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REFINEMENT OF HOUSNER'S MODEL AND ITS APPLICATION FOR THE OVERTURNING ACCELERATION SPECTRA

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Abstract

Housner published his classical paper in 1963 for the calculation of a rocking block. He assumed that impact occurs at the corners of the block and identical angular momentum before and after the impact. Based on these assumptions he derived expressions for both the energy loss, and the change in velocity.

This model is widely used for modeling of stone and masonry columns and arches. Researchers also developed the so called 'overturning acceleration spectra' for the verification of blocks, columns and arches subjected to earthquakes. In all these cases the basic element of the calculation is Housner's classical model. Note, however that experiments – as published in several papers – show lower energy loss during impact than it is predicted by Housner's model. As a consequence, the overturning acceleration spectra based on Housner's model may be unsafe.

In this paper we discuss the effect of the overprediction of the energy loss of Housner's model. Furthermore, the effect of the shape of the applied acceleration is investigated.

Keywords: Housner's model, overturning spectra, rocking block

1. Introduction

Modelling of masonry and stone columns and arches must include the possible openings and closings of the cracks between the elements which require the use of an impact model. In most of the cases Housner's model is applied.

Housner published his classical paper more than five decades ago [1], in which he presented a simple model for the rocking block (Fig. 1). He investigated a block which rotates around corner *A*, then – when the block reaches the vertical position – impact occurs, and the block rotates further around corner *B*. Assuming identical angular momentum on corner B before and after the impact (Fig. 1c), he arrived at the following expression for the angular velocity of the block:

$$\omega_{\mathrm{a}} = \mu_{\mathrm{Hous}}\omega_{\mathrm{b}}, \quad \mu_{\mathrm{Hous}} = \frac{2h^2-b^2}{2h^2+2b^2}, \tag{1}$$

where ω_{b} and ω_{a} are the angular velocities before and after rocking, h and b are the dimensions of the block, μ is the angular velocity ratio.

The rocking block was investigated experimentally by several researchers ([2], [3], [4] and [5], [6]). In almost every case it was found that in the experiments the energy loss (and the decrease in angular velocity) is smaller than the one predicted by Housner's model.

In spite of the inaccuracies, Housner's model is widely applied to determine overturning acceleration spectra or stability maps to analyze the stability of a single rocking block [1], [3], [7]–[15]. [16] extended this for the investigation of arches and [17], [18] defined stability maps for impulse-ground motions. Housner's model was also extended to investigate non-symmetric monolith blocks [8], [19] and two [20], [21] or multi degree of freedom structures [22].

In this paper we focus on the investigation of a single block, the analysis of columns and arches will be treated in companion papers. Since the overturning acceleration spectra (OAS) is calculated on the basis of Housner's model – which over predicts the energy loss – its usage is unconservative. Furthermore, the OAS is always calculated from an impulse of a given shape, however the effect of the shape is not investigated in a systematic way.

2. Problem statement

A simple rigid block of arbitrary aspect ratios (b/h) is considered which is subjected to a pulse-like ground motion shown in Fig. 2. The OAS curves are calculated taking into account the impact due to Housners's model, and also, as an approximation on the safe side, assuming zero energy dissipation during impact.

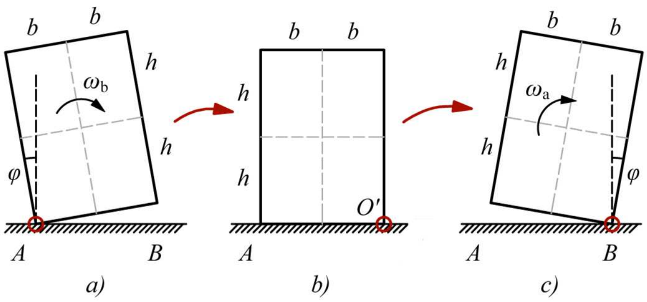

Fig. 1 – Rocking block in Housner's model

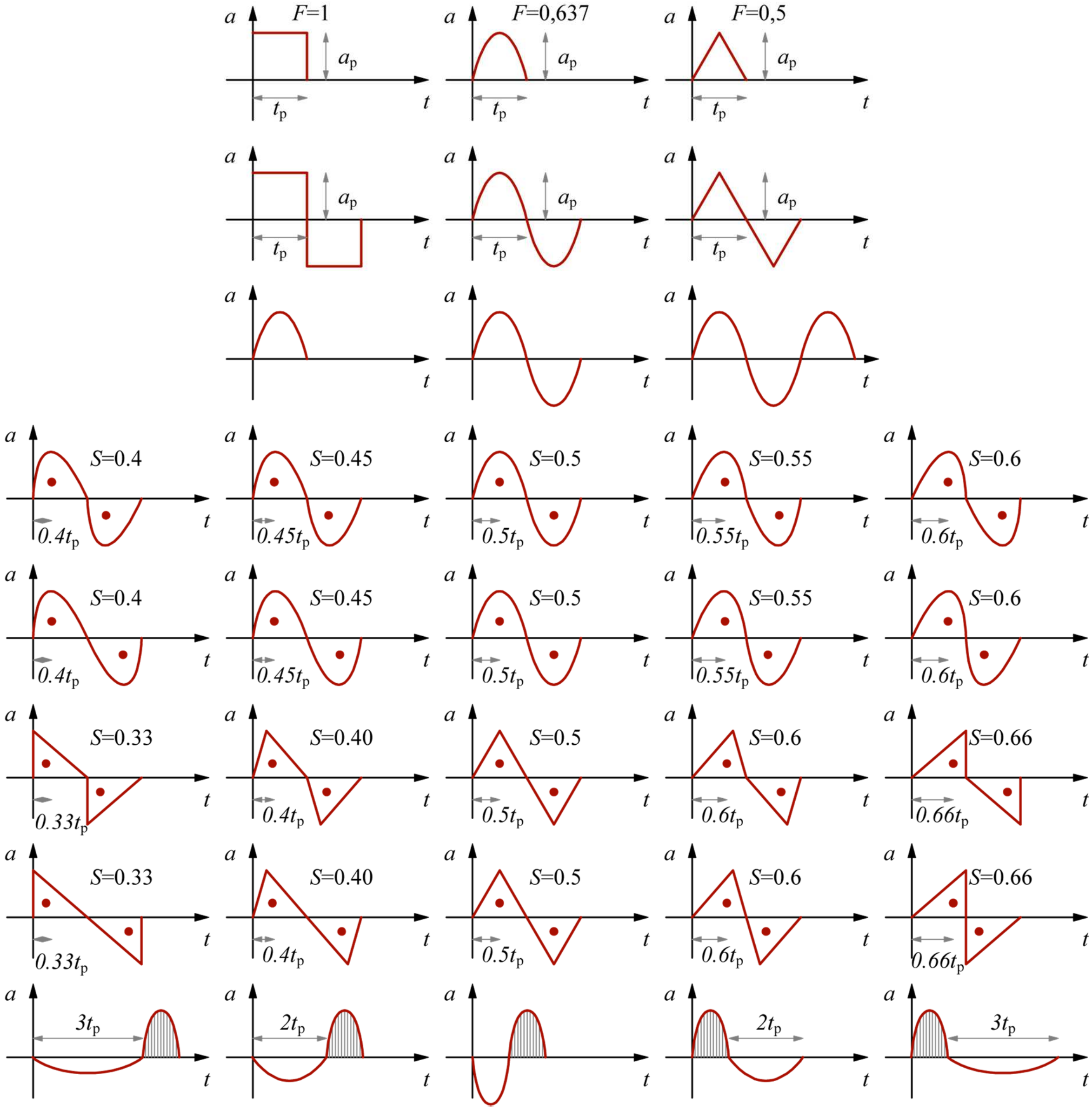

Fig. 2 – The pulse-like ground motions considered in the article. a_p is the maximum acceleration and t_p is the corresponding duration. Fullness is defined as $F = \int_0^{t_p} a_p dt / a_p t_p$, while skewness as $S = \int_0^{t_p} t a_p dt / \int_0^{t_p} a_p dt$. (The skewness is 0.5 in every case, where it is not given.) The consecutive impulses have the same shape (with opposite sign) in every case except in the last row.

3. Model

A model was developed which is capable to calculate the rocking motion of a column made of rigid blocks. In this paper the motion of the monolithic column i.e. a single block is investigated. During the motion, the geometry is updated, hence the second order effects are taken into account. The only considered damping effect is the one which occurs during the impact. It is assumed that the motion is 2D, there is no sliding between the ground and the element, and the strength of the masonry is not investigated.

4. The shape of the overturning acceleration spectra

The unnormalized OAS curve is a plot in the a_p-t_p coordinate system, where a_p is the maximum acceleration of the given impulse, and t_p is the duration of this impact. The curve separates the safe and unsafe areas, i.e. where overturning does not or does occur. Three examples are shown in Fig. 3, the unsafe areas are shaded, while the safe areas are white. Below $a_{p,min}$ defined as

$$a_{p,min} = gb/h \tag{2}$$

no rocking happens, hence it is always safe.

When there is only one impulse (Fig. 3a), the OAS is monotonic: higher a_p and longer pulse more likely cause overturning than lower a_p or shorter t_p. According to [14], this boundary is called Mode 2 failure.

If there are two consecutive impulses with different signs (Fig. 3b) there is a narrow safe area within the unsafe zone. According to [14], this boundary is called Mode 1 failure. These zones were investigated by several researchers ([11], [12], [14], [15]).

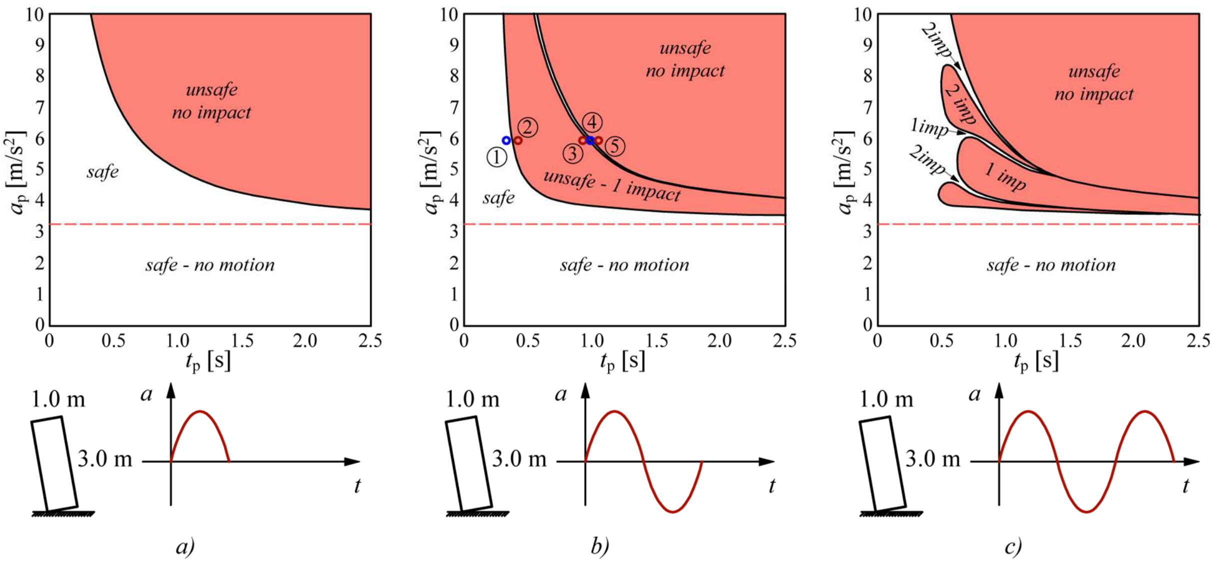

Fig. 3 – Typical overturning acceleration spectra for ground motion with 1, 2 and 3 consecutive impulses

In Fig. 4 the motion of a block is given for 5 different impulses defined by number 1 to 5 in Fig. 3b. In all the cases the maximum accelerations are identical, however the lengths of the impulses are different.

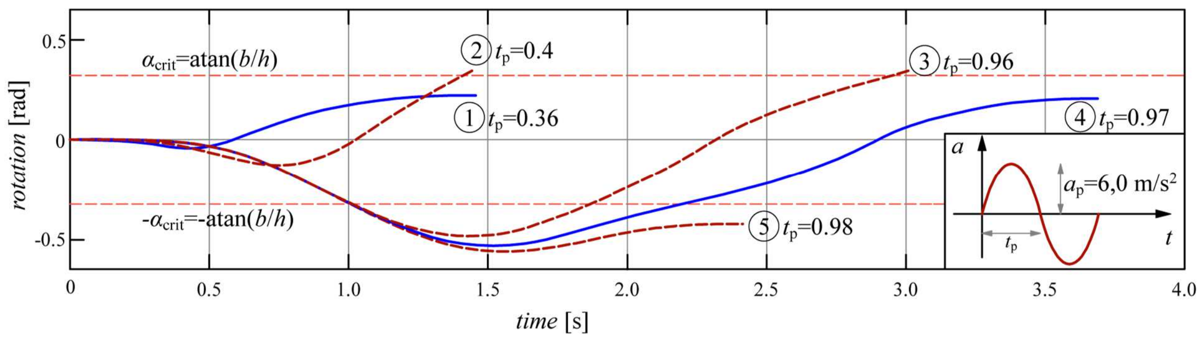

Fig. 4 – Motion of the rocking block for different impulses

When there are three consecutive impulses even more narrow areas or “bays” may occur (Fig. 3c). Since these narrow areas has no practical importance, in the following sections we will investigate only the outer envelope of the OAS curves.

5. Results

In the analysis the following parameters were investigated

- slenderness ration h/b=3, 5, 8, 10
- shape of the pulse: rectangular, sinusoidal, triangular (Fig. 2)
- skewness of the shape of the pulse
- parameters of the second impulse: $a_2=a_1$; $a_2=a_1/2$; $a_2=a_1/3$, $a_2=2a_1$, $a_2=3a_1$ while $\int_0^{t_1} a_1 dt = \int_0^{t_2} a_2 dt$ and $a_2=0$ (see the last row of Fig. 2)
- the parameter range of each stability map is a_p= 0 - 10 m/s^2, t_p= 0 - 2.5 s

Note that larger elements move more slowly than smaller ones, hence there is a size effect ([1], [8], [11], [15], [20]). However, if we normalize the horizontal axis by the square root of the size, we obtain size independent results. In the following plots the vertical axis is normalized by $a_{p,min}$ (Eq. 2) while the horizontal axis by $\sqrt{2b/g}$, where g is the acceleration of gravity and $2b$ is the width of the block.

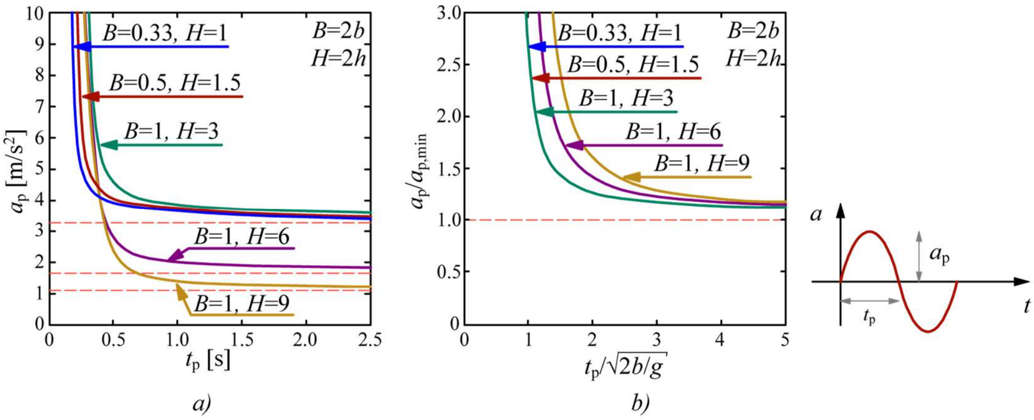

Fig. 5 – The unnormalized and the normalized overturning acceleration spectra of blocks with different slenderness and size

The effect of energy dissipation is shown in Fig. 6. The difference between Housner’s model and the case, when there is no energy loss during impact can be 16%. In reality, since the reported energy loss of the rocking block is roughly the half of that of Housner’s model, the difference can be around 8-10%.

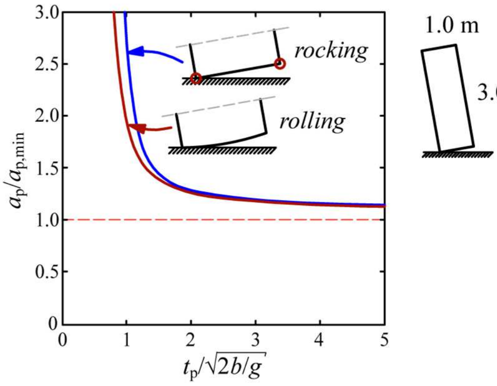

Fig. 6 – The envelope of the overturning acceleration spectras based on Housner's model and the model when no energy dissipation is considered

The effect of impulse shape is shown in Fig. 7a and Fig. 7b. We presented the results both as a function of the length of the pulse and as a function of the impulse defined as

$$I = \int_0^{t_p} a\, dt. \tag{3}$$

In the first case the rectangular shape results smaller safe areas, then the other two, and the curve due to the sinusoidal pulse is between the other two. If we plot the results as a function of the impulse *I*, the three curves intersect. It can be seen that there is a minimum value of impact, which – at a given value of acceleration – is capable to turn over the block.

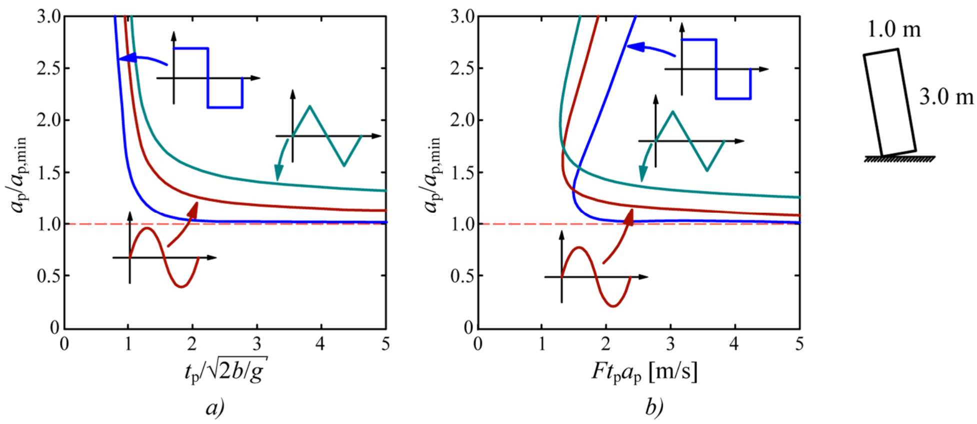

Fig. 7 – The effect of the impulse shape

The skewness of the shape is investigated in Fig. 8 and Fig. 9. The "fullness" and "skewness" are defined as

$$F = \frac{I}{a_p t_p},\;\; S = \frac{1}{I t_p}\int_0^{t_p} t a_p dt. \tag{4}$$

In Fig. 8 skewed sinusoidal shapes are investigated, where the fullness are identical (F=0.637), while the skewnesses are S=0.4, 0.45, 0.5, 0.55, 0.6. In Fig. 9 a triangular pulse shape is investigated (F=0.5), the skewnesses are S=0.33, 0.4, 0.5, 0.6, 0.66.

It is an important observation that the asymmetry of the pulse has an important effect even if the fullness, a_p, t_p, and I are identical.

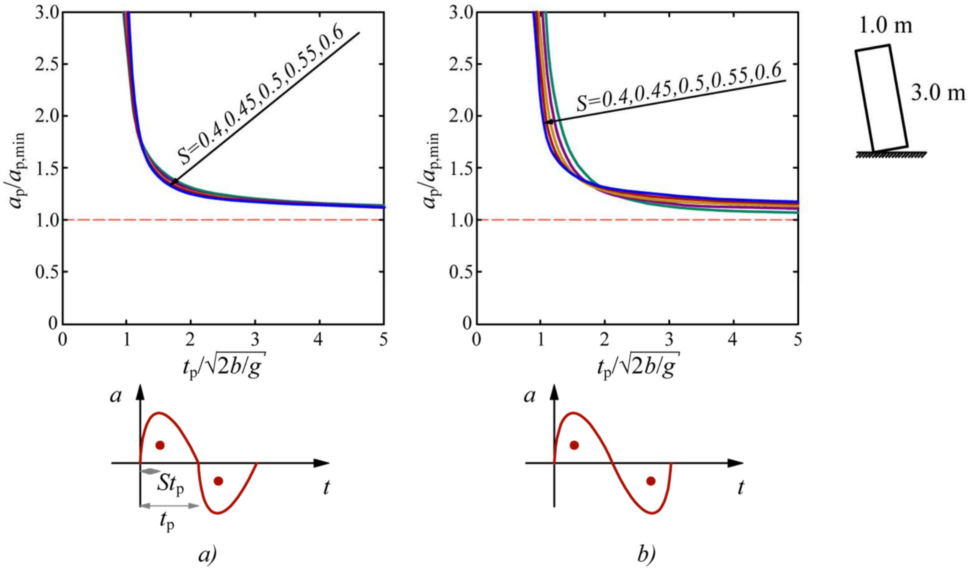

Fig. 8 – The effect of the symmetry and asymmetry of the skewness (modified sinusoidal shape)

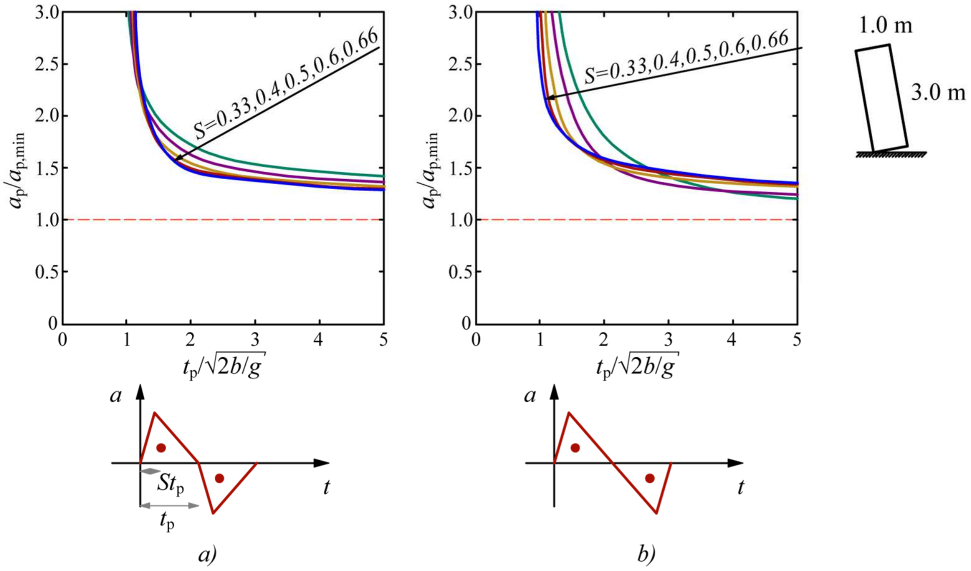

Fig. 9 – The effect of the symmetry and asymmetry of the skewness (triangular shape)

In Fig. 10 the effect of the shape of the impact (before or after the main impact) is investigated. In every case the secondary impulse has the same impulse as the main impulse, but the secondary acceleration may be smaller, however with a longer duration. The results in Fig. 10 shows that these effects are important and an envelope can be recommended for practical applications.

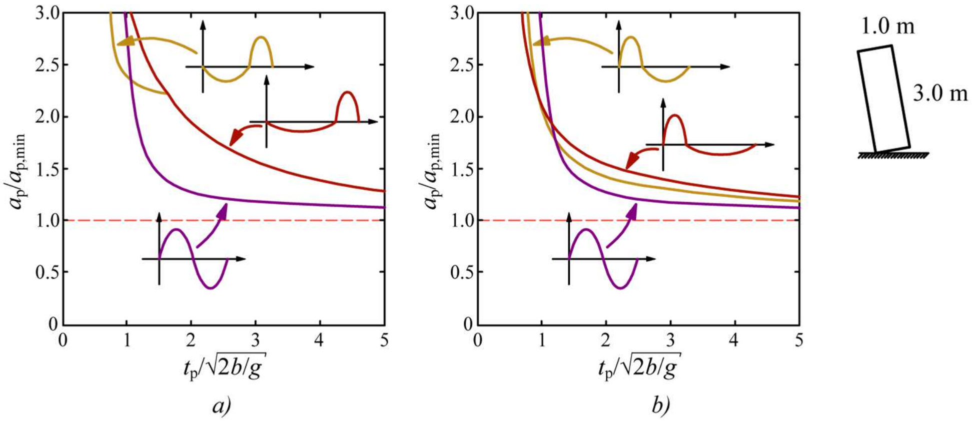

Fig. 10 – The effect of the secondary impulse

6. Discussion

In this paper the overturning acceleration spectra curves of single blocks were determined. The effects of the shapes of the impulse and the effect of Housner's impact model were investigated. It was found that taking the classical Housner model into account the results can be around 8-10% on the unsafe side. We also showed that the skewness of the pulse and the shape of the secondary pulse significantly affect the results. Since real earthquakes the pulse is generally not symmetrical, and the main pulses are followed by secondary pulses, these effects must be taken into account.

7. Acknowledgement

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