16th World Conference on Earthquake, 16WCEE 2017
Santiago Chile, January 9th to 13th 2017
Paper N° 3932 (Abstract ID)
Registration Code: S-A1464677271

ESTIMATION OF PLASTIC DEFORMATION OF VIBRATIONAL SYSTEMS USING THE HIGH-ORDER TIME DERIVATIVE OF ABSOLUTE ACCELERATION

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Abstract

Much research has been conducted in the area of structural health monitoring and damage detection, in order to develop methods which are capable of detecting aging deterioration and damage due to earthquakes as well as estimating the maintenance time. When a steel structure experiences a strong earthquake, damage such as yielding, fracture, recontact, and bolt slip in beams and columns, as well as anchor bolt extension and uplift at the column base may occur. These phenomena appear as nonlinearity in the load-displacement relationship. Detection of these changes can result in significant and valuable information regarding the evaluation of damage and residual performance. This study presents a method which is capable of detecting nonlinearity and damage from the absolute acceleration response time history.

In a previous paper, we have presented criteria to detect nonlinearity in the restoring force using the second time derivative of the absolute acceleration, which is often referred to as snap. In order to investigate the capability of snap in detecting changes in the stiffness of the system, earthquake response analysis of a single degree of freedom system was conducted. The analysis results show that discontinuous jumps, which coincide with the yielding locations, can be observed in the snap time histories. It is possible to detect the yielding point by comparing a predefined threshold value with the snap time history calculated from the second order central difference approximation of the second time derivative of the absolute acceleration response time history. A limitation of the method is the inability to quantitatively estimate the damage of the system.

The method presented in this paper overcomes the previous limitation by enabling the estimation of the plastic deformation using snap. In order to validate the estimation accuracy from the proposed method, plastic deformation computed from earthquake response analysis were obtained and compared to those estimated from the proposed method. The model is a single degree of freedom vibrational system with a bilinear elasto-plastic hysteretic restoring force. The results show that it is possible to estimate the plastic deformation with good accuracy without noise. On the other hand, in the presence of noise, the estimated value has larger error than the case without noise. In particular, it is observed that the error seems larger in the range of smaller plastic deformation. But it is possible to estimate the plastic deformation with approximately 20% error if the value of the plastic deformation becomes larger than 20% of the yield deformation.

Keywords: snap, jerk, plastic deformation, earthquake response analysis, bilinear-restoring force

1. Introduction

Much research has been conducted in the area of structural health monitoring and damage detection, in order to develop methods which are capable of detecting aging deterioration and damage due to earthquakes as well as estimating the maintenance time [1,2,3]. When a steel structure experiences a strong earthquake, damage such as yielding, fracture, recontact, and bolt slip in beams and columns, as well as anchor bolt extension and uplift at the column base may occur. These phenomena appear as nonlinearity in the load-displacement relationship. Detection of these changes can result in significant and valuable information regarding the evaluation of damage and residual performance. This study presents a method which is capable of detecting nonlinearity and damage from the absolute acceleration response time history.

In a previous paper, criteria to detect nonlinearity in the restoring force using the second time derivative of the absolute acceleration, which is often referred to as snap, has been presented [1]. In order to investigate the capability of snap in detecting changes in the stiffness of the system, earthquake response analysis of a single degree of freedom system was conducted. The analysis results show that discontinuous jumps, which coincide with the yielding time, can be observed in the snap time histories. It is possible to detect nonlinearity due to change in stiffness by comparing a threshold value with the snap time history calculated from the second order central difference of the absolute acceleration response time history. Moreover, the effect of noise contained in the absolute acceleration response time history on snap was estimated and a method to treat the noise has been proposed [2].

The method presented in this paper overcomes the limitations of previous damage detection theory by enabling the estimation of the plastic deformation using snap and jerk.

2. Definition of jerk and snap

2.1 Relationship between system response, jerk, and snap

In this paper, the first and the second time derivative of the absolute acceleration is referred to as jerk and snap, respectively. A single degree of freedom dynamical system with nonlinear restoring force $Q(x)$ and no damping under earthquake excitation can be represented as:

$$m\ddot{x} + Q(x) = -m\ddot{x}_0 \tag{1}$$

Here, m is the structural mass. The terms $\ddot{x}$, x, and $\ddot{x}_0$ represent the relative acceleration, relative displacement, and the ground acceleration, respectively. By defining the absolute acceleration $a = \ddot{x} + \ddot{x}_0$, the equation above can be rewritten as follows:

$$a = -\frac{1}{m}Q(x) \tag{2}$$

The first time derivative of the absolute acceleration a yields the jerk $\dot{a}$ as:

$$\dot{a} = \frac{\mathrm{d}}{\mathrm{d}t}\left\{-\frac{1}{m}Q(x)\right\} = -\frac{1}{m}\frac{\mathrm{d}Q(x)}{\mathrm{d}t} = -\frac{1}{m}\frac{\mathrm{d}Q(x)}{\mathrm{d}x}\frac{\mathrm{d}x}{\mathrm{d}t} = -\frac{1}{m}k(x)\dot{x} \tag{3}$$

Here, $k(x)$ is the tangent stiffness of the restoring force, $\mathrm{d}Q(x)/\mathrm{d}x$ and the term $\dot{x}$ represents the relative velocity. The second time derivative of the absolute acceleration a yields the snap $\ddot{a}$ as:

$$\ddot{a} = -\frac{1}{m}\left\{\dot{k}\dot{x} + k(x)\ddot{x}\right\} = -\frac{1}{m}\left\{k'(x)\dot{x}^2 + k(x)\ddot{x}\right\} \tag{4}$$

Here, $\dot{k}$ is defined as the derivative of $k(x)$ with respect to the time, and $k'(x)$ is defined as the derivative of $k(x)$ with respect to the relative displacement.

When the system behaves linear elastically with initial constant stiffness k_0 , jerk and snap can be obtained from Eq. (3) and Eq. (4):

$$\dot{a} = -\frac{k_0}{m}\dot{x} = -\omega_0^2\dot{x} \tag{5}$$

$$\ddot{a} = -\frac{k_0}{m}\ddot{x} = -\omega_0^2\ddot{x} \tag{6}$$

Here, ω_0 is the fundamental circular frequency. As seen in Eq. (6), in a linear system with constant initial stiffness, snap will harmonically oscillate proportional to the relative acceleration. On the other hand, in a system with variable stiffness, such as an elasto-plastic system, k' can become non-zero. Eq. (4) shows that a sudden change in the stiffness can result in an instantaneous large value of snap. If this change in snap can be properly detected, it can lead to a damage detection method for systems. However, it is not an easy task, since the first term in Eq. (4) is the product of k' and the square of the relative velocity. The maximum value of k' does not always correspond to the maximum of snap, since the relative velocity can be small. When the relative velocity is almost zero, the contribution of the first term is small compared to the second and the change in stiffness is difficult to detect.

2.2 Snap and jerk for discrete data

In the previous section, snap and jerk were calculated from the absolute acceleration for continuous time. But the accelerogram data obtained from acceleration sensors are recorded as discrete data. Therefore, snap and jerk must be computed from a finite difference equation. Here, formulas to calculate snap and jerk from discrete absolute acceleration data is presented and the relationship between the formulas and the system response is organized in the case of discrete data.

Snap can be computed from the second order central difference of the absolute acceleration using the following formula:

$$\ddot{a}_i = \frac{1}{\Delta t^2}\left(a_{i+1} - 2a_i + a_{i-1}\right) \tag{7}$$

Here, Δt represents the data time interval.

From Eq. (2), the absolute acceleration at the *i*-th time step, a_i can be represented with the restoring force $Q(x_i)$ as:

$$a_i = -\frac{1}{m}Q(x_i) \tag{8}$$

By defining the quantities in Fig. 1 and using Eq. (7) and (8), the discrete version of snap can be expressed as:

$$\ddot{a}_i = -\frac{1}{m}\left(\frac{{}_{\mathrm{f}}k_i - {}_{\mathrm{b}}k_i}{\Delta t}\cdot {}_{\mathrm{f}}\dot{x}_i + {}_{\mathrm{b}}k_i \cdot {}_{\mathrm{c}}\ddot{x}_i\right) \tag{9}$$

Here, ${}_{\mathrm{f}}k_i$ and ${}_{\mathrm{b}}k_i$ are secant stiffnesses, the relative velocity ${}_{\mathrm{f}}\dot{x}_i$, ${}_{\mathrm{b}}\dot{x}_i$, and the relative acceleration ${}_{\mathrm{c}}\ddot{x}_i$ are computed from the backward, the forward, and the central difference, respectively. Eq. (9) is the discrete version of Eq. (4). The first term is the product of the stiffness variation and the relative velocity at the *i*-th time step. When ${}_{\mathrm{f}}k_i = {}_{\mathrm{b}}k_i$, the term becomes zero, and if ${}_{\mathrm{f}}k_i \neq {}_{\mathrm{b}}k_i$, the term becomes a large value. However, if the time interval Δt is large, the term becomes smaller, since the term is inversely proportional to the time interval.

Jerk can be computed from the backward difference of the absolute acceleration using the following formula:

$$\dot{a}_i = \frac{1}{\Delta t}(a_i - a_{i-1}) \tag{10}$$

Using the quantities in Fig. 1 and Eq. (8) and (10) the discrete version of the jerk defined in Eq. (3) can be expressed as:

$$\dot{a}_i = -\frac{1}{m}\,{}_{\mathrm{b}}k_i \cdot {}_{\mathrm{b}}\dot{x}_i \tag{11}$$

When the system behaves linear elastically with initial constant stiffness k_0, the discrete versions of snap and jerk in Eq. (9) and (11) are reduced to:

$$\ddot{a}_i = -\frac{k_0}{m}\,{}_{\mathrm{c}}\ddot{x}_i = -\omega_0^2 \cdot {}_{\mathrm{c}}\ddot{x}_i \tag{12}$$

$$\dot{a}_i = -\frac{k_0}{m}\,{}_{\mathrm{b}}\dot{x}_i = -\omega_0^2 \cdot {}_{\mathrm{b}}\dot{x}_i \tag{13}$$

Eqs. (12) and (13) are the discrete versions of Eqs. (6) and (5).

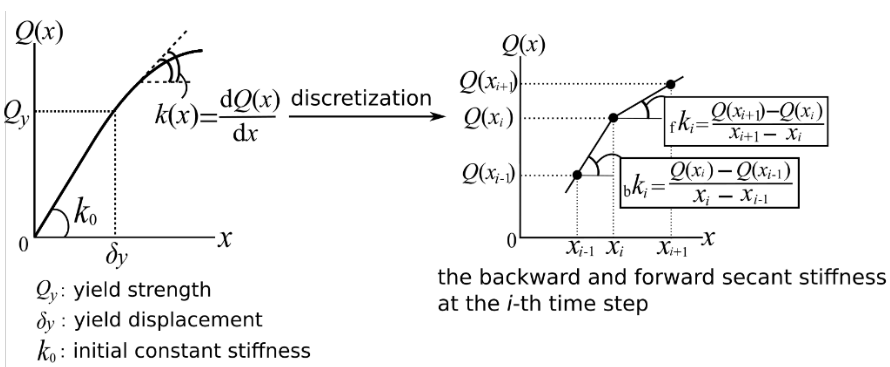


Fig. 1 – The stiffness of discrete data

3. Estimation method of plastic deformation

3.1 Calculation method of snap and jerk with noise reduction

The proposed method regards the absolute acceleration response a and the ground motion $\ddot{x}_0$ of the vibrational system, and the fundamental circular frequency ω_0 of the system as known values.

Snap and jerk are calculated from Eq. (7) and (10) using the absolute acceleration measurement. The absolute acceleration measurement contains noise, which affects the evaluation of snap and jerk. A previous paper [2] proposes a method to reduce the unnecessary noise in accelerograms. The method is summarized as follows.

In order to control the noise in the second time derivative of the absolute acceleration, the time interval Δt has to satisfy the following criterion:

$$\Delta t \geq \Delta t_{\mathrm{m}} = \frac{2}{\omega_0}\sqrt{\frac{m}{Q_y}N_{\max}} \tag{14}$$

Here, $N_{\max}$ is the maximum absolute value of noise. After the absolute acceleration time history is filtered with a low pass filter which has a cut-off frequency $1/(2\Delta t_{\mathrm{m}})$ corresponding to the Nyquist frequency for sampling time Δt_{m}, it is down sampled to the time step Δt_{m}.

3.2 Detection of nonlinearity in vibrational systems through snap

In the previous paper [1], when the system behaves linear elastically, the following bound for the snap value was presented:

$$-\omega_0^2\left(\frac{Q_y}{m}-\ddot{x}_0\right)\leq \ddot{a}\leq \omega_0^2\left(\frac{Q_y}{m}+\ddot{x}_0\right) \tag{15}$$

It is possible to detect the nonlinear behavior by using the maximum and minimum values of the bound as a threshold. It should be noted that the threshold value varies with time, as it depends on the ground acceleration $\ddot{x}_0$.

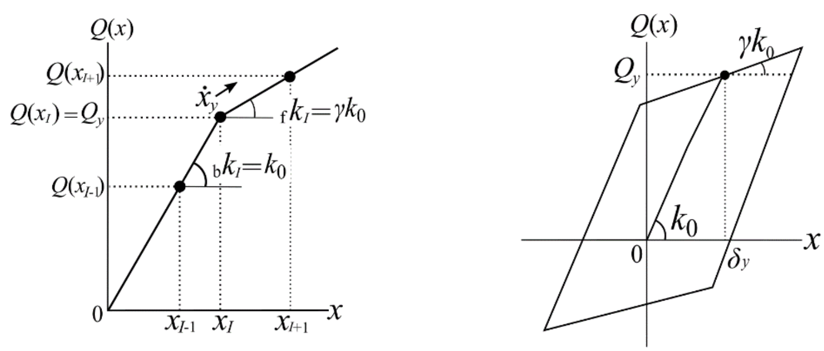
3.3 Calculation of the plastic deformation

First, the relative velocity when the system yields $\dot{x}_y$ is calculated from jerk and ω_0. As shown in Fig. 2(a), if the system were to yield at the *I*-th time step, $\dot{x}_{y,I}$ can be calculated from Eq. (13) as:

$$\dot{x}_{y,I}=-\frac{1}{\omega_0^2}\dot{a}_I \tag{16}$$

Second, the relative acceleration can be calculated from the absolute acceleration response and the ground motion, and the relative velocity can be calculated by numerical integration of the relative acceleration from the *I*-th time step until the *n*-th time step ($n\geq I$), where *n* is defined as the first time step at which the relationship $\dot{x}_{y,I}\cdot\dot{x}_n<0$ holds. The *n*-th time step corresponds to the unloading point.

Finally, the plastic deformation is obtained as the relative displacement calculated from numerical integration of the relative velocity from the *I*+1-th time step to the *n*-th time step.



(a)the backward and forward secant stiffness when the sysytem yields (b)hysteretic behavior

Fig. 2 – Bi-linear restoring force characteristic

4. Validation of the method with earthquake response analysis

4.1 Validation protocol flow-chart and analysis model

Fig. 3 shows the validation protocol flow-chart. In order to investigate the estimation accuracy of the proposed method, earthquake response analysis was conducted. The model is a single degree of freedom elasto-plastic vibrational system which has a bi-linear restoring force characteristics shown in Fig. 2(b). The term γ

representing the post yield stiffness ratio in Fig. 2(b), has two types, $\gamma = 0,\ 0.2$. The system has a predominant period of 1.0 seconds and a damping coefficient of 0%.

The yield strength Q_y is defined as:

$$Q_y = 0.3 Q_e \tag{17}$$

Here, Q_e is the maximum elastic force response of the system under the given ground motion. The Newmark Beta method is used to integrate the equations of motion with a fixed time step of $\Delta t = 0.004$ seconds.

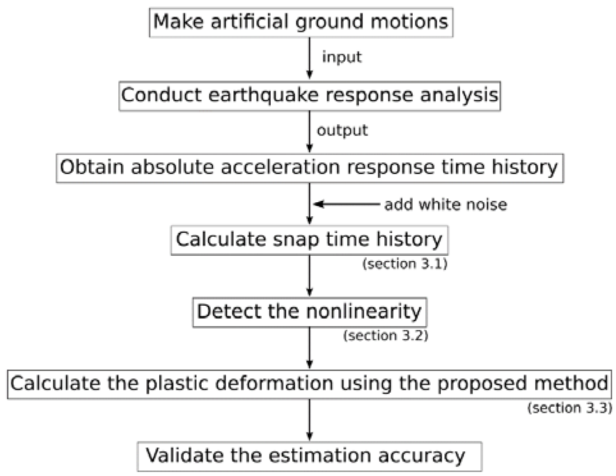


Fig. 3 – Validation protocol flow-chart

4.2 Input earthquake motion

An artificial ground motion of oceanic type is used. The artificial earthquake motion is generated by an inverse Fourier transform of the Fourier phase and Fourier amplitude spectrums. The Fourier amplitude spectrum is the given as the “Kanai-Tajimi spectrum” [3], where the predominant period is set to 1.0 seconds and the coefficient h_g determining the spectral shape to 0.6. The standard deviation of the Fourier phase difference distribution is $0.50 \times 2\pi$, which corresponds to the Fourier phase spectrum of an oceanic type earthquake [4]. The earthquake data is sampled at 50Hz, and the number of data points is 4096. This results in a ground motion with a duration of 81.92 seconds. Fig. 4 shows the Fourier amplitude spectrum and the acceleration time history. The earthquake data is linearly interpolated to match the time step in the earthquake response analysis.

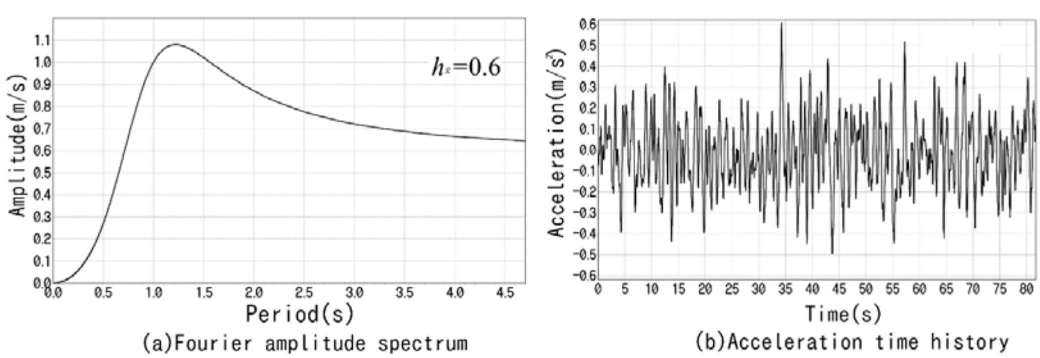


Fig. 4 – Artificial ground motion

4.3 Obtain absolute acceleration time history including noise

The sample absolute acceleration data used to validate the proposed method is obtained by adding white noise to the acceleration response records from the response analysis results. The white noise *N* generated from random number, has a maximum absolute value N_{max} equal to 2% of the maximum absolute value of the acceleration response record. In the following, this modified absolute acceleration time history is called the noisy acceleration time history.

4.4 Detection of the yielding point through snap time history

In order to reduce the effect of noise, the noisy acceleration time histories were down sampled to the time step $\Delta t_{\mathrm{m}} = 0.048$ s calculated from Eq. (14). Fig. 5 shows the snap time histories which were calculated from the method referred in section 3.1. The horizontal axis represents the time and the vertical axis the snap. Moreover, the gray line shows the threshold calculated from Eq. (15) and the black circles denote yielding.

In Fig. 5, regardless of the post yield stiffness ratio γ, the extremum points of snap exceeded the threshold whenever the systems yielded. Thus the method is able to detect the time when the systems yielded using snap and the threshold as long as the noise is properly reduced.

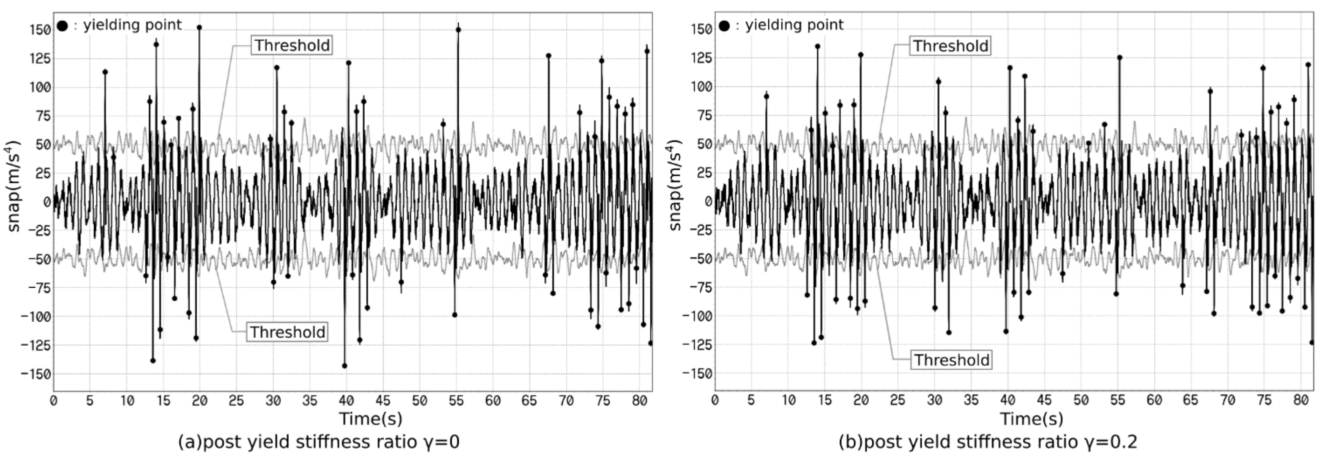


Fig. 5 – Snap time history

4.5 Validation of the estimation accuracy

In the previous section, it was shown that the extremum points of snap exceeding the threshold corresponds the yielding points. So, the plastic deformation can be estimated from the method proposed in section 3.3. Fig. 6 shows the estimation accuracy of the plastic deformation at each occurrence of yielding. Fig. 6 also shows the estimation accuracy of the relative velocity at the yielding point, since the plastic deformation estimated from the proposed method is affected by the relative velocity calculated from jerk. The horizontal axis represents the estimated value calculated from the method and the vertical axis the analytical value obtained from the response analysis. For comparison, the estimation accuracy in the case of the absolute acceleration without noise is also shown.

The relative velocity of the yield point and the plastic deformation in the case without noise is estimated with reasonable accuracy, regardless of the post yield stiffness ratio. In most cases the estimated value is larger than the analytical value. This is assumed to be due to the difference in the way the relative velocity at the initiation of yielding $\dot{x}_y$ is evaluated between the proposed method and directly from the response analysis. The proposed method calculates $\dot{x}_y$ from Eq. (16) by taking a backward difference of the absolute acceleration as opposed to the direct method from response analysis where it is obtained naturally through the Newmark Beta method time integration procedure. On the other hand, in the case with noise, the estimated value has larger error for both the relative velocity and the plastic deformation. In particular, it is observed that the error seems larger in the range of the smaller value of both the relative velocity of the yield point and the plastic deformation. Since the snap time history was obtained from the absolute acceleration which is down sampled, it is assumed that time lag would occur between the extremum times of snap exceeding the threshold and the time when the systems yielded.

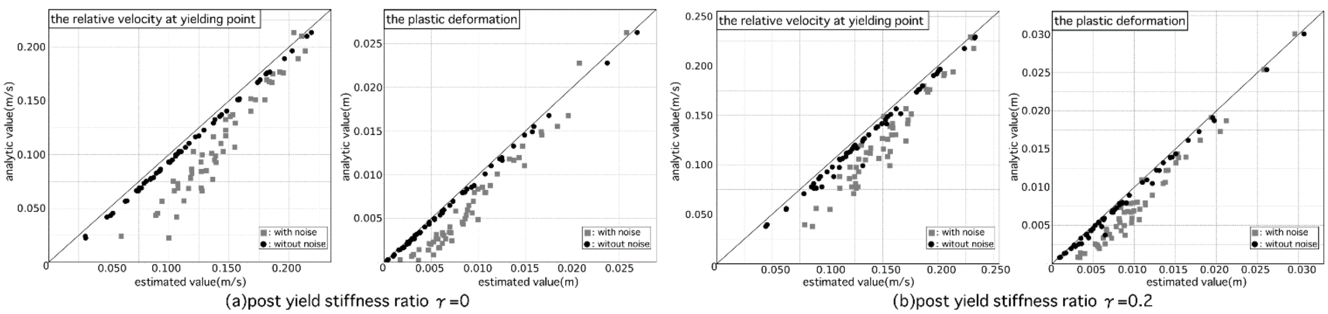


Fig. 6 – Comparison between estimated and analytic value

Fig. 7 shows the relationship between the ductility factor and the relative error of the plastic deformation. Here, the ductility factor represents the ratio of the actual plastic deformation to the yield displacement of the systems. The horizontal axis represents the ductility factor and the vertical axis the relative error of the plastic deformation. Regardless of the presence of noise and post yield stiffness ratio, the estimation accuracy increases, as the ductility factor increases.

Table 1 shows the average of the relative error in the prediction of the plastic deformation for three ranges of ductility factors. In the case with noise, regardless of the ductility factor, it is possible to estimate the plastic deformation with a good accuracy. However, in the case without noise, it is difficult to estimate the plastic deformation for ductility factors in the ranges of 1.0 to 1.1. But it is possible to estimate the plastic deformation with approximately 20% errors for ductility factors in the range 1.2 and larger.

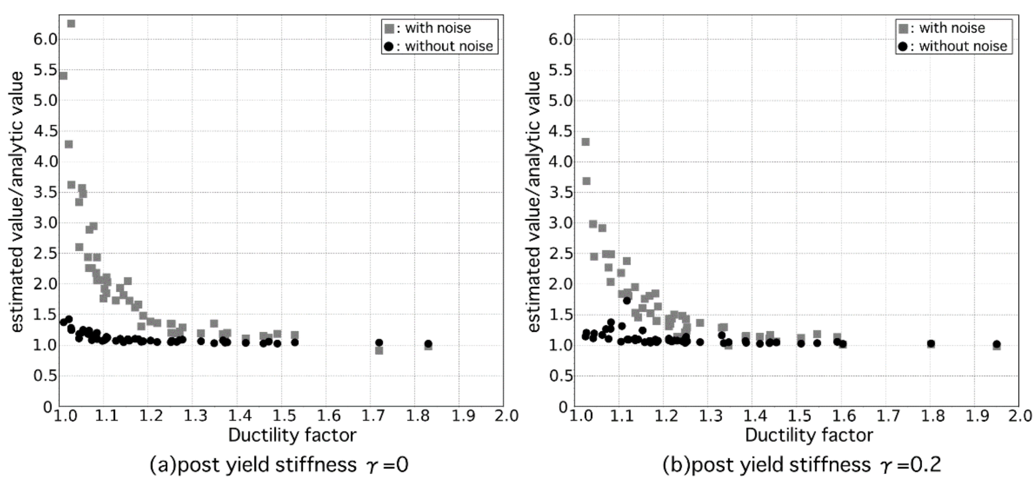


Fig. 7 – Relationship between ductility factor and estimation accuracy

Table 1 – The average of the relative error in the prediction of plastic deformation for different ranges of ductility factors

Noise	Post yield stiffness ratio γ	Ductility factor		
		from 1.0 to 1.1	from 1.1 to 1.2	1.2 and larger
Without	0	1.11	1.07	1.05
	0.2	1.11	1.09	1.06
With	0	2.02	1.43	1.20

	0.2	1.66	1.41	1.22

5. Conclusion

This paper presents a method to estimate the plastic deformation of a single degree of freedom system using snap and jerk, which are the second and first time derivative of the absolute acceleration respectively. The onset of yielding can be detected using snap, and the plastic deformation after yielding of the system can be calculated using jerk.

The method is validated for single degree of freedom elasto-plastic systems of the bi-linear restoring force characteristics with two different post yield stiffness ratios, and no damping. Dynamic response analysis is performed and the response accelerogram is obtained for the two cases considered, where in one case white noise is added to the accelerogram to simulate the measuring device noise. Then using the two accelerograms, plastic deformation at each yielding is estimated by the proposed method. By comparing the estimated plastic deformation with those from the dynamic response analysis, the follow conclusions are obtained:

1. The results show that it is possible to estimate the plastic deformation with good accuracy in the case without noise.
2. In the case with noise, the average error in the prediction of plastic deformation is large when the plastic deformation is small compared to the yield deformation. The error decreases as the plastic deformation increases and falls below 20% when the plastic deformation is larger than 20% of the yield deformation. This is due to the decrease in error of evaluating the relative velocity at the point of yielding.

This paper verifies the case of no damping. And the threshold value and the time interval for reducing noise effects are calculated using the yield strength. But an existing full scale structure has damping and it is difficult to measure the yield strength for the existing full scale structure. In the future work, the proposed method will be verified to systems with damping and a method obtained the threshold value and the time interval will be proposed without the yield strength.

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