



# QUANTITATIVE ESTIMATION OF SITE EFFECTS IN THE CITY OF ARICA, BY THE SPECTRAL ELEMENTS METHOD

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## **Abstract**

Chile lies on the contact between the Nazca and South American plates, making it one of the most seismic countries in the world. In recent years, several studies have been carried out with the purpose of characterizing the dynamic properties of the soils of the most populated cities in the north of Chile. The purpose of this work is to estimate, by numerical modeling, the seismic amplification of a dense populated area of the city of Arica. Spatial distribution of main soil dynamic properties have been obtained from a detailed geophysical survey, including surface-wave based methods and gravimetry. To estimate the site effects, we solve the wave propagation equation in full heterogeneous media by the Spectral Element Method (SEM). This method allows including the topography, the irregular contact between soils and bedrock and heterogeneities of main materials properties along the computational domain. The inelastic behavior of the soil has been considered, using stiffness degradation and damping curves. The computations were done using the high performance open-source numerical code “SPEED” [1], which can solve the problem in parallel computers. Results of the distribution of the Peak Ground Acceleration (PGA) and other strong-motion indicators are compared against standard 1D horizontally layered modeling.

*Keywords: Site effects, Spectral Elements Method, Surface-wave geophysics*

## 1. Introduction

Before the April 1, 2014 earthquake (A01-2014), northern Chile had been characterized by a seismic gap of 136 years [2]; nevertheless, the total energy released by the 8.2 Mw A01-2014 was only 1/6 of the accumulated seismic energy. For this reason a future strong earthquake in the area is still imminent. The distribution of damage to buildings observed after a strong earthquake is strongly related to local geological conditions i.e. associated with local amplification effects known as site effects. Some factors affecting site effects are: the local geology, the distribution of soil properties, the topography irregularities, the depth to the basement, the water table depth, the soil-city dynamic interaction and many others. The first order factor that contributes to the severity of ground motion is the proximity to the hypocenter, but it is well known that local site effects can be critical during a strong earthquake [3]. In some cases, these local conditions could generate significant amplification of the seismic waves; for this reason the quantitative estimation of site effects of dense populated areas is very important for Earthquake Engineering. Seismic microzoning is a popular tool to assess the earthquake hazards and to predict site amplification susceptibility within in a city. During a previous research project (FONDEF D10I1027Project), qualitative seismic microzoning was performed in two main cities of

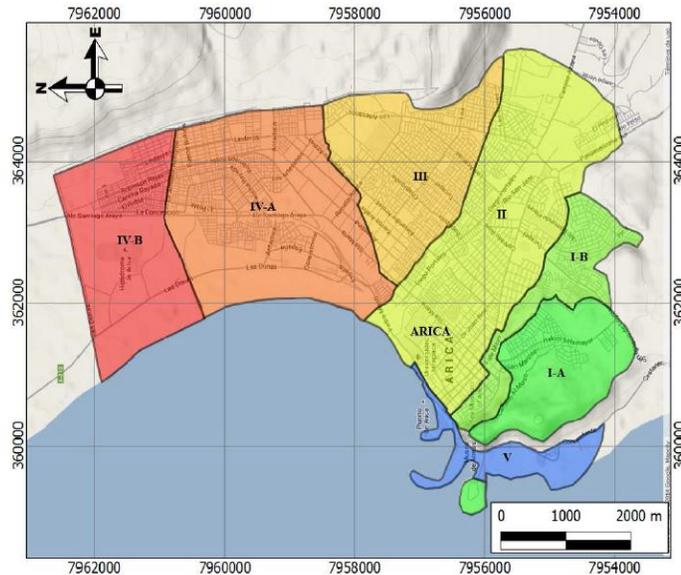


Fig. 1 - Microzoning of Arica [4].

Northern Chile, Arica and Iquique [4], based on geophysical surveys, boreholes and available geological information. Each city was subdivided into different zones with similar expected site effects. The microzoning of the city of Arica is shown in Fig. 1; Table 1 presents main characteristics of each zone. According to this Microzoning map, the city is characterized by relatively large velocities, except to the South, near to Morro Hill (zones I-A and I-B). This area corresponds to a rock outcrop, where no-amplification is expected. Soils in the north area are a mixture of materials controlled by the different sedimentary process. The polygons in this zone were defined to limit areas with similar values of the  $V_{s30}$  parameter and similar site predominant frequencies. The next step of the investigation is to take advantage of the available data and to estimate quantitatively the soil response under realistic dynamic excitations.

In the last few years, physics-based numerical methods have gained more popularity in seismic research to simulate different earthquake scenarios for a great number of applications [5]. Numerical methods such as the Finite Difference, Boundary Integral, Finite Elements and Spectral Elements methods are useful to solve the wave equation in continuous media for many engineering problems. A complete model of seismic wave propagations should include the seismic source, the attenuation in the earth media, and local geology, but the great quantity of parameters that are required to solve this problem demands a high computational cost, and in many occasions, the incorporation of those three components is prohibitive [6]. In this work, we take advantage

of the Spectral Element Method (SEM) implemented in the open-source numerical code, SPEED (“Spectral Elements in Elastodynamics with Discontinuous Galerkin”) [1], to obtain a quantitative estimation of the site effects in the city of Arica. The main objective is to compare the results obtained by a traditional 1D model for seismic wave propagation against results from a 3D heterogeneous media derived from natural variability of dynamic soil properties. The analysis included local topography, irregular contact between the soil and the bedrock, natural material stiffness distribution along the city and a 3D equivalent linear implementation to take into account inelastic soil dynamic behavior. The SEM solves the Elastodynamics equations, in its weak form, on a tridimensional mesh composed by hexahedral and quadrilateral elements, where Dirichlet and Neumann conditions could be applied. Plane wave sources with a perfect vertical incidence direction and medium attenuation are considered. A tridimensional mesh of 3x1x0.2 km was developed to evaluate the dynamic response of the dense urban area in northern Arica.

Table 1- Microzoning details, adapted from [4].

Color	Zone	Avg. $V_S^{30}$ (m/s)	$F_0$ [Hz]	A0	Description
	I-A	1093	-	[2.6-9.2]	Predominance of bedrock composed of andesite and limestone deposits.
	I-B	623	[2.12-8.85]	[3.4-3.8]	Foot of the hill composed of alluvial deposits. Bedrock is located between 5 to 20 m deep.
	II	462	[0.92-1.97]	[2-5.75]	Fluvial deposits, consisting of rounded coarse gravel in a matrix of sand and silt, bedrock at deep levels.
	III	530	[1.3-2.4]	[3-8.9]	Foot of the Chuño Hill mixture of river deposits and gravels from the hill, bedrock at deep levels.
	IV-A	409	[0.91-1.6]	[2.1-7.55]	Marine deposits, composed of mostly done sand. Bedrock at deep levels.
	IV-B	423	[1.07-5.43]	[2.2-6.65]	Mixture of marine deposits and diatomite, bedrock at deep levels.
	V	-	-	-	Artificial landfill from the port of Arica.

## 2. Geotechnical Background

### 2.1 Seismic Context

Chile lies in the subduction contact between the Nazca and South American plates. For this reason, the country has been subjected to large destructive earthquakes repetitively throughout its history. The convergence rate is nearly 6.5 cm/year, and until A01-2014, the estimated accumulated displacement was around 10m. The city of Arica is located in the area known as Arica’s elbow that defines the south limit of a marine plain that extends from Morro hill to Perú. Arica experienced several earthquakes in the past; many of them also produced destructive tsunamis. The last two big earthquakes were in 1768 and 1877; Fig.2 shows the comparison between the released moment of A01-2014 event and the accumulated moment deficit.

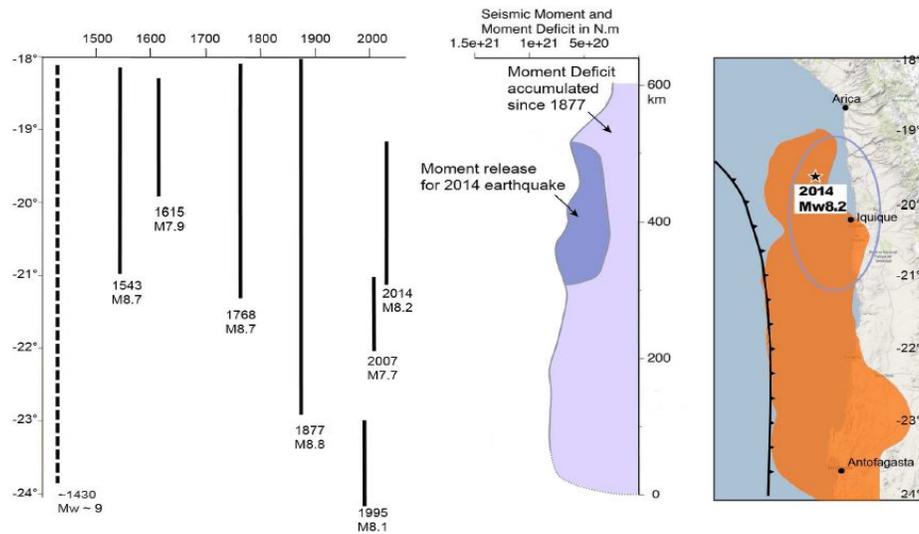


Fig. 2 - Seismic context of northern Chile [4].

Under this scenario, it's feasible to expect the occurrence of a strong and destructive earthquake in northern Chile. Seismic hazard assessment is mandatory in the Chilean context and site effects are a crucial factor that must be included in the national building design regulations, for seismic site classification.

## 2.2 Geomorphology and the characteristics of Arica's soil foundations

The city of Arica was founded to the north of Morro Hill next to the San José River. Regionally, this area is defined by two fluvial valleys with east-west direction from the pre-Cordillera to the Pacific Ocean. These are the Azapa Valley and the Lluta Valley. Those valleys are enclosures by sedimentary and volcanic deposits from the Azapa, Oxaya and El Diablo formations, and avalanche sediments. Arica's soil corresponds to a mixture of sedimentary deposits from fluvial, gravitational, eolian and marine origin, which had filled the sedimentary basin for thousands of years. The area under study, as shown in Fig. 3a, is located north of the San José River, beneath El Chuño Hill. The local topography is fairly regular, limited to the west by the actual coastal line and to the east by a raised topography that is part of the piedmont of Andina pre-Cordillera. Some authors [7] suggest that this flat surface was originated by sea action, classifying it as a marine origin relief. This plain is part of a sedimentary deposit where the materials origin is diverse. The first refers to a marine regime, composed by sediments that shaped marine terraces. The second origin is continental, supplied from the fluvial regime of the San José and Lluta rivers. Finally, marine abrasion is the factor that maintains and preserves the flat surface. Geophysical data doesn't expose a well-defined horizontal stratification. In this research we were able to take that into consideration with the distribution of materials in the model we developed.

## 2.3 Gravity anomaly

Gravity geophysical tests were carried out in the city of Arica. The gravity anomaly is useful, in order to know the distribution materials' density and it also provides a general idea of the geometry of the basement under the soil deposit. Gravity anomalies were measured in several points to obtain the regionalized tendency shown in Fig. 3b.

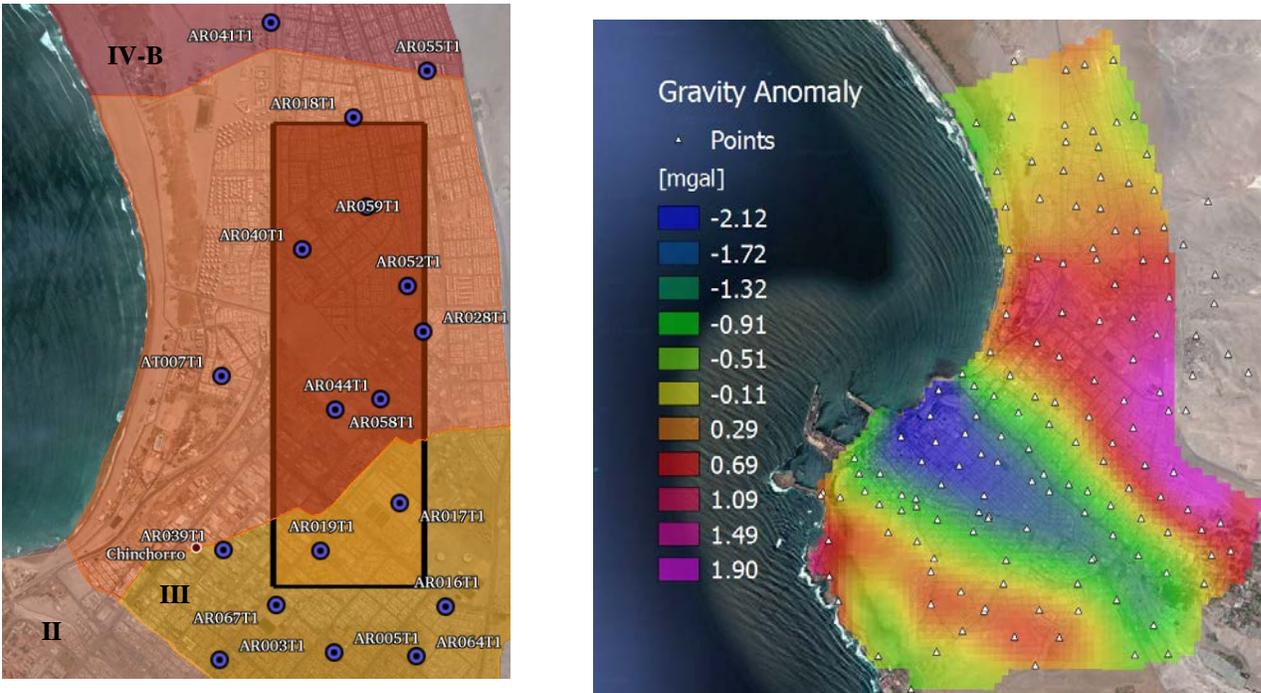


Fig.3a - (left): Zone for 3D Model. Markers indicate the locations where dispersion curves were measured by geophysical methods based on surface wave dispersion. Fig. 3b - (right): Gravity Anomaly. Markers indicate places where gravity anomalies were measured. Lowers anomalies indicate a deeper bedrock and high anomalies indicate a shallower one.

These results agree with the geomorphology, in the sense that a deeper rock is expected in the line that follows the San Jose River, where the sedimentary basin is composed of successive fluvial deposits. In the nearest zones of the Morro and El Chuño Hills, shallower rock is indicated. A distinct contrast is observed between El Chuño Hill and the San Jose River, which is used as a natural limit to select the data to the model generation. To the north, bedrock becomes deeper, corresponding to another filled basin from different sediments origin, as gravitational landslides from the rivers, carried by wind, marine deposits and tsunamis. For this study, we used the gravity information only as supplementary data to check if there is agreement against the estimation of bedrock depth variation from surface-wave based geophysics.

#### 2.4 Dynamic soil properties from surface-wave based geophysics experiments and soil profile extension

To characterize the dynamic properties of the foundation soils of the city of Arica, seismic geophysical methods based on the dispersion of surface waves were used to estimate shear wave velocity ( $V_s$ ) profiles and predominant site frequencies on several points along the city. To characterize  $V_s$  profiles of the studied area, a combination of passive (f-k and SPAC) and active surface-wave based experiments (MASW) were conducted to estimate the soil dispersion curve at different locations in Arica [8]. The purpose of this exploration is to generate a confident set of shear wave velocity profiles on the upper 30 meters to compute standard  $V_{s30}$  parameter for seismic site classification. The Nakamura technique [9] was applied to three components ambient noise records to estimate the predominant frequency from the peaks of the Horizontal to Vertical ratio curve. Although neither velocity neither profiles nor boreholes could reach the basement, this depth is required to model the dynamic soil amplification. Nakamura's technique, together with a shallow  $V_s$  profile, could be used as an indirect estimation of the depth from the surface to bedrock (or other relatively dense/-to-stiff material interface defining a high impedance contrast). In that case, a low frequency indicates a deep location of significant impedance contrast, while a high frequency suggests a shallow one. In Table 1, relative low

frequencies were measured in northern Arica, which is in agreement with a deep sedimentary basin. Fig.4a displays the correlation between the gravity anomaly and the predominant frequencies from Nakamura measurements, suggesting a direct relation between both quantities.

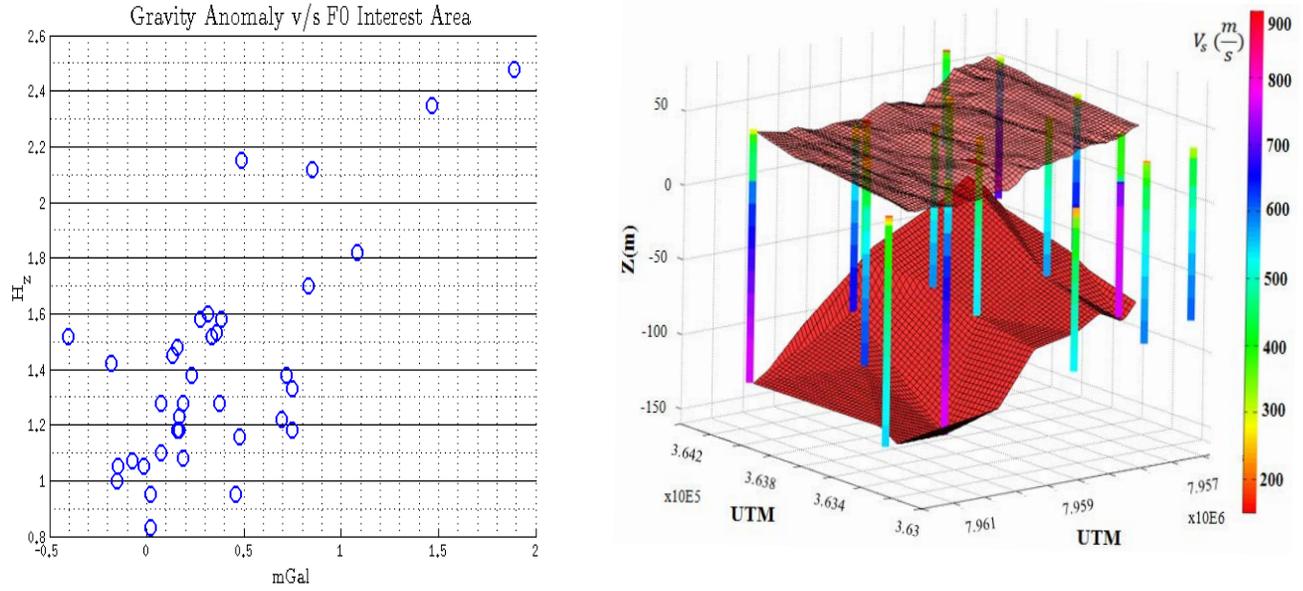


Fig.4a - (left) Relation between F0 and the gravity anomaly. Fig. 4b - (right): Profile extension with topography and bedrock surfaces.

All profiles indicate an increasing  $V_s$  with depth, thus we use Eq. 1 to add successive layers iteratively to obtain a 1D soil column with an analytic natural frequency close to the ones measured throughout the Nakamura's technique.  $V_s(z)$  corresponds to the shear velocity at a depth  $z$ ;  $k$  is a calibration parameter based on the available shallow values of  $V_s$ .

$$V_s(z) = k * \sqrt[4]{z} \quad (1)$$

Once extended the available shear wave velocity profiles, an estimated bedrock surface could be defined with simple interpolation methods (Fig.4b).

### 3. 3D Model

#### 3.1 The Spectral Element Method

The Spectral Elements Method (SEM) is an effective and powerful approach for solving three-dimensional seismic wave propagation problems in highly heterogeneous media [1]. The SEM works like the Finite Element, solving the weak form of the wave equation. The main difference is that SEM uses high-order shape functions sampled with the Legendre-Gauss-Lobato(LGL)quadrature points. The same quadrature points are used to perform the integration process, so this technique implies a diagonal mass matrix thereby reducing the computational cost. Another advantage of SEM is that, compared with traditional FD or FEM where 8-10 integration points are required to correctly characterize a wave length; in SEM 4-6 spectral nodes are enough to represent the minimum wave length to be propagated. The formulation of the Spectral Elements Method requires considering an elastic heterogeneous medium occupying an open bounded region  $\Omega \subset R^3$  with Lipschitz boundary  $\Gamma = \partial\Omega$ . The boundary can be divided into the usual portions  $\Gamma_D$ , where Dirichlet's condition are prescribed,  $\Gamma_N$  where external surface load or Neumann's conditions apply, and  $\Gamma_{NR}$  where non-reflecting boundary conditions are imposed to simulate the unbounded media. For details of the implementation in SPEED please see [10].

### 3.2 Non-linear soil behavior

Soils' non-linear behavior is a great challenge for time schemes like SEM. Experimental evidence supports that the inelastic dynamic response of soils can be approximately modeled by two curves: "stiffness-degradation curve" and "damping curve". Borehole loggings are useful to identify appropriate modulus reduction and damping curves from the literature. We implemented an external code to SPEED that introduces a standard Equivalent-Linear approach [11] performing several iterations in which stiffness and damping are updated depending on a reference strain  $\gamma_{eff}$  value defined by equations Eq. 2-4, where  $\varepsilon_1$ ,  $\varepsilon_2$  and  $\varepsilon_3$  are the principal strains. Iterations continue until stabilized values are obtained for the soils properties. Fig.5 shows three different sets of stiffness degradation and damping curves used in this investigation. Each soil element has been assigned to one of these curves, depending on the initial value of  $V_s$ . For softer materials  $V_s < 350 \left(\frac{m}{s}\right)$  the lower limit curve was selected, if  $350 \left(\frac{m}{s}\right) \leq V_s < 550 \left(\frac{m}{s}\right)$  the material follows the mean curve; and for stiffer materials  $V_s \geq 550 \left(\frac{m}{s}\right)$  the upper limit curve was assigned.

$$\gamma_{max} = \max\{|\varepsilon_1 - \varepsilon_2|, |\varepsilon_2 - \varepsilon_3|, |\varepsilon_3 - \varepsilon_1|\} \quad (2)$$

$$\gamma_{eff} = R_w * \gamma_{max} \quad (3)$$

$$R_w = \frac{M_w - 7}{10} \approx 0.67 \quad (4)$$

SPEED models visco-elastic media by the modified equation of motion Eq.5. At each time step, shear modulus  $G$  and damping ratio  $\zeta$  are updated in SPEED by the effective strain  $\gamma_{eff}$ . If  $\xi [s^{-1}]$  is the decay factor, in the frequency domain an elastic attenuation operators are implemented using the quality factor of the medium  $Q$ ; this is defined in Eq. 6,

$$\rho u_{tt} + 2\rho\xi u_t + \rho\xi^2 u - \nabla \cdot \sigma(u) = f, \quad in \Omega x(0, T]. \quad (5)$$

$$Q = \frac{\pi f}{\xi} = \frac{q_0 f}{f_0} = \frac{1}{2\zeta} \quad (6)$$

where  $q_0 = \pi f_0 / \xi$  is the quality factor of the representative frequency value to be propagated  $f_0$ . Our implementation uses the same formulation, but stiffness and damping are updated iteratively at the end of individual runs as standard Equivalent-Linear method. Hence, these parameters are constant during the loading.

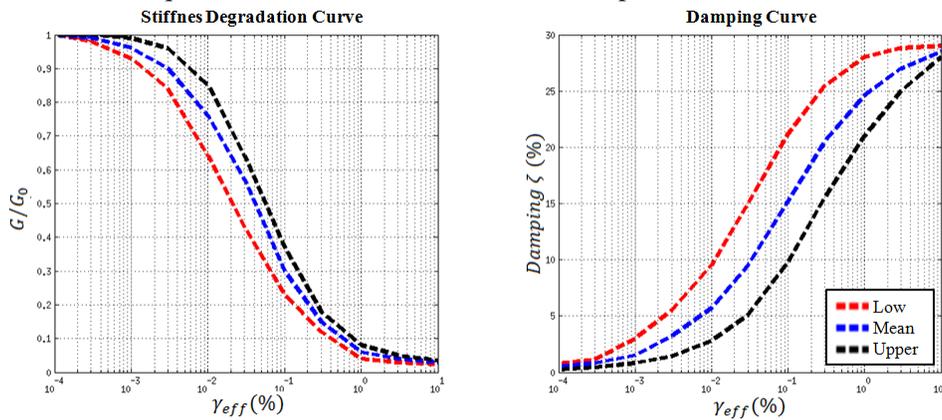


Fig.5–Stiffnessdegradationcurve and Damping curve [12]

### 3.3 Simulations

We focused in the northern area of Arica, located in  $x := [363215 \ 364215]$  and  $y := [795745 \ 796045]$ , UTM Zone -19, because this is an area with dense geophysical information useful for developing the model and

because this is a region with a high demographic expansion [13] where preventive actions could be applied. According to Microzoning map Fig.1, this area includes sites prone to important site effects. SRTM 90m digital elevation data were considered to obtain the topography of the city of Arica. Both the topography and bedrock surface were exported to the mesh generation software Trellis® and were used to build a tridimensional mesh composed by hexahedral elements for soils elements and quadrilateral element for the faces, where Dirichlet and absorbing boundaries are applied [14]. Material assignment and SPEED's input generation were performed in MATLAB®. Fig 6a shows the tridimensional mesh and the  $V_s$  distribution across the model. To cut down the running time, keeping frequency contents compatible with Chilean earthquakes, a synthetic Ricker wavelet of 5s long (Fig.6b) has been selected as preliminary input. The maximum frequency to be propagated is around ~10 [Hz]. This input motion was imposed as perfect vertical incident S-wave, polarized across X direction (short dimension of the model).

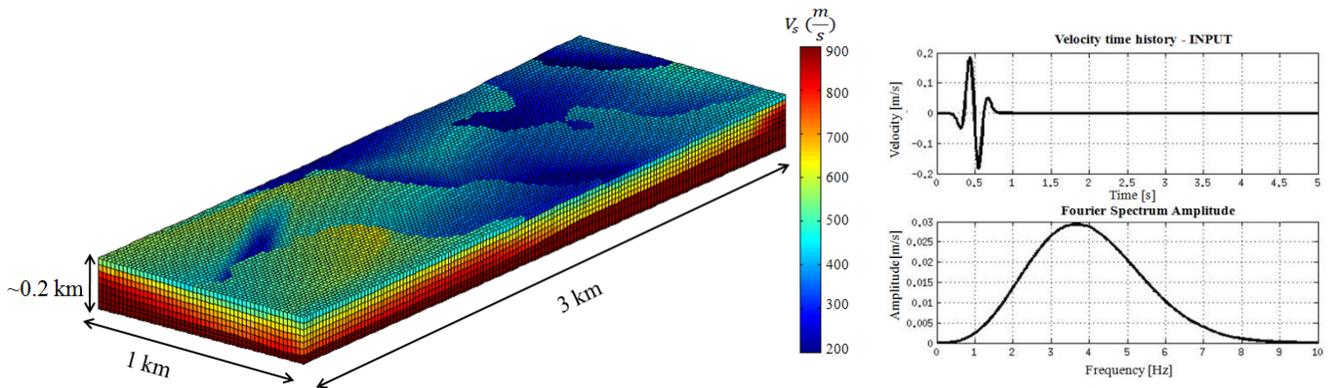


Figure 6a - (left) Material distribution in the 3D model. Figure 6b - (right) Ricker signal used as input in SPEED.

## 4. Results

The main purpose of this work is to evaluate if tridimensional site effects induced by material heterogeneities are important in the zone under study. In Fig. 7a, two sections are displayed; where the shear wave velocities of initial materials are compared with the degraded ones obtained by our linear equivalent approach. A significant reduction in stiffness of the materials is appreciated in both sections (55-85%). The Peak Ground Acceleration (PGA) for the linear equivalent results are shown in Fig. 7b. A high variability of the surface ground motion can be seen, obtaining PGA values between 0.2 to 2g depending on the area. The dashed line in Fig.7b represents the limit between areas IV-A and III from seismic microzoning in Fig. 1. In general terms, higher values can be noted on the northern part (zone IV-A) compared to southern part of the model (zone III). Nevertheless, the maxima and minima observed in each zone of the model are very similar. To study 3D site effects due to natural soil variability, two representative columns “c2” and “c4” (Fig.8b and 8d) were selected to perform simple 1d wave propagation analysis, and compare their results against 3d simulations. The surface motion results for elastic and equivalent linear analysis for “c2” and “c4” columns are shown in Fig. 8a and Fig. 8d. Some minor differences in terms of acceleration amplitude or Arias Intensity can be noted in these figures. The column “c4” was selected in the deeper part of the sedimentary basin (Fig. 8c). According to our results for this control point, the assumption of horizontal stratification seems accurate enough to properly evaluate the dynamic soil behavior in these conditions. In terms of seismic energy at the surface, both elastic and equivalent linear models provide slightly larger energy (+15%) at the surface for the 3D case. Likewise, in control point “c2” (Fig. 8a), more heterogeneity exists around the column. In this case, results show more influence of three-dimensional effects. Higher peaks of acceleration and large spectral ordinates over a wide range of periods can be noted on the Pseudo Acceleration Response spectrum. In terms of Arias Intensity, elastic 3D “c2” models indicate a larger value of about 50%, but after application of Equivalent Linear approach the opposite is true. In Fig. 8b and Fig. 8d, the initial elastic columns and the degraded ones are displayed. In general terms, the shear velocity becomes more uniform in the column after the application of the Equivalent Linear Method.

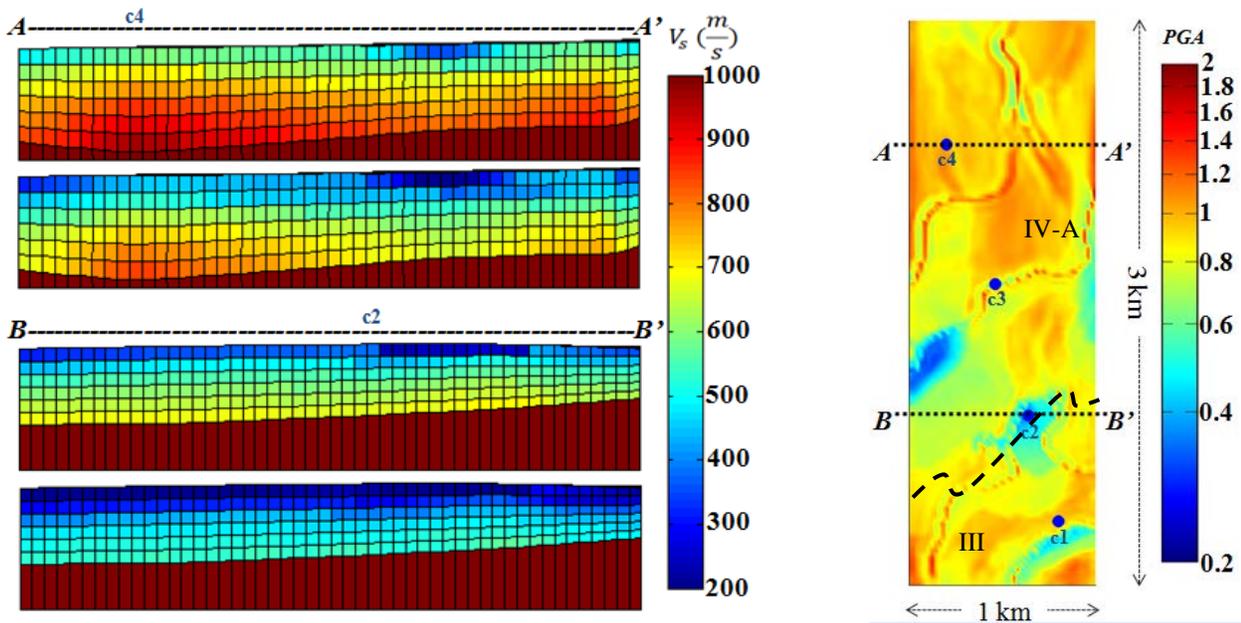


Fig.7a - (left) Sections A-A' and B-B': the upper 2d profile expose the original materials and the bottom indicate the degraded materials by the Equivalent Linear approach. Fig.7b - (right) Peak Ground Acceleration (PGA), blue dots indicate the location of the representative columns to evaluate 3d amplification; dashed line represents the limit in microzoning.

## 5. Conclusions

SEM proved to be a useful tool to study and evaluate 3D seismic amplification effects. Reproducing the non linear soil behavior in the time and frequency domains is a useful method to assess the seismic amplification in soil deposits. The intensity distribution of strong seismic motion on the surface of northern Arica is related to the characteristics of the materials in the shallow soil layers and not to basin-edge effects from the irregular contact between the soil and rock. The surface response is highly dependent on the quality of materials in the shallow layers, so the  $V_{s30}$  parameter is useful when evaluating the seismic amplification of a particular site when there are no major differences in the quality of surrounding materials. The results for the nonlinear range show that the spatial distribution of the dynamic properties can generate amplification effects which are determined by vertical and lateral impedance contrast between materials. The relevance of this geophysical test was confirmed as a method to understand and characterize the variability of geological material at a regional scale. Understanding this phenomenon is crucial in the national context, as seismic amplification must be considered when evaluating the location of structures and human settlements. The current Chilean seismic code is only based on the  $V_{s30}$  parameter, which omits the natural frequency of the soil; in this regard, 3D models help to understand the seismic propagation in complex media, highlighting clear differences between sites with almost uniform values of  $V_{s30}$ . High variability of PGA values at the surface is related to the natural distribution of material parameters related to the complex geology of Arica city. Deterministic 3D models are very influenced by the basement depth and shape. A better estimation of the transition between the soil and apparent bedrock using the gravity anomaly in combination with seismic geophysical data is under development.

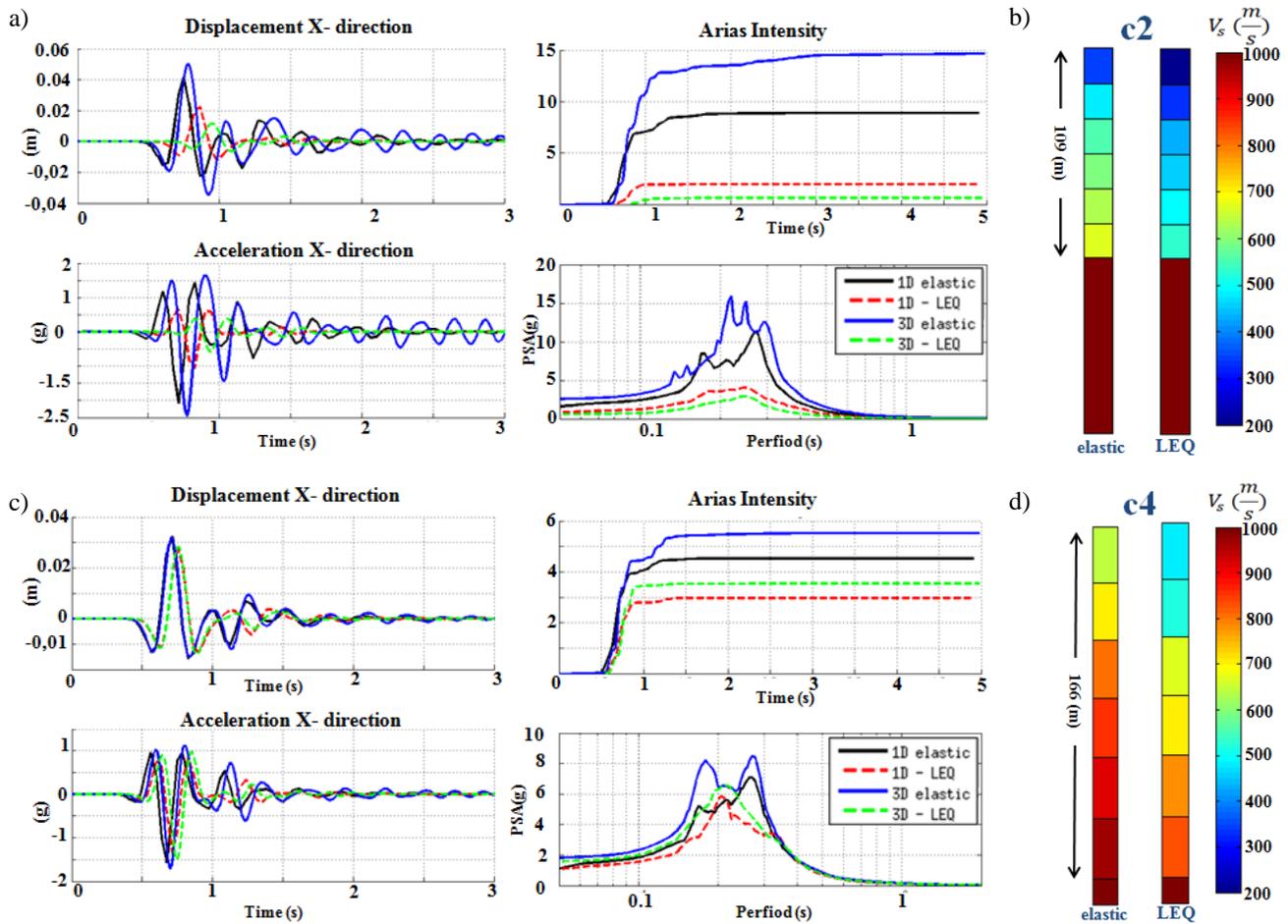


Fig.8a - (up-left): Results at the surface for c2. Fig. 8b - (up-right): Column "c2" and the elastic and Linear Equivalent materials. Fig. 8c - (down-left): Results on the surface for c4. Fig. 8d - (down-right): Column "c4" and the elastic and LEQ materials

## 6. Acknowledgements

This research was partially supported by a Grant from the Chilean National Commission For Scientific and Technological Research, under FONDEF+ANDES Award Number D10I1027 and partially by the National Research Center for Integrated Natural Disaster Management CONICYT/FONDAP/15110017. The authors would like to thanks Dr. Ilario Mazzieri for his support in the use of SPEED code.

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