



## Response of RC Buildings with Weak Beam-Column Joints Subjected to Different Types of Ground Motions

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### Abstract

The seismic assessment of older-generation reinforced concrete buildings is of great concern because they constitute the bulk of the building stock in earthquake prone areas in many countries. Previous research has revealed that reasonable estimates for the structural response are obtained through the use of current analytical tools, though local deformation parameters are more difficult to pinpoint in such response estimates. However, the behavior of beam-column connections has usually been ignored in modeling process due to its complexity. This study investigates the contribution of joint flexibility to the lateral response of reinforced concrete buildings subjected to different types of ground motions and emphasizes the significance of modeling properly unreinforced beam-column joints. Here, the beam-column connections without transverse reinforcement are referred to as unreinforced joints. Nonlinear dynamic analyses of a prototype building frame are performed through two analytical models with rigid and flexible beam-column joints. The analytical models are identical to those previously used by the authors in simulation of reinforced concrete frames tested on shake tables. In these models, joint flexibility is simulated through joint rotational springs. These inelastic rotational springs simulate shear deformation at the joint and member-end rotations due to bond-slip behavior. The hysteretic behavior of the rotational spring is represented by a multilinear hysteretic material (Pinching4) available in OpenSees. Variation in the response of selected building is studied under different types of ground motions representative of large magnitude-large distance and large magnitude-short distance records. The selected response quantities for the considered reinforced concrete structure are the global response quantities such as roof displacement, distribution of the story drifts along the height of the structure, and local response quantity such as joint rotation. Results of the analyses are discussed in comparative fashion for the two analytical models subjected to different types of ground motions. It is shown that computations must take into account the role played by joint deformations in case of RC buildings having unreinforced beam-column joints.

*Keywords: beam-column joint; older generation RC buildings; analytical model; dynamic analyses; nonlinear demands*

## 1. Introduction

Post-earthquake reconnaissance indicates that reinforced concrete beam-column joints with inadequate transverse reinforcement sustained severe damage and even contributed to the collapse of structural systems. Northridge, California, 1994; Tehuacan, Mexico, 1999; Kocaeli, Turkey, 1999; Athens, Greece, 1999; Chi-Chi Taiwan 1999; Wenchuan China, 2008; Abruzzo, Italy, 2009 and Haiti, 2010 are instances of earthquakes that involved beam-column joint damage. Fig.1 illustrates a collapsed building from the Kocaeli, Turkey earthquake.



Fig. 1 – Collapsed building from the August 17, 1999 Kocaeli Eq. [1]

Several experimental and analytical studies on RC columns and frames with unreinforced beam-column joints have been conducted. Alath and Kunnath [2] modeled joint shear deformation through a rotational spring model, with degrading hysteresis empirically. Biddah and Ghobarah [3] modeled joint shear and bond-slip deformations through rotational springs, and in a subsequent study, Ghobarah and Biddah [4] used the model to demonstrate that joint deformations resulted in increased flexibility and drifts under earthquake loading. Youssef and Ghobarah [5] proposed a joint element comprising 14 springs; twelve translational springs located at the panel zone interface to represent bond slip and other inelastic actions, and two diagonal springs to simulate joint shear deformation. Pampanin et al. [6] simulated beam-column joint through a panel zone region including a joint rotational spring and panel zone rigid elements. Other joint models have also been proposed [7, 8, 9, 10]. Celik and Ellingwood [11] reports available models for computer simulation of RC beam-column joints and calibrates parameters to model joint shear stress-strain relationships using the results of full-scale beam-column joint tests.

As mentioned above various analytical models simulating joint flexibility are available but actual global response of buildings with unreinforced joints has not been widely investigated. In the current study, we analytically investigate the effect of unreinforced joint flexibility to the seismic response of multi-story building frames. The analytical models with or without joint modeling are subjected to earthquake simulations in a comparative manner. A simple, representative and formerly verified joint model from the literature is used in the analyses, thus practicing engineers who usually have refrained from modeling the behavior of joints are motivated to incorporate joint nonlinearity in their future analyses.

## 2. Description of the Prototype Building Frame

The reference frame is identical with the original details of the exterior frame of Van Nuys Hotel Building [12], except for the shear reinforcement in the columns and beams. Here, this exterior frame modified and studied by Park and Mosalam [13] is of concern, but different from their model, the effective slab width and contribution of slab reinforcement to the strength of the beam are taken into account. It is aimed to investigate “solely” the effect of unreinforced beam-column joint flexibility to the seismic response of an older-type multistory buildings subjected to different types of ground motions. The effect of shear critical columns or beams, splice and foundation problems to the response is undesirable. Beams and columns of reference buildings are assumed to have enough shear reinforcement to prevent their shear failures. The reference frame, shown in Fig.2, is a strong

column-weak beam system where the failure of beam-column joints is expected to occur at the ultimate flexural strength of beams. The dimensions and material properties of the building is given in Table 1.

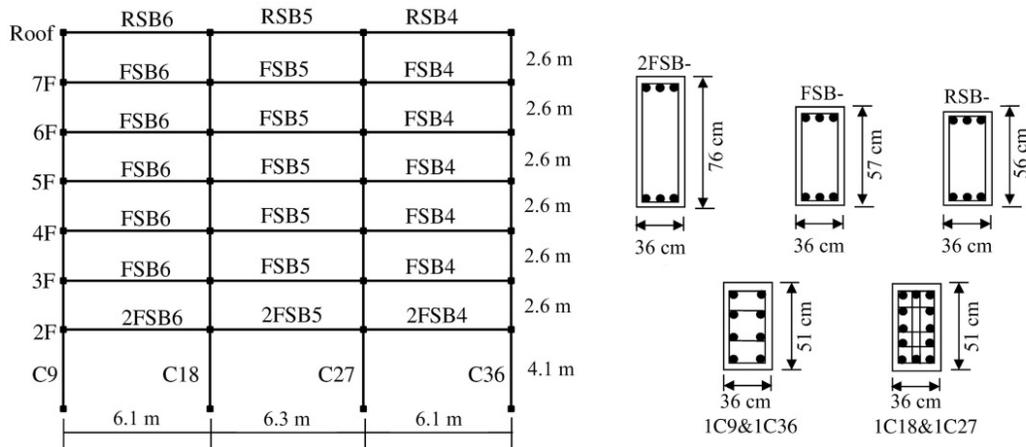


Fig. 2 – The building frame and cross sections of the beams and columns (adapted from [13])

Table 1 – Dimensions and material properties of the column and beam sections

Dimensions b x h		$f_y$	$f_c$	Floor	C-9		C-18		C-27		C-36	
(cm x cm)		(MPa)	(MPa)		# of bars	Bar Area (cm <sup>2</sup> )	# of bars	Bar Area (cm <sup>2</sup> )	# of bars	Bar Area (cm <sup>2</sup> )	# of bars	Bar Area (cm <sup>2</sup> )
<b>Columns</b>	36 x 51	414	34.5	<b>1st</b>	8	6.45	12	6.45	12	6.45	8	6.45
			27.6	<b>2nd</b>	6	3.87	8	6.45	8	6.45	6	3.87
			20.7	<b>3rd</b>	6	3.87	8	6.45	8	6.45	6	3.87
			20.7	<b>4th</b>	6	3.87	6	6.45	6	6.45	6	3.87
			20.7	<b>5th-7th</b>	6	3.87	6	3.87	6	3.87	6	3.87
Dimensions b x h		$f_y$	$f_c$	Floor	B4		B5		B6			
(cm x cm)		(MPa)	(MPa)		# of bars	Bar Area (cm <sup>2</sup> )	# of bars	Bar Area (cm <sup>2</sup> )	# of bars	Bar Area (cm <sup>2</sup> )		
<b>Beams</b>	36 x 76	276	27.6	<b>2nd (2FS-)</b>	Top	3	6.45	3	6.45	2	6.45	
				Bot.	2	6.45	2	5.10	2	6.45		
	20.7		<b>3rd-4th (FS-)</b>	Top	3	8.19	3	8.19	3	8.19		
			Bot.	2	5.10	2	5.10	2	5.10			
	20.7		<b>5th-6th (FS-)</b>	Top	3	6.45	3	6.45	3	6.45		
			Bot.	2	5.10	2	5.10	2	5.1			
	20.7		<b>7th (FS-)</b>	Top	2	6.45	3	5.10	2	6.45		
			Bot.	2	5.10	2	5.10	2	5.1			
36 x 56	20.7	<b>Roof (RS-)</b>	Top	2	5.10	3	3.87	2	5.1			
		Bot.	2	3.87	2	3.87	2	3.87				

### 3. Analytical Models

Two different analytical models of the reference frame with (i) rigid and (ii) flexible joints were conducted. In the first model, the beam-column joints are defined as infinitely rigid as shown in Fig.3a. In the second model, the shear deformations at beam-column joints and member-end rotations due to bond-slip behavior are simulated through rotational joint springs (Fig.3b). The backbone curve proposed by Park and Mosalam [13], is used in implementing the moment-rotation relationship of the joint springs to the model. The effective slab width to account for slab reinforcement in tension was included in flexural strength calculation of beams.

A variety of approaches [2,3,4,5,7,8,9,14] have been proposed for modeling flexible joints. In this study, the scissors model proposed by Alath and Kunnath [2] was used because of its simplicity. This model was also verified by the authors in their former studies [15,16]. In the scissors model, the finite size of the joint is modeled by two rigid links interconnected by an inelastic rotational spring. When the spring is subjected to a moment, the links rotate relative to one another at an angle that represents the shear distortion of the beam-column joint and slip at the beam and/or column interface.

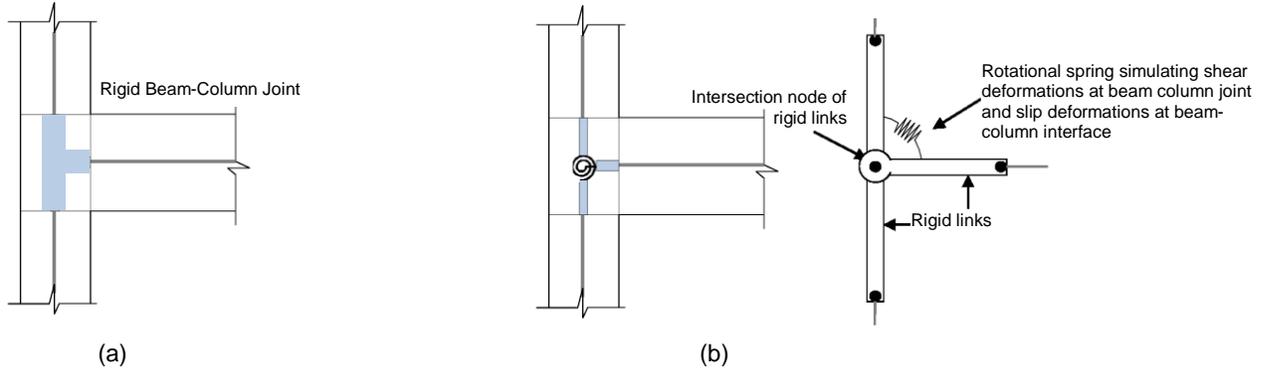


Fig. 3 – (a) Model 1: Rigid beam-column joint (b) Model 2: Flexible beam-column joint; rotational spring simulating shear deformations and slip deformations at beam-column interface

The hysteretic behavior of the rotational spring is represented by Pinching4 hysteretic material (Fig.4) which is a uniaxial material proposed by Lowes and Altoontash [7] and implemented in OpenSees [17]. It has a multi-linear envelope exhibiting degradation and a tri-linear unloading and reloading path representing a pinched hysteresis.

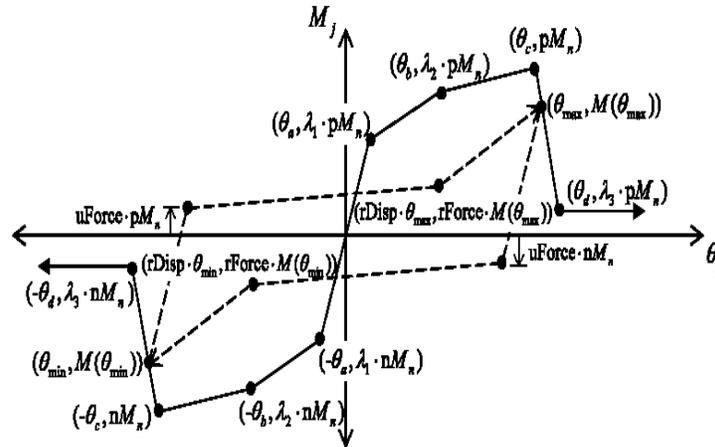


Fig. 4 – Pinching 4 hysteretic material model (adapted from Park and Mosalam, 2013)

The moment-rotation envelope relationship for the Pinching4 material was determined from the backbone curve (Fig.5) proposed by Park and Mosalam [13]. The first point represents initial joint cracking, the second represents either beam reinforcement yielding or significant opening of existing joint crack, the third corresponds to the peak strength and further propagation of existing joint crack or additional joint crack opening, and the fourth corresponds to the residual strength and rotation when the joint damage is severe. Here, joint rotation is assumed as the sum of joint shear strain and rotation due to slip at the beam-joint interface, that is:

$$\theta_j = \gamma_{xy} + \theta_{slip} \quad (1)$$

The rotation of the joint spring,  $\theta_j$ , is calculated based on the equations given in Fig.5. Park and Mosalam [13] indicates that shear strength of a beam-column joint and joint rotation are insensitive to joint aspect ratio and beam longitudinal reinforcement before peak strength is reached.

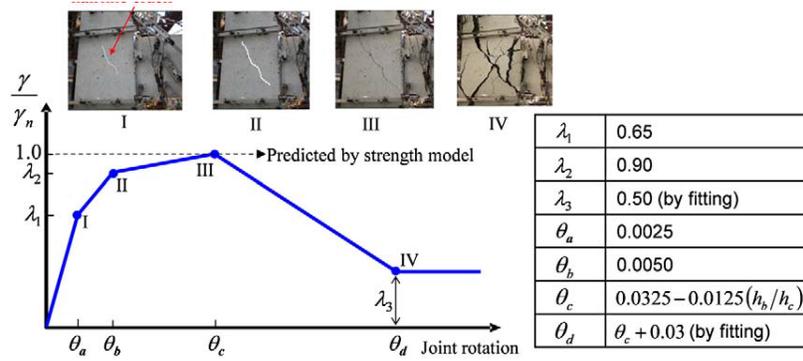


Fig. 5 – Backbone curve for joint shear and rotation relationship proposed by Park and Mosalam (2013)

Park and Mosalam [18] propose that shear strength of a beam-column joint is related to aspect ratio of the joint section and tensile reinforcement ratio of the beam. They conclude with the Eqns. (2) and (3) where the effect of these two parameters is taken into account through semi-empiric and analytical claims.

$$V_n = k \left[ \Gamma \sqrt{f'_c} b_j h_c \frac{\cos \theta}{\cos(\pi/4)} \right], \quad \theta = \tan^{-1} \left( \frac{h_b}{h_c} \right) \quad (\text{MPa}) \quad (2)$$

$$SI_j = 12 \left( \frac{A_s f_y}{b_j h_c \sqrt{f'_c}} \right) \left( 1 - 0.85 \frac{h_b}{H} \right) \quad (\text{MPa}) \quad (3)$$

In Eq. (2)  $b_j$  is the effective width of the beam-column joint;  $h_c$  is the height of the column,  $h_b$  is the height of the beam,  $f'_c$  is the compressive strength of concrete;  $\theta$  is angle based of the beam-column joint geometry;  $\Gamma$  is the shear strength coefficient dependent on the location of the beam-column joint (i.e. exterior, interior, roof).  $\Gamma$  is taken as 1 for the mid-story exterior frame; 0.66 for the roof exterior frame; 1.67 for the mid-story interior frame; 1 for the roof interior frame.  $k$  is a coefficient dependent on the longitudinal beam reinforcement ratio and takes values between 0.4 and 1. The variable  $k$  changes according to Shear Index ( $SI_j$ ) value which is a coefficient correspondent to the shear demand at the beam column joint when the beam tensile reinforcement yields.  $f_y$  is the yield strength of steel reinforcement.  $A_s$  is the area of beam tensile reinforcement;  $H$  is the distance between the zero-moment locations.  $0.85h_b/H$  is an approximate ratio of column shear force to the axial force in tensile reinforcement of the beam.

Using equations 2-3 and Fig.5, the shear strength-rotation ( $V_n-\theta$ ) relationship is converted into moment-rotation ( $M_n-\theta_j$ ) backbone curve by using Eqns. (4) and (5).

For the mid-story beam-column joints:

$$M_{jn} = V_{jn} \frac{1}{\frac{1-b_j/L_B}{jd} - \frac{1}{L_C}} \quad (4)$$

For the roof beam-column joints:

$$M_{jn} = V_{jn} \frac{1}{\frac{1-b_j/L_B}{jd} - \frac{2}{L_C}} \quad (5)$$

In calculation of Eqns. (4) and (5), axial forces at beams are neglected.  $V_{jn}$  is the shear strength of beam-column joint;  $M_{jn}$  is the moment value when the shear strength value is reached at the beam-column joint.  $b_j$ , the effective width of the beam-column joint;  $L_B$  and  $L_C$  are the distance between contraflexure points,  $d$  is the effective depth of the beam;  $jd$  is the moment arm between the tensile and compressive forces at the beams. For the cases in which the beam longitudinal reinforcement yields,  $j$  is accepted as 0.9 [19] and for the cases without yield,  $j$  is accepted as 0.875d [20].

At the last step of introducing beam-column joint,  $M_j-\theta_j$  backbone curves are constructed through aforementioned pinching hysteretic material (Fig.4) defined by Lowes and Altoontash [7]. Beam elements are introduced to the analytical model through linear elements with effective rigidity and lumped plasticity idealization at the ends. Elastic-perfectly-plastic (EPP) material model, predefined in OpenSees [17] is used for the definition of moment-rotation relationship at beam ends. The effective rigidity values are calculated from the moment-curvature analyses of the sections. The columns are introduced with nonlinear beam-column elements with 3 or 4 integration points. The number of integration points is selected considering the plastic hinge length and integration weights based on Gauss-Lobatto rules. The sections are defined with steel and concrete fibers. The material models used are given in Fig. 6.

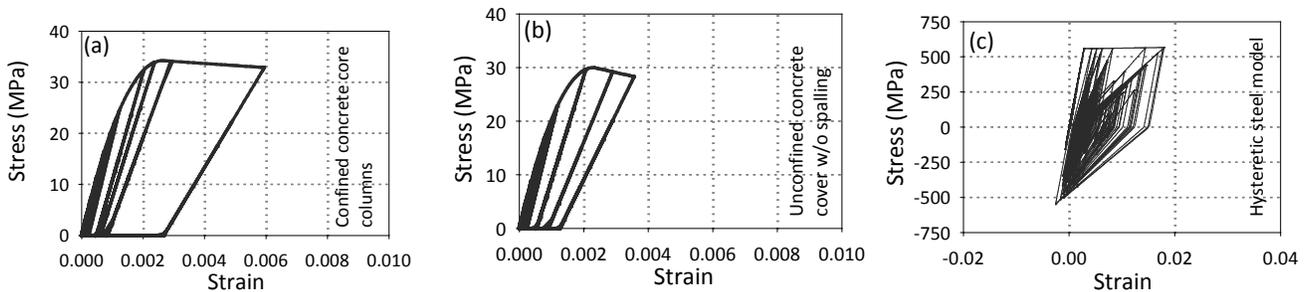


Fig. 6 – (a) confined concrete mat. (b) unconfined concrete mat. (c) hysteretic steel material [15]

### 3. Selection of Ground Motions

Six earthquake ground motions were selected from the study of Warn and Whittaker [21] and downloaded from the Pacific Earthquake Engineering Research (PEER) database, <http://ngawest2.berkeley.edu/> [22] (Table 1). The selection was based on distance to fault and moment magnitude. All records are from the earthquakes with magnitude  $M_w$  larger than 6.5. Three of six records are from near-field (smaller than 20 km, Somerville [23]) and the rest are from far-field (distance larger than 30 km) ground motions. As a note, none of the records exhibits any directivity effects.

Table 1 – Ground Motions ( $M_w$ : Moment Magnitude; NF: Near-field; FF: Far-field; Distance: Closest distance to fault rupture; Site class: Classification according to NEHRP 2000)

Record	Event	Year	$M_w$	Station	Orientation	PGA (g)	Distance (km)	Site Class
CNP106	Northridge	1994	6.7	90053 Canoga Park	106	0.36	15.8 (NF)	C
RIO270	Cape Mendocino	1992	7.1	89324 Rio Dell Over Pass FF	270	0.39	18.5 (NF)	B
STG000	Loma Prieta	1989	6.9	58065 Saratoga Aloha Ave	0	0.51	13 (NF)	B
H-VCT345	Imperial Valley	1979	6.5	6610 Victoria	345	0.17	54.1 (FF)	C
TDO000	Kobe	1995	6.9	Tadoka	0	0.29	30.5 (FF)	C
CAS000	Northridge	1994	6.7	Compton Castlegate St.	0	0.09	49.6 (FF)	C

The spectral acceleration spectra for the selected near- and far-field ground motion records are shown in Fig.7a and Fig.7b respectively. The fundamental periods of the analytical models with rigid and flexible beam-column joints are indicated with vertical lines. It is obvious that the spectral acceleration demand on the analytical model varies with the fundamental period of the model.

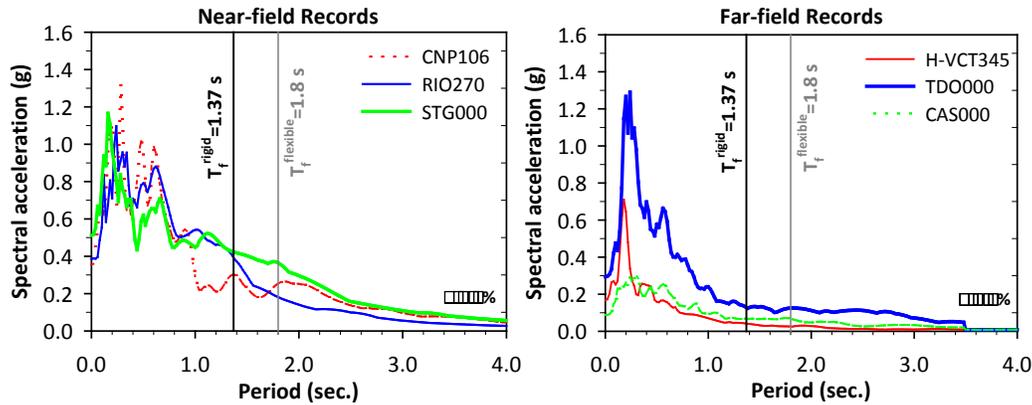


Fig. 7 – Acceleration response spectra for the selected ground motion records with 5% damping: (a) near-field records and (b) far-field records.  $T_f^{rigid}$  is for the fundamental period of the analytical model with rigid joints.  $T_f^{flexible}$  is for the fundamental period of the analytical model with flexible joints

#### 4. Nonlinear Dynamic Analyses

Two different analytical models, namely with rigid and flexible beam-column joints were carried out. The fundamental periods of the analytical models with rigid and flexible beam-column joints were calculated as 1.37s and 1.80s, respectively. From the Fig.7 it can be inferred that the analytical models with different fundamental periods will be subjected to different dynamic forces even for the same acceleration record of a ground motion. Three near-field and three far-field ground motions with moment magnitude ( $M_w$ ) larger than 6.5 were applied to the analytical models in order to investigate the global demands such as the ratio of maximum base shear to the weight of the structure and the ratio of maximum roof displacement to the height of the building.

Fig. 8. shows the analytical model with rigid beam-column joints are subjected to higher base shear compared to that with flexible beam-column joints as expected; however, this may vary dependent on the relationship between the fundamental period of the structure and response spectra of the ground motion record, such as the CAS000 case shown in Fig.8.

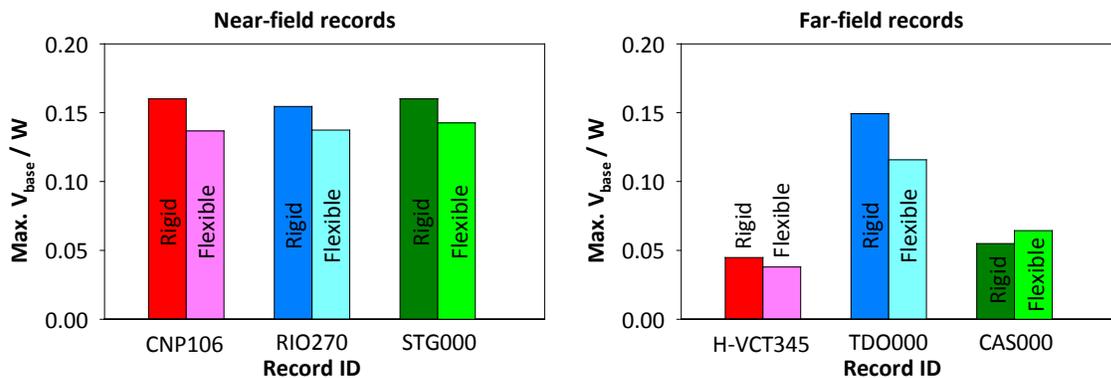


Fig. 8 – Ratio of maximum base shear to the weight of the frame obtained from the nonlinear dynamic analyses of the analytical models with rigid and flexible beam-column joints

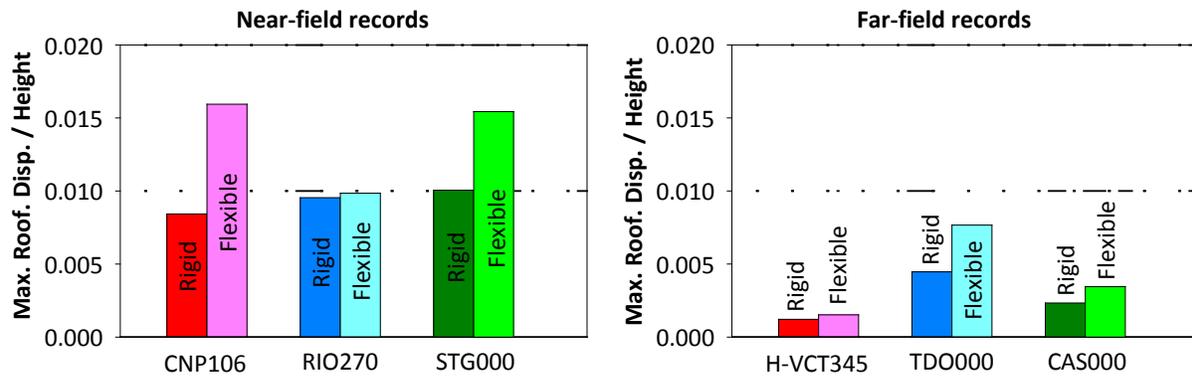


Fig. 9 – Ratios of maximum roof displacement to the height of the frame obtained from nonlinear dynamic analyses (considering near-field and far-field records) of the analytical models with rigid and flexible beam-column joints

The ratios of maximum roof displacement to the height of the frame were obtained from the dynamic analyses of the frames, considering near- and far-field records (Fig. 9). Higher values were calculated for the analytical models with flexible joints; however, the difference is very low for the RIO270 case.

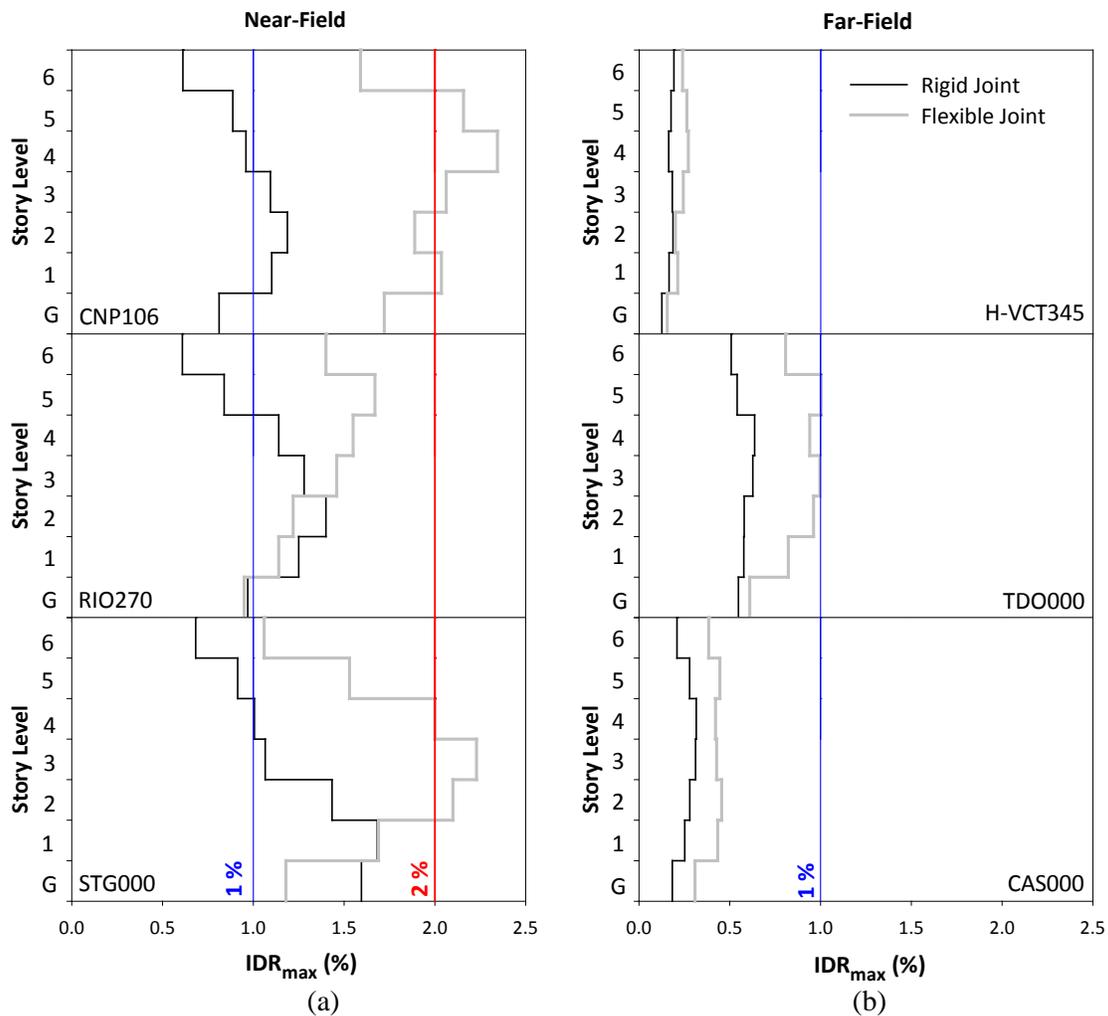


Fig. 10 – Maximum inter-story drift ratios obtained from dynamic analyses of the two different analytical models considering (a) near-field records and (b) far-field records

Fig. 10 reveals the maximum inter-story drift ratios obtained from the dynamic analyses of the two analytical models subjected to near- and far-field ground motion simulations. The analytical model with flexible joints yield higher inter-story drift-ratios compared to the model with rigid joints. When the performance levels of the buildings based on inter-story drift ratio values are investigated, far-field simulations indicate an inter-story drift ratio value lower than 1% that corresponds to a level of “immediate occupancy” based on current guidelines. When the near-field simulations are considered (CNP106 and STG000), a sharp increase is observed, even changing the performance level of the frame from “damage control” level to the “collapse prevention” level.

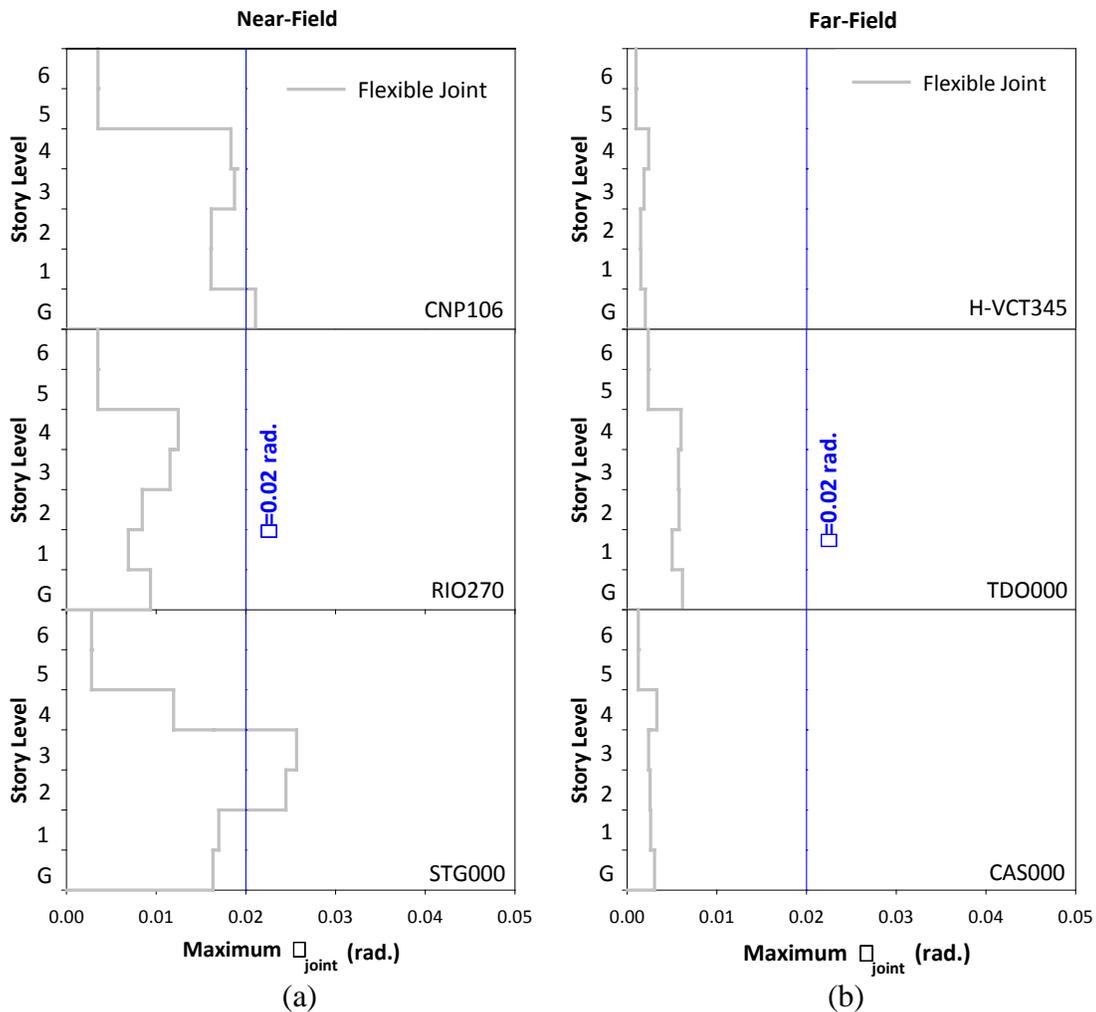


Fig. 11 – Maximum joint rotations obtained from dynamic analyses of the analytical model with flexible joints considering (a) near-field records and (b) far-field records

Fig.11 shows the maximum joint rotation values at each story level obtained from dynamic analyses of the analytical model with flexible joints. In parallel to the inter-story drift ratio results, joint rotation values regarding the records CNP106 and STG000 are equal to or higher than a rotation value of 0.02, that is roughly corresponding to the shear strength of the beam-column joints.

An inter-story drift level of 2% or a joint rotation value of 0.02 rad. may cause severe damage in a structure, so with these results it can be inferred that ignorance of shear deformation at beam-column joint and slip deformation at beam-joint interface may result in an incorrect performance assessment of a structure. Any frame may be concluded to have a performance level of “damage control” or “life safety” based on the analytical model with rigid joints whereas in fact, it may be in “collapse prevention” or “structural stability” performance

level. As a result, unrealistic modelling of beam-column joint may unsafely change the expected performance level of a structure. This should be seriously taken into consideration especially in determination of seismic performance level of an older-type building with unreinforced joints.

## 5. Conclusions

Nonlinear dynamic analyses of a three-bay, seven-story reinforced concrete frame have been performed. Two analytical models with rigid and flexible beam-column joint assumptions considering possible shear deformation at the joint and slip deformation at the beam-joint interface have been conducted. The results of the analyses indicate that the performance level estimations are very sensitive to the modelling assumptions of beam-column joints in buildings with unreinforced beam-column joints and may unsafely change the seismic performance level of the structure to be calculated. It is recommended to take into account the deformations occurring at beam-column joints in the assessment of nonductile older-type buildings with weak beam-column joints. In future research, frames with different number of stories will be subjected to numerous ground motion records with different types in order to investigate if the increase in deformation demand of an older-type structure with unreinforced joints is always the case with any kind of structure or ground motion record.

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