BAYESIAN UPDATING OF DAMAGED BUILDING DISTRIBUTION IN POST-EARTHQUAKE ASSESSMENT

A. Kusaka\(^{(1),(2)}\), H. Nakamura\(^{(3)}\) and H. Fujiwara\(^{(4)}\), H. Okano\(^{(5)}\)

\(^{(1)}\) Manager, Kobori Research Complex Inc., kusakaa@kobori-takken.co.jp
\(^{(2)}\) Visiting Researcher, National Research Institute for Earth Science and Disaster Resilience
\(^{(3)}\) Senior Researcher, National Research Institute for Earth Science and Disaster Resilience
\(^{(4)}\) Director, National Research Institute for Earth Science and Disaster Resilience
\(^{(5)}\) Professor, Chiba University

Abstract

This paper proposes a method to have a whole picture of earthquake damage rapidly and accurately as a post-earthquake damage assessment.

In Japan, strong ground motion observation station networks, such as K-NET and KiK-net are in place across the country. With the networks, the observed records are collected by information and communication technology as soon as they are recorded. Japan Real-time information System for earthquake (J-RISQ) has been developed, which estimates the strong motion distribution by spatial interpolating the collected records in a very short time. Given fragility functions, which relate ground motion intensity and damage probability, and building statistics are prepared, distribution of damaged buildings is able to be estimated immediately. Such estimation will help governments and private sectors to grasp the whole damage situation and to make their disaster management more effective. However, it should be noted that the estimation includes errors those associated with interpolating strong ground motion observation and those in evaluating fragility functions.

Meanwhile, actual damage will be gradually informed, although it may be for limited areas, because local governments etc. start to collect and report it as a part of disaster response activities. Then, by using such information on actual damage, it is possible to make the estimation more realistic even for the region where damage information is not reported yet.

This paper proposes to apply Bayesian inference for merging information described above. The procedure of the proposed method is summarized as following: the target area, where might be subject to strong ground motion, is divided into geography mesh. In each mesh, safety margin, which is difference between seismic load effect and resistance of building, is evaluated for each category of building and each damage state by use of the estimated strong ground motion distribution, and the parameters of the prepared fragility functions. Here, the categories of building are determined by the types of building structure, seismic design codes etc. Damage states are classified in such as ‘major damage’ or ‘heavy damage’ etc. Then, the errors in evaluating safety margins are modeled as normal distributed random variables. The damage probability is evaluated as the probability that safety margin is negative with consideration of the errors. The parameters of the normal distributed variables are also dealt as random variables, whose prior distributions are supposed by use of the accuracy of the estimated ground motion distribution and the fragility functions. Once, the actual number of buildings in each damage state is available for some parts of the target area, it is used as the ‘observation’ in Bayesian updating protocol to induce the posterior distribution of the parameters. Finally, the updated damage probability is calculated as the weighted expectation of damage probability with respect to the posterior distribution of the parameters.

As an illustrative example, a numerical study with past earthquake disaster records is presented so as to examine the effectiveness and characteristics of the presented method.

Keywords: Bayesian inference, damage management, post-earthquake assessment, damage distribution, uncertainty
1. Introduction

In Japan, strong ground motion observation station networks, such as K-NET and KiK-net are in place across the country. With the networks, the strong ground motion records are automatically collected by information and communication technology as soon as they are recorded. Japan Real-time information System for earthquake (J-RISQ) [1] has also been developed, which estimates the strong motion distribution by spatial interpolating the collected records in a very short time. Using the strong motion distribution as input data of fragility functions, which relate ground motion intensity and building damage probability, the distribution of damaged buildings can be estimated. Such estimation as we say the “immediate estimation” will help governments and private sectors to rapidly grasp the whole situation and to make their disaster management more effective.

However, it should be noted that the immediate estimation includes errors those caused by interpolating strong ground motion observation and those in evaluating fragility functions. Meanwhile, information on actual damage will be gradually available because local governments etc. start to collect and report it as a part of disaster response activities. Then, by using the actual damage information, it is possible to make the estimation more accurate even for the region where damage information is not reported yet.

Bayesian updating is one of well-known frameworks to correct estimating models with observation. A framework to update fragility functions is proposed for reinforce concrete columns [2]. In the framework, some random variables as correction terms are added to capacity models which are results of other research, and the parameters of probability distribution of the random variables are updated with new experimental data. Another approach [3] is proposed; it supposes that the disaster area is divided into some areas where statistical characteristics of input ground motion intensity and structural capacity are uniform, then the damage probability in each area, which is considered as the parameter of binomial distribution, is updated with the damage information in a part of the area. The former approach which describes damage process with structural models can more easily deal estimation errors by categorizing them based on structural viewpoint rather than the latter approach which updates damage probability directly. However, the former may need to identify not a few parameters, and may not work well especially at an early stage of disaster when limited data are available.

This paper presents a method to merge the information provided by damage inspections into the post-earthquake damage assessment to consider the problem mentioned above. The method uses the numbers of damaged buildings in limited areas as observation in Bayesian updating protocol, to update estimation error in safety margin, which is difference between load effect and resistance, of buildings all over the disaster area. Finally, as an illustrative example, a numerical simulation is shown that the method is applied to the data of the 2011 off the Pacific coast of Tohoku Earthquake.

2. Updating method for estimation error in numbers of damaged houses

2.1 Framework description

The framework supposed in this paper is that fragility functions and statistics of buildings are prepared all over the area and, that the distribution of $\hat{\delta}$, the estimation of seismic ground motion strength $\hat{S}$, is given in short time after an earthquake occurs. Using $\hat{\delta}$ with fragility functions, and statistics of buildings, the numbers of damaged buildings are estimated, and then they are updated with the actual numbers in limited areas, which are brought successively. Fig. 1 shows the overview of the presented method. It should be noted that the capital letters mean random variables in this paper.

The framework is still applicable for general post-earthquake damage assessments with fragility functions. The fragility functions by reference [4], which is shown in Fig.2, are considered in the following discussions because details of updating procedure such as what parameters and how many parameters should be updated depend on what fragility functions are used. The fragility functions are determined by normal distributions for seven categories by building years for wooden houses and for three categories for non-wooden houses. They use JMA (Japan Meteorological Agency) instrumental seismic intensity $I_s$ as explanatory variables. The degrees of
damage, “Heavy” and “Major” are referred to the reference [5]. In addition, although any types of buildings can be theoretically applicable, in the following, residential houses are supposed because of availability of statistics.

Now, let \( R \) denote the resistance of houses, which is expressed by the seismic intensity. Then, the probability that \( R < S \) means damage rate. When we consider a random variable, \( M = R - S \), the fragility functions are represented by normal distribution of mean \( \hat{R} \), the best estimate of the resistance of house, and same variance of \( M \).

Then, as shown by Eq. (1), \( M \) is supposed to be modeled as the summation of \( \hat{M} = \hat{R} - \hat{S} \), difference between the best estimates for \( R \) and \( S \), and normal variables, \( X_0 \) and \( X_k \) (\( k = 1, \ldots, n \)), which express estimation errors of \( M \).

\[
M = \hat{M} + X_0 + \sum_{k=1}^{n} X_k \tag{1}
\]

where \( X_0 \) denotes the average error in estimating \( M \) for all houses, and \( X_k \) denote residual errors which are assorted by attributes such as structural type, ground condition, area etc. \( X_k \) are to be updated in \( n_k \) groups.

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**Fig. 1 – Overview of Bayesian updating of damaged house distribution by use of post-earthquake inspection**

**Fig. 2 – Fragility functions to be considered [4]**
In order to reduce the parameters. That is, information during the period after a short time an earthquake occurrence, we choose only means to be updated in the minimum numbers required to correct the errors. It is considered with the actual disaster data of the 2011 off the Pacific coast of Tohoku Earthquake in the following section.

By replacing fragility functions with the probability of being negative in Eq. (1), the variables to be updated, \( \Theta \) shown in Fig. 1, are the parameters of probability distribution of random variables \( X_0 \) and \( X_k \). In general, to determine normal random variables, the parameters are mean, variance, and correlation coefficient. A past study [6] is presented for Bayesian updating all of the parameters above for fragility functions of electric building facilities. However, required information is increased so as to increase the parameters to be updated. From the viewpoint of supporting disaster management, in order to have stable results even with limited information during the period after a short time an earthquake occurrence, we choose only means to be updated in order to reduce the parameters. That is, \( \Theta \) is the following equation.

\[
\Theta = \left( M_0, M_{1,1}, \ldots, M_{1,n_1}, \ldots, M_{n,1}, \ldots, M_{n,n_n} \right)
\]  

where \( M_0 \) denotes mean of \( X_0 \), and \( M_{k,l} \) denotes mean of \( X_k \) for the \( l^{th} \) group \((l = 1, \ldots, n_k)\).

2.2 Assumptions

In order to formulate the problem, the followings are assumed.

1. The target area is geographically divided into \( n_d \) units of mesh. Each mesh is assigned to either of \( n_d \) districts \((n_d < n_a)\), and each district is assigned to either of \( n_r \) regions \((n_r < n_d)\).

2. For each mesh, information on ground amplification is provided as the Japan engineering geomorphologic classification (in the following, just written as the geomorphologic classification).

3. Houses are categorized in \( q \) categories from the viewpoint of earthquake resistant difference. For each category, fragility functions are modelled with normal distributions for major and heavy or more damage, respectively. The mean and standard deviation of the fragility functions are \( \hat{\theta}_{h,j}, \hat{\sigma}_{h,j} \) for major damage, and \( \hat{\theta}_{m,j}, \hat{\sigma}_{m,j} \) for heavy or more damage \((j = 1, \ldots, q)\). \( n_{h,ij} \), the number of houses of category \( j \) in mesh \( i \), is known \((i = 1, \ldots, n)\).

4. When an earthquake occurs, seismic intensity \( s_i \) for mesh \( i \) is estimated. Then, the number of damaged houses is estimated using \( s_i \) and the assumption above. It is called as the “immediate estimation.”

5. After the immediate estimation is given, the actual numbers of houses with major and heavy damage in each district are to be available one by one.

2.3 Bayesian updating procedure

In Bayesian updating framework, as shown in Eq. (3), the probability distribution of \( \Theta \) is updated from the prior distribution \( p(\Theta) \) to the posterior distribution \( f(\Theta) \) by multiplying likelihood function \( L(\Theta) \), which is proportional to the probability that the event is observed in the condition of \( \Theta = \Theta \), and \( p(\Theta) \).

\[
f(\Theta) = C \cdot L(\Theta) \cdot p(\Theta)
\]

where \( C \) denotes the constant of integration, which is determined to normalize \( f(\Theta) \), that is, to be a unit when it is integrated for all domain.

Likelihood function \( L(\Theta) \) for an event that the numbers of major and heavy or more damage of houses are reported as \( d_{h,m} \) and \( d_{m,m} \) respectively in \( n_{g,obs} \) districts is calculated as following, where suffix \( m \) means that the numbers are concerned with district \( m \) \((n_{g,obs} \geq 1, m = 1, \ldots, n_{g,obs})\). Using the model shown by Eq.(1), when \( \Theta \) is determined as \( \Theta, p_{h,ij}, \) major damage rate of house category \( j \) in mesh \( i \), and \( p_{m,ij} \), heavy or more damage rate of house category \( j \) in mesh \( i \), are given by Eq.(4) and Eq.(5) respectively.

\[
\frac{n_{h,ij}}{n_{d}} \leq \frac{d_{h,m}}{n_{d}} \leq \frac{d_{m,m}}{n_{d}}
\]

The problem is how to specify proper \( X_k \) and groups for each \( X_k \), in the minimum numbers required to correct the errors.
\[ p_{h,ij} = \Phi \left( -\frac{\hat{r}_{h,j} - \hat{s}_i + \mu_0 + \sum_{k=1}^{n} \mu_k}{\hat{a}_{h,j}} \right) \]  
\[ p_{m,ij} = \Phi \left( -\frac{\hat{r}_{m,j} - \hat{s}_i + \mu_0 + \sum_{k=1}^{n} \mu_k}{\hat{a}_{m,j}} \right) \]

where \( \mu_k \) is appropriately selected from \( \theta \) as the mesh \( i \) and house category \( j \), and \( \Phi(\cdot) \) denotes cumulative distribution function of standard normal distribution.

In order to calculate \( L(\theta) \) with \( p_{h,ij} \) and \( p_{m,ij} \), consider all of the cases that \( d_{h,m} \) (or \( d_{m,m} \)) equals to the summation of the numbers of houses estimated to suffer major (or heavy or more) damage for all categories in all meshes associated with district \( m \) and calculate the probabilities for the cases on the condition that \( \theta = \hat{\theta} \), then take the total summation of the probabilities. However, in general, possible combinations of the numbers of damage can be enormous, strict calculation may require too much resource and time to prepare on the disaster scene. Additionally, worthwhile information for disaster management would be rough prospect on relatively high damage rate, say, more than around 1\%, than the precision of probability evaluated with the tales of probability distribution. Thus, in this research, \( L(\theta) \) is evaluated with the approximation that uses \( \bar{p}_{h,m} \) and \( \bar{p}_{m,m} \), weighting average damage rates of the numbers of house in each mesh, which are calculated by Eq.(6) and Eq.(7) respectively.

\[ \bar{p}_{h,m} = \frac{\sum_{i=1}^{n_{a,m}} \sum_{j=1}^{q} n_{b,ij} p_{h,ij}}{\sum_{i=1}^{n_{a,m}} \sum_{j=1}^{q} n_{b,ij}} \]  
\[ \bar{p}_{m,m} = \frac{\sum_{i=1}^{n_{a,m}} \sum_{j=1}^{q} n_{b,ij} p_{m,ij}}{\sum_{i=1}^{n_{a,m}} \sum_{j=1}^{q} n_{b,ij}} \]

where \( n_{a,m} \) denotes the number of mesh in district \( m \).

\( L(\theta) \) is evaluated by Eq.(8) which is derived from multinomial distribution with the observation \( d_{h,m} \), \( d_{m,m} \), and \( n_{b,m} = \sum_{i=1}^{n_{a,m}} \sum_{j=1}^{q} n_{b,ij} \), the total number of houses in district \( m \), and the damage rates, \( \bar{p}_{h,m} \), and \( \bar{p}_{m,m} \), calculated on the condition of \( \theta \).

\[ L(\theta) \propto \prod_{m=1}^{n_{a,obs}} \bar{p}_{h,m}^{d_{h,m}} \cdot (\bar{p}_{m,m} - \bar{p}_{h,m})^{d_{m,m} - d_{h,m}} \cdot (1 - \bar{p}_{m,m})^{n_{b,m} - d_{m,m}} \]

\( \tilde{d}_{h,i}, \tilde{d}_{m,i} \), the posterior numbers of major and heavy or more damage in mesh \( i \) are calculated by Eq. (9) and Eq. (10), respectively.

\[ \tilde{d}_{h,i} = \sum_{j=1}^{q} n_{b,ij} \cdot \int p_{h,ij}(\hat{r}_{h,j}, \hat{s}_i, \theta) f(\theta) d\theta \]  
\[ \tilde{d}_{m,i} = \sum_{j=1}^{q} n_{b,ij} \cdot \int p_{m,ij}(\hat{r}_{m,j}, \hat{s}_i, \theta) f(\theta) d\theta \]
where $p_{h_{ij}}(\hat{r}_{h_{ij}}, \hat{s}_{ij}, \theta)$, and $p_{m_{ij}}(\hat{r}_{m_{ij}}, \hat{s}_{ij}, \theta)$ are those calculated by Eq.(4) and Eq.(5), and explicitly express the dependence on $\theta$.

The domain of the integrations in Eq. (9) and Eq. (10) are for all domain of $\theta$. Their integral degrees are the same as degree of $\theta$, and are usually high enough to have numerical difficulty in computation. Thus, this research applies Metropolis-Hastings algorithm [7], a representative technique of MCMC (Markov Chain Monte Carlo) method, to directly generate samples of $\theta$ according to the probability distribution by Eq.(3) and calculate $\bar{d}_{h,t}$ and $\bar{d}_{m,t}$ with the samples.

3. Illustrative example

3.1 Description of example

This section shows an illustrative example, for the purpose of examining the feature of the presented method. The example is an application for the data of the 2011 off the Pacific coast of Tohoku Earthquake ($M_w$ 9.0), which occurred on March 11, 2011. In the disaster, more than 400,000 houses were suffered from heavy or more damage, and more than 18,000 persons were killed or missed, and not a few of them were by tsunami. The presented method is for house damage caused by strong ground motion, so thus to avoid the influence of tsunami, the applied area is limited for non-coastal area of three prefectures (Iwate, Miyagi and Fukushima) in Tohoku region.

A map of Tohoku region of Japan is shown in Fig. 3. In the figure, seismic intensity distribution is also shown. The distribution is obtained by interpolating the records of by K-NET, KiK-net, and those at strong ground motion observation stations by JMA and local governments. The interpolating procedure is; seismic intensities $I_s$ at observation stations are calculated from the time histories recorded at ground surface. They are translated to those at engineering bedrock by use of AVS30 (average shear-wave velocity in the upper 30m), then, are interpolated at the center points of 250m mesh with considering the average trend of attenuation. Finally, the interpolated values are put back to those at ground surface.

The distribution of $I_s$, input data of fragility functions, can be translated to damage rates of houses. The immediate estimates of the numbers of damaged houses are calculated with the damage rates and the house statistics based on the Housing and Land Survey of 2008 and the Population Census of 2005. The estimated numbers are summed up by cities, towns and villages and shown in Table 1 in comparison with the facts, the numbers in the 151st report by FDMA (Fire and Disaster Management Agency) on March 9, 2015. The cities/towns/villages in Table 1 are ordered by that the numbers of major damage in the FDMA reports reached at 90% to 110% of the figures shown in Table 1.

The example shows that the presented method updates the immediate estimates with “information on actual damages”, the numbers of major damage and heavy or more damage in each city/town/village, in the order shown in Table 1. Here, city/town/village is corresponding to “district”, and prefecture to “region” as referred in subsection 2.2. For the location of city/town/village in Table 1, see Fig.7 with the numbers showing the order (it is referred in details in 3.3.)

3.2 Error model

For this example, $X_k$, the attributes of an error model by Eq. (1), are the followings; damage level (major/heavy), the geomorphologic classification, and prefecture.
Fig 3 - Distribution of estimated seismic intensity

Building year may be a good attribute because it shows which code the seismic design is accorded to. However, the difference in distribution of building year is so limited when it is summed up in city/town/village, that it is judged to be improper to differentiate estimating errors. The geomorphologic classification is selected because the geomorphologic classification data have been prepared for 250m mesh of the all over Japan, and it is expected that the continuity of geographic characteristics can alternatively express the continuity of estimation error in each local area. Moreover, seismic intensity $I_s$ dose not explicitly express frequency content of strong motion, so that the errors in estimates by use of $I_s$ may have correlation within those where the characteristics of surface ground amplification are similar to.

Fig. 4 shows the distribution on the geomorphologic classification in the target area. The description of the ID is common to Fig.4 and Fig.5. The error model shown in Fig.5 is set as that each divided group has almost same amount of houses, in order to avoid the posterior estimates excessively influenced by minor, but extremal data. Fig. 5 also shows the cumulative probability of the number of houses with respect to the geomorphologic classification where $I_s$ is estimated to be equal to or more than 5.0. The order of the geomorphologic classification is rearranged as the order of the average shear-wave velocity[8]. Five, the number of the groups, is determined by trials and errors as to distinguish the geomorphologic classification with which relatively large portion of the houses associates.

In the immediate estimation, errors in Iwate and Miyagi prefecture and that in Fukushima prefecture have different trend. Thus, prefecture is selected to correct regionally-varying errors, although the origin of difference has been unidentified. However, for the early steps, in order to update the error models with at least one or more data for every group, two models expressed by Eq. (11) and Eq. (12) are in parallel adopted. Namely, before the
information on Takizawa town (the 8th entry in Table 1) is available, and at least one datum is given to every prefecture, both models are updated. That is, the damages in city/town/village in Miyagi prefecture are estimated by Eq. (12) from the first step, those in Fukushima prefecture are by Eq. (11) at the first step and by Eq. (12) at the second and after steps, and those in Iwate prefecture are by Eq. (11) until the 7th step, and by Eq. (12) at the eighth and after steps.

$$M = \tilde{m} + X_0 + X_b + X_g \quad (11)$$

$$M = \tilde{m} + X_0 + X_b + X_g + X_r \quad (12)$$

where $X_b$ expresses the error by damage level, $X_g$ does the error by the geomorphologic classification, and $X_r$ does the error by prefecture.

The prior probability distribution of $\theta, p(\theta)$ is assumed for all parameters as normal distribution with zero mean, 0.1 of standard deviation, and no correlation of coefficient. In M-H procedure, 15000 samples are generated and first 5000 samples are abandoned as burn-in in each step.

3.3 Results and discussions

Fig. 5 shows that how the total summations of absolute error in each city/town/village change by merging information on actual damages in comparison of the case of “replacing.” In Fig.5, the left-hand is for major damage, and the right side is for heavy or more damage. “Merging” hereby means both of replacing and updating estimates, e.g. when actual numbers of damaged houses are available for some cities/towns/villages, replacing the estimates of the cities/towns/villages with the numbers, and also updating those of other cities/towns/villages by the presented method.

It is shown in the figure that “merging information” with the presented method resolves the total numbers of the errors more effectively than just “replacing,” which are expressed by differences between the solid lines
and the dashed lines. It is also shown that the presented method is more effective in the first step, that is, the first information brings more “surprise” into estimation has more value. In contrast, in after around 25th steps, the differences between “merging” and “replacing” are slight because, in those steps, information has already been enough to adjust the supposed error model, thus less value of information is brought.

As indicated in Fig. 5, the effect on resolving errors is different between that for major damages and for heavy or more damages, as latter is less effective. It is considered to be particular to this example. To understand the difference in details, change of the errors in estimating the numbers of damaged houses is described in Fig. 6 for each city/town/village where more than 100 houses are estimated to be damaged at the immediate estimation. In the figure, the errors of Koriyama city (#28) are remarkable, and it occupies the majority especially in estimating heavy or more damages. For understanding the geography, in Fig. 7, errors in estimating the numbers of major damages are described by comparing those at the immediate estimation and those at the ninth step. In the figure, each circle shows the error in each city/town/village. The circle area expresses the absolute values of error, and color does sign; pink is for overestimated and blue is for underestimated. Here, Koriyama city (#28) is located in midland area of Fukushima prefecture. Near the blue circle on Koriyama city (#28), several blue circles are also drawn (such as for Fukushima city (#27), Sukagawa city (#29) etc.) in both figures in Fig.7. As shown in Table 1, until the twenty-seventh step, the actual damage information on the cities mentioned above is not merged, and, in addition, the cities have relatively large populations in this region. Thus, the errors are not resolved by the presented method until merging the information on those cities.
4. Conclusions

This paper has proposed a method to have a whole picture of earthquake damage rapidly and accurately as a post-earthquake damage assessment in order to help the authorities to manage the disaster, e.g. they may be sure to appoint resources such as manpower to the areas estimated to be more severely damaged. The method applies Bayesian inference to update the estimating errors of safety margins, which is the difference between seismic intensity and resistance of houses, by use of actual damage information in limited areas. Then, a procedure by the presented method is numerically simulated for the data of the 2011 off the Pacific coast of Tohoku Earthquake. The conclusions are summarized as follows:

1. The numerical example illustrates that the presented method has possibility to resolves the total numbers of the errors more effectively than replacing the estimates with actual information, with an error model which use damage level, the Japan engineering geomorphologic classification and prefecture to express the errors in estimating safety margins.

2. It is indispensable to build an error model which can express the error characteristics so that the presented method works. How efficient it works, however, also depends on the available information for updating the model. The example shows that the errors are limitedly dissolved in case that the information on correlated damages is unavailable. It means that the error model should be built considering easiness of data acquisition at the time of disaster as well as accuracy to describe the error characteristics.

Fig. 7-Comparison of estimating errors in immediate estimation and those after merging information on actual damage of 9 cities/towns/villages
3. For future work, the presented method will be applied to other earthquake disaster data in order to examine its features and limitations in details. Proper attributes will be sought for the error model other than those examined in this paper, in considering with locally available data as well.

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6. References