of the structure (b)Frame on the shake table (c)1st to 4th mode shape

Fig. 2 – Multi-story frame and the mode shape

3.2 Track NES design and realization

A series of optimal calculations are conducted in designing the track NES that be applied to the test frame. The goal of the optimization is to find appropriate shape of the track. Considering the fundamental frequency of the frame, El Centro seismic wave is used to exicte the frame in the optimization. To evalueate the performance of the track NES with different shape, dynamic responses at roof of the frame are inspected when the frame coupled with the NES is excited by 0.15g El Centro wave. The inspectation targets include displacement, accleration and their r.m.s values. The optimiztion is implemented in matlab, eq. (1) -(7) that define the numerical mode of the frame with the NES are calculated through use of ODE45 integral function. The mass of the NES and the coefficient parameter a of function $h(u_N)$ are set in a resonable range and calculated one by one. Parameters that lead to minimum dynamic responses are selcted by comparisons of the computation results.

To make the experimental NES identical to the analysis model defined in section 2 , it is important to take consideration of the assumptions in the dynamic equations. The NES is not allowed to rotate when it moves along the tracks according to the defination of the assumptions. In order to achieve this, the design of the NES composes of four tracks with identical shape. The structure of the track NES is shown in Fig. 3(a) and Fig. 3(b) shows the NES is fixed on the roof of the test frame by bolted connections.



(a) Stucture of the track NES (b) Instllation of the track NES

Fig. 3 – Structure of NES and installation on the frame

To verify different damping effect on vibration control performance of the NES , two set of wheels are employed in the test. One is steel wheels which were made of Q345 steel. And the other type is rubber wheels ,of which middle part are covered by rubber material.

(a) Steel wheel (b) Rubber wheel

Fig. 4 – Steel and rubber wheels of the NES

3.3 Instrumentation for shake table tests

As shown in Fig. 2(a), a large-scale shake table employed for the experiments of this paper is a earthquake simulator which locates at State Key Laboratory of Disaster Reduction in Civil Engineering of Tongji university and are produced by MTS. The shake table has dimension of $4m \times 4m$. And, the operation frequency range of the shake table is 0.1~50 Hz when the preload dose not exceed 25 tons.

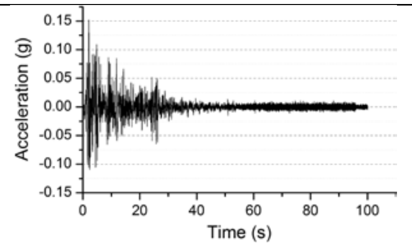
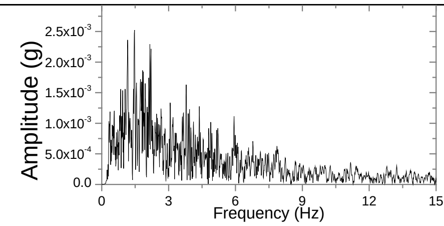
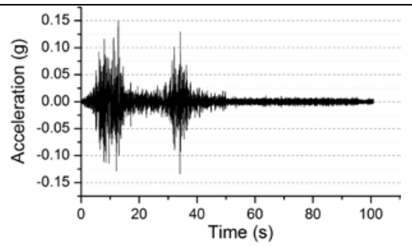
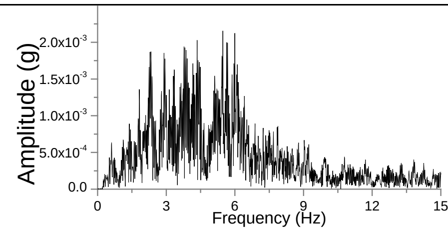
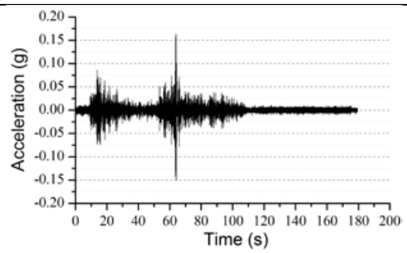
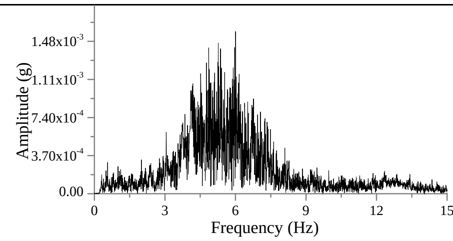
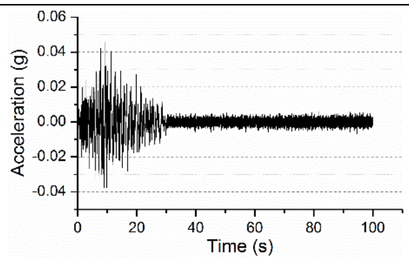
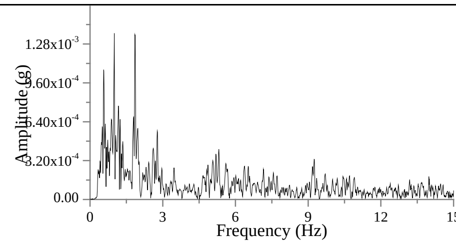
IEPE accelerometers and wire displacement sensors are attached to the test frame to collect dynamic responses. There are two accelerometers and two displacement sensers on each floor, and the measuring direction of these sensors paralleles to the weak axis of the test frame. The NES is attached with a IEPE accelerometers to monitor its acceleration response. Unaxial strain gages are attached to each columns of the first floor to measure the strain of the column during the test. A data aquisition system with 128 signal channels manufacted by MTS is employed to record the singles of these sesors. In the experiments, the sampling frequency of each signal channel is 256 Hz.

3.4 Excitations for shake table test

Four earthquake time histories of ground motions are selected as the input data in the shake table test to evaluate the capability of the NES. Three of the earthquake waves are natural seismic records including El Centro (1940,NS), Wenchuan (2008, NS) and Japan 311 (2011, NS). The fourth earthquake wave is Shanghai design code specified artificial earthquake accelerogram (SHW2). The time interval of the earthquake time history is 0.02s and all the seismic time histroy of accelerations are excited in only one direction of the the test frame. The peak value of the accelerations increases from 0.05g to 0.15g. Time histories and response spectrums of all earthquake waves employed in the experiments are shown in Table. (1). The responses of test frame without the

NES under 0.1g SHW2 wave is so large that it may result in the collapse. Hence, the experiment of SHW2 wave is only conducted at 0.05g PGA.

Table 1 – Time history and response spectrum of the earthquake waves

Seismic name	Acceleration time history	Acceleration response spectrum
El Centro 0.15g		
Wen Chuan 0.15g		
Japan 311 0.15g		
SHW2 0.05g		

4. Results and discussion

4.1 Peak response

Peak responses at roof of the test frame are selected to evaluate the performance of the NES. Maximum displacement responses (X_5) at roof of the test frame and root mean square value (σ_5) are shown in Table2 for all experiments. Similar to the linear vibration absorbers, such as tuned mass dampers, the NES has delayed behavior in vibration attenuating. In this regard, it's incomplete that only maximum displacement are employed in the evaluation of the NES. The overall vibration characteristics of the frame in the entire time history should be inspected. The r.m.s value of the displacement is an index of the frame's vibration energy, and it can be used as a description of overall performance of the NES. To directly represent the vibration attenuating capability of the NES, reduction effect is defined by the following expression:

$$re = \frac{r - r_n}{r} \times 100\%$$

where re is the reduction effect, r is response of the frame without the NES and r_n is response of the frame with the NES.

It can be seen from Table 2 and Table 3 that: (1) the peak displacement responses of the frame with the NES is smaller than that of the frame without the NES; (2) the vibration reduction performance of the r.m.s is more remarkable than that of the maximum displacement; (3) in the displacement control, the NES is not sensitive to its own damping feature; (4) similar to the case of displacement control, the NES has better performance in acceleration r.m.s value reduction than acceleration peak values; (5) the NES is sensitive to the damping factor in reducing the peak values of the acceleration on the roof.

Table 2 – Maximum displacement and their r.m.s values (mm) at the roof of the frame

Seismic input	PGA (g)	Max displacement and r.m.s values (mm)						Reduction effect (%)			
		Without NES		Rubber wheel NES		Steel wheel NES		Rubber wheel NES		Steel wheel NES	
		X_5	σ_5	X_5	σ_5	X_5	σ_5	X_5	σ_5	X_5	σ_5
El Centro	0.05	43.9	13.42	39.04	5.54	39.25	5.52	11.1%	58.7%	10.6%	58.9%
	0.1	102.1	30.5	82.94	12.47	83.48	12.75	18.8%	59.1%	18.2%	58.2%
	0.15	153.2	44.06	138.1	22.87	134.7	23.68	9.9%	48.1%	12.1%	46.3%
Wen Chuan	0.05	9.45	2	7.23	1.51	7.53	1.53	23.5%	24.5%	20.3%	23.5%
	0.1	32.8	6.56	16.47	3.1	18.43	4.77	49.8%	52.7%	43.8%	27.3%
	0.15	29.14	6.72	24.3	5.33	24.68	5.46	16.6%	20.7%	15.3%	18.8%
Japan 311	0.05	7.1	1.4	5.6	0.96	5.58	0.91	21.1%	31.4%	21.4%	35%
	0.1	13.89	4.19	11.6	2.08	11.65	1.96	16.5%	50.4%	16.1%	53.2%
	0.15	24.3	9.31	20.65	3.33	20.2	3.89	15%	64.2%	16.9%	58.2%
SHW2	0.05	91.5	42	59.6	9.2	60.4	8.41	34.9%	78.1%	34%	80%

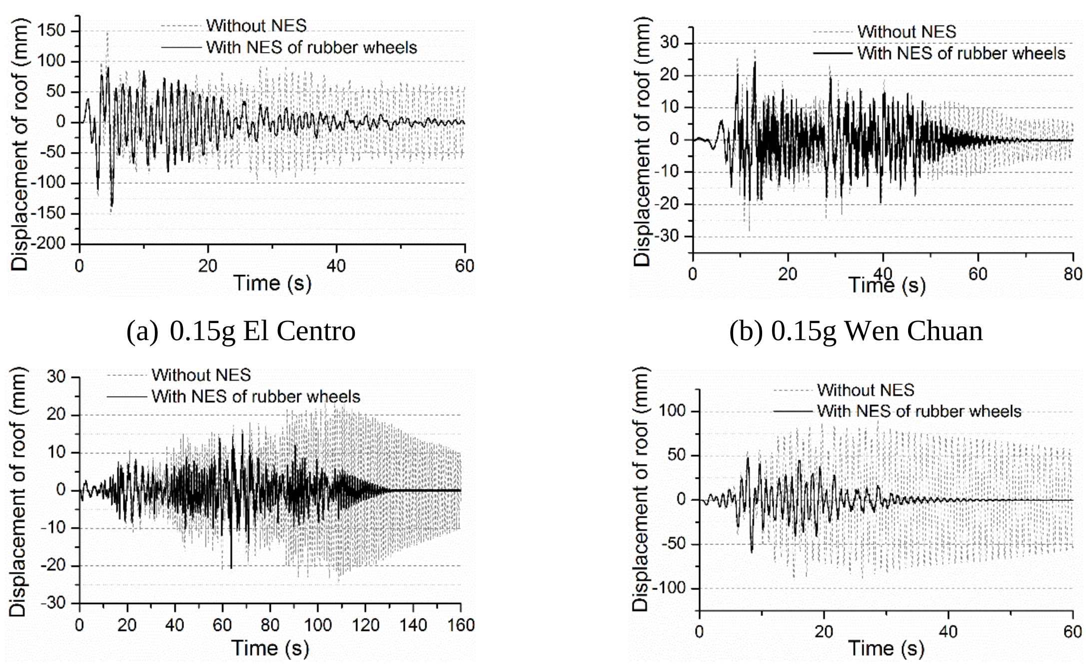
Table 3–Maximum acceleration and acceleration r.m.s (g) values at the roof of the frame

Seismic input	PGA(g)	Max acceleration and r.m.s values (g)						Reduction effect (%)			
		Without NES		Rubber Wheel NES		Steel wheel NES		Rubber Wheel NES		Steel wheel NES	
		A_5	σ_{a5}	A_5	σ_{a5}	A_5	σ_{a5}	A_5	σ_{a5}	A_5	σ_{a5}
El Centro	0.05	0.22	0.057	0.13	0.02	0.14	0.02	41%	65%	36.4%	65%
	0.1	0.5	0.13	0.3	0.054	0.48	0.063	40%	58.5%	4%	51.5%
	0.15	0.744	0.186	0.607	0.1	0.73	0.114	18.4%	46.2%	1.9%	38.7%
Wen Chuan	0.05	0.21	0.046	0.151	0.02	0.146	0.023	28.1%	56.5%	30.5%	50%
	0.1	0.455	0.106	0.34	0.061	0.39	0.085	25.3%	42.5%	14.3%	19.8%

	0.15	0.674	0.159	0.53	0.11	0.55	0.14	21.4%	30.8%	18.4%	11.9%
Japan 311	0.05	0.12	0.026	0.099	0.011	0.1	0.011	17.5%	57.7%	16.7%	57.7%
	0.1	0.27	0.063	0.24	0.03	0.255	0.033	11.1%	52.4%	5.6%	47.6%
	0.15	0.46	0.112	0.45	0.058	0.458	0.067	2.2%	48.2%	0.4%	40.2%
SHW2	0.05	0.41	0.17	0.32	0.048	0.37	0.048	22%	71.8%	9.8%	71.8%

4.2 Time history

Fig.4 shows the time history of the displacement at roof of the frame with the NES under 0.15g El Centro, Wen Chuan, Japan 311 and 0.05g SHW2 wave. From the diagrams of displacement time history, not only the peak values are clearly decreased, but also the displacement of the whole time history are attenuated quickly after the amplitude of the shake table excitation decreased. The displacements at roof of the frame with the NES are close to that of the frame without the NES at initial stage of each time history. The delay in vibration attenuating of the NES is similar to the TMD (Turned Mass Damper). The reason for the delay is that amplitude of relative motions between the NES and the frame is relative small. Although energy of the frame is partly transferred to the NES, kinetic energy of the NES remains at relative low level in the initial stage of each time history and the NES cannot dissipated much energy. With increase of time or earthquake amplitude, relative motions between the NES and the frame becomes faster and faster and the energy dissipated by the NES increased simultaneously for the damping force is proportional to the speed. In addition, the reacting force on the frame increases with damping force of the NES. Based on these factors, the more energy is transferred to the NES the better vibration controlling performance it shows.



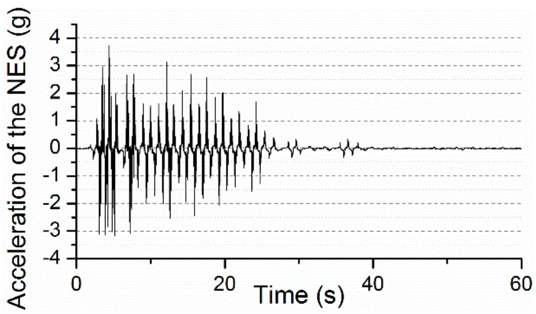
(a) 0.15g El Centro (b) 0.15g Wen Chuan

(c) 0.15g Japan 311 (d) 0.05g SHW2

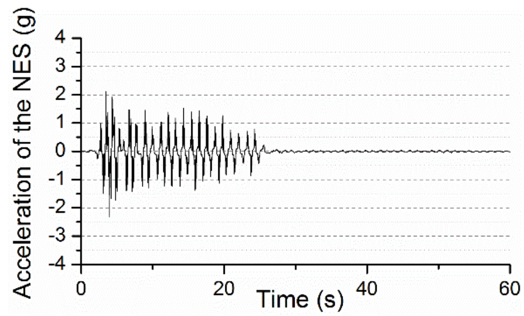
Fig. 5 – Displacement time history at roof of the frame

4.4 The effect of damping

From the displacement data of the experiment results listed in Table 2, the vibration attenuating performance of the NES with rubber wheels is close to that of the NES with steel wheels. However, the NES with two types of wheel shows different capability in the reduction effect of acceleration peak value at the roof. Obviously, the performance difference resulte from different damping feature of wheels. In the experiment data listed in Table 3, the differences are larger between the experiments of 0.15g El Centro and 0.05g SHW2 wave. To analyze the phenomenon and make comparison between different acceleration time histories of the NES with different wheels, acceleration time history at roof of the frame and the NES acceleration time history in the experiment of 0.15g El Centro wave are selected. These two time history are shown in Fig. 6 and Fig.7. The acceleration peak value of the NES with rubber wheels is about half of that of the NES with steel wheels, but its performance is better. In the interaction of the NES and the frame, rubber wheels not only dissipates more energy but also provids larger damping force. In the case of the NES of steel wheels, the interaction force makes the acceleration of the NES larger in the absence of sufficient damping force.

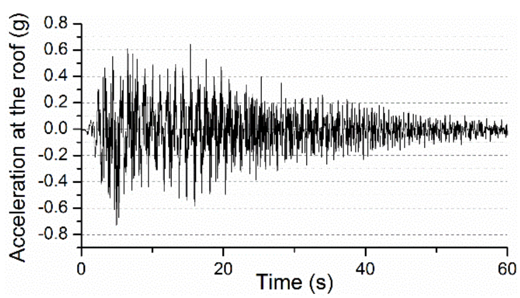


a) NES with steel wheels

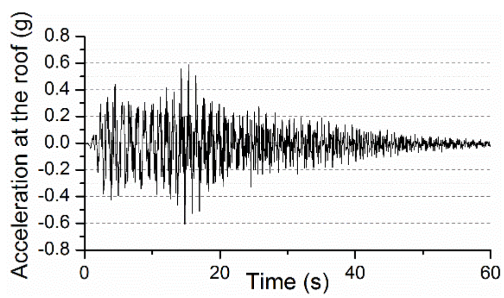


(b) NES with rubber wheels

Fig. 6–Comparison of acceleration time history of NES under 0.15g El Centro wave



(a) Frame with the NES of steel wheels



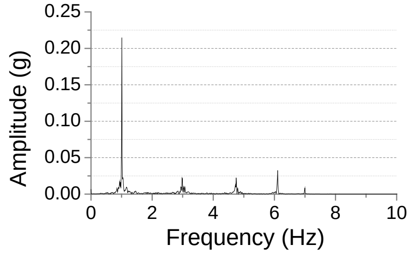
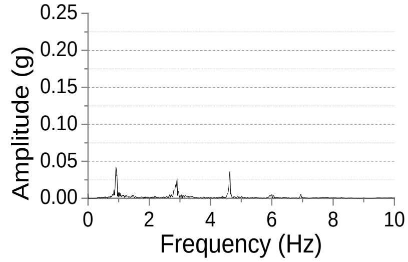
(b) Frame with the NES of rubber wheels

Fig.7–Comparison of displacement time history at roof of the frame under 0.15g El Centro wave

4.4 Frequency domain analysis

The vibration attenuating capability of the NES is analyzed in time domain on the above content. To evaluate its broadband vibration control feature in frequency domain, Fourier transforms are conducted on the acceleration records of the NES and the roof. In the design and numerical optimization on the shape parameters of the NES, the goal of the optimization is to control the first mode vibration of the frame. As Table 1 shows, the spectrums composition of El Centro and SHW2 wave mainly consists of frequency around 1 Hz, the spectrum of Japan 311 wave is mainly from 3 to 5 Hz, the frequency of Wen Chuan wave is mainly concentrated in the range of 2 to 6 Hz. For the analysis in frequency domain, response spectrums of the acceleration at the roof of the frame without the control of the NES and controlled by the NES are listed in Table 4 respectively.

Table 4–Response Spectrum of the frame

<table>
<tr><th>waves</th><th>Frame without the NES</th><th>Frame with the NES of rubber wheels</th></tr>
<tr><td>El Centro
0.15g</td><td>
</td><td>
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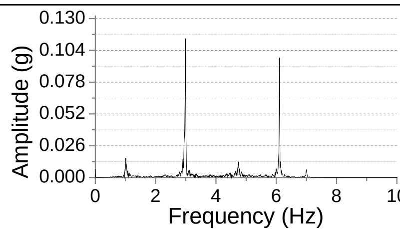
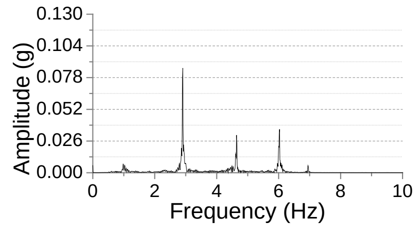
<tr><td>Wen Chuan
0.15g</td><td>
</td><td>
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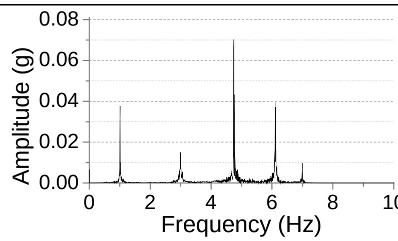
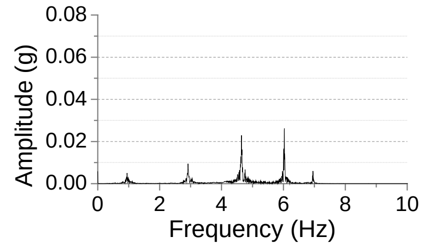
0.15g</td><td>
</td><td>
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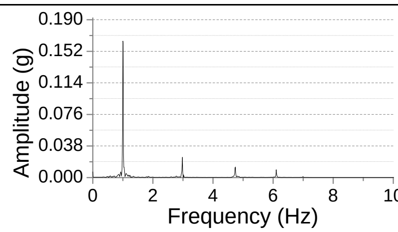
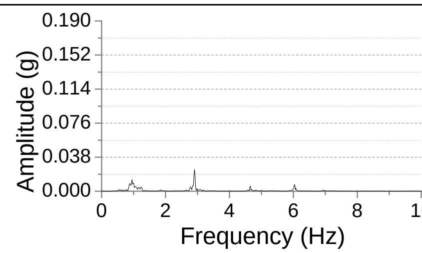
0.05g</td><td>
</td><td>
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</table>

The reason lead to the different performance of the NES is the installation position and the dynamic response features of the frame. Mounting position of the NES is the roof of the frame which has the maximum displacement at the first mode shape. When the vibration of the frame is mainly composed of the first mode, interactions between the NES and the roof is more intense than the other vibration mode. The stronger interactions are able to transfer more vibration energy from the frame to the NES, and the performance of the NES will be better. In the experiment of 0.15g Wen Chuan wave, the primary vibration frequency of the frame is the second mode frequency, and the position with maximum displacement of this mode shape is not roof. So, the performance of the NES is relative more poorly than that in the experiment of El Centro wave.

5. Conclusions

This paper presents experimental results of a track type NES system device for seismic mitigation on a large scale structural model. Three historic and one artificial earthquake seismic are implemented by a shake table. All the seismic waves have different spectrum and are scaled to different level in the experiments. The experimental results obtained from the tests shows that the NES not only reduces peak values of the displacements and the acceleration but also their r.m.s values, and the reduction effect of the NES on r.m.s values is the better than peak values.

Comparisons of the testing records of the NES with two types of wheels clearly indicates that the damping feature of the NES has an effect on its performance in vibration attenuating. The NES demonstrated it is capable of working in broader frequency range based on the analysis results in frequency domain. But it should be noted that the NES with established shape of the tracks still has main working frequency range which should be assigned in design procedure, because the modal shapes of the structure and the mounting position of the NES has an influence on the process of energy transferring.

According to the experimental and analysis results, the NES which has a low mass ratio to the structure can effectively reduce the response of a light damped the structure under the seismic excitations.

6. Acknowledgements

The financial support of State Key Laboratory of Disaster Reduction in Civil Engineering through grant SLDRCE14-A-007 and National Natural Science Foundation of China (NSFC, Grant No. 91315301-4 and 51478361) are gratefully acknowledged. The findings, observations and conclusions in this paper are those of the authors and do not necessarily represent those of the sponsors.

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