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# On rocking core-moment frame design

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***Abstract-*** The paper introduces a number of new analytic techniques and practical technologies that lead to the development of an efficient earthquake resistant system. It is capable of preventing severe damage to its columns and footings and can be equipped to prevent catastrophic collapse and re-center itself due to strong ground motion. The analytic development is based on global, design led analysis and results in closed form, exact solutions that are highly suitable for manual as well as spread sheet computations.

*Key words: earthquakes, design led analysis, collapse prevention, self-centering, damage avoidance, uniform drift*

## 1. Introduction

The vast majority of earthquake resisting structures consist of fixed base Moment Frames (MF) combined with fixed base shear walls and/or braced frames. While these systems have served their functions rather well, they are not free from technical flaws and socio-economic drawbacks. To address some of these issues, the authors propose a new dual earthquake resisting system consisting of Grade Beam Supported MFs (GBSMF) in combination with Rigid Rocking Cores (RRC) that promises greater utility and futuristic options than its conventional counterparts. The theoretical bases of the proposed development have been compiled in sections 1-4 of the current article. Sections 5-6 discuss the practical design aspects of the proposed solutions. A brief discussion of the proposed structural system is presented in section 7.

### 1.1 Bases for structural system development

All codes of practice, e.g. ASCE (2007) recommend design and construction features that are important to seismic performance. With these guidelines in mind an attempt is made to propose a viable alternative that is free from the known flaws of fixed base moment frame-core combinations. The evolution of a new earthquake resisting involves the fulfillment of the following developmental steps;

- Definition of system objectives and functional requirements based on technical knowledge and observed data. E.g., a system that can sustain the prescribed drift ratio nearly uniformly along its height with built in provisions for self-centering, damage reduction, and collapse prevention. Grigorian (2015a, b).
- Analytic or experimental verification of the ability of the system to meet performance level objectives
- Verification of suitability of new components and supplementary devices, e.g., gap opening and closing beams tend to expand their spans by as much as the gap. Span growth not only induces additional moments on the adjoining columns but also damages column-diaphragm interfaces and results in high drift concentration Dowden (2011). Similarly the interface between the RRC and floors should be carefully detailed so as not to compromise the integrity of the connection. The important lessons learned from these sources, for developing a new earthquake resisting system, can be summarized as that;
- The assumed seismic load distribution should be as close to that generated by the structure as possible.
- The kinematics and boundary conditions of the MF should be selected in such a way as to reduce seismic demand and to prevent damage to columns (especially at supports) and footings.
- RRC can prevent soft story failure, minimize damage to columns, footings and their connections.

- The RRC, BRBs and LBs can be used as self-centering and collapse prevention mechanisms. These ideas have been combined to develop a new RRC-GBSMF system, also known as rocking wall/core moment frame, (RWMF) / (RCMF) respectively. The practical applications and limitations of these ideas are briefly discussed in the forthcoming section.

## 2 Basic attributes of the proposed dual system

When designed as part of the RCMF, the MF tends to exhibits some of the attributes of structures of uniform shear, such as minimum self weight and uniform demand-capacity ratios close to unity. Seismic loading is a function of building mass and stiffness. The conventional triangular distribution suggested by most codes of practice, e.g. ASCE/SEI 7-10, may or may not correspond to the physical realities of a given structure. In the proposed structural system, the RRC forces the RCMF to adapt a linear deformation profile and a similar normalized displacement function. While the rigid body rotation of the system changes with monotonic changes in the applied loading, its deformed shape remains the same. Furthermore the first natural mode is similar in shape to the displacement profile and suppresses all higher modes of vibrations. The concurrence of the load function and the displacement profiles is significant in that it leads to a number of findings which in turn help formulate a simple but accurate solution to an otherwise difficult problem:

- The dominance of the first mode qualifies the system for nonlinear static analysis, (FEMA 2005). The similarity of the load and deformation profiles, relates member forces and deformations, the failure load and the mode of collapse to the same normalized straight line.
- The force-displacement relationship of the RCMF can be expressed as the function of a single variable for all loading conditions.
- The uniformity of the drift function allows the structure to be treated as a SDOF system. *This makes the structure ideally suited to equal energy treatment for base shear computations.*
- The structure can be designed as a MF of Uniform Shear (MFUS). A description of MFUS is presented in section 6.2 below.
- The system lends itself well to design led manual analysis. I.e., *groups of similar elements such as beams, columns, braces, links and their connections can be studied as independent groups of members.*
- The normalized displacement function is a straight line.
- Uniform drift reduces the *P*-delta effects and residual displacements.
- Some of the technical advantages of GBSMF over fixed base frames can be summarized as follows;
- The drift ratios of GBSMF at incipient collapse are smaller than those of identical frames with fixed and pinned boundary conditions. *For fixed base MFs the abrupt change of stiffness from base to first level produces a sharp concentration of the drift ratio and increases the corresponding angle of rotation.*
- Being more flexible, GBSMF tend to attract smaller seismic forces than their fixed base counterparts. *The global stiffness of* GBSMF *is smaller than those of the geometrically similar fixed base systems.*
- GBSMF attract substantially less residual stresses and deformations due to strong ground motion. *The stiffnesses of the grade beams are selected in such a way as not to cause drift concentration at the base*
- In GBSMF overturning moments are transmitted to the footings through axial reactions only. *No anchor bolt, base plate and footing damage can occur due to seismic moments.*
- The grade beams prevent the formation of plastic hinges at column supports and provide means of controlling column base rotation and the overall drift.
- Uniform drift or tilt causes all points of contraflexure to move towards mid spans.
- The physical behavior of RCMF can best be visualized by the frame restraining the RRC in place, and the core imposing uniform or near uniform drift along the height of the frame, Figs.1 (a) and 2(g).
- The wall causes wall attached supplementary devices to absorb proportional amounts of energy.
- The RRC tends to rotate as a rigid body without significant in-plane deformations. If needed, a post tensioned core can be activated to prevent collapse and initiate self-centering.
- The RRC helps reduce axial forces in the frame. This increases moment capacities and reduces costs. *A large proportion of axial forces due to overturning moments is absorbed by the RRC.*

- The RRC tends to redistribute seismic moments evenly between groups of similar member such as beams and columns of equal lengths respectively. In MFUS the core absorbs the entire seismic force and exerts a self-balancing reaction on top of the frame.
- The RRC can enforce the desired mode of failure, but cannot increase the carrying capacity of the free standing structure.
- The RRC tends to bend as an upright simply supported beam rather than a vertical cantilever. The base pivot provides two degrees of restraints. The MF adds one more lateral restraint and makes the core act as a statically determinate beam.
- RRCs need not be overwhelmingly rigid. However they should be stiff and strong enough to enforce an almost uniform drift and prevent soft story failure. *The ultimate strength of the* RRC *should be greater than that of the* MF *and the supplementary devices. The stiffness of the* RRC *should be selected in such a way as to reduce its own maximum drift to less than a fraction of the prescribed design drift ratio.*
- Post-tensioned rigid cores such as concrete or steel shear walls and braced frames can be designed to remain entirely elastic after frame failure. The lateral restoring capabilities of the energized RRC mechanism are defined by its ultimate strength and rotational stiffness respectively.
- The function of the LB is to transfer diaphragm shear, add stiffness to the structure, dissipate seismic energy, provide leverage for self-centering and collapse prevention. The conventional LB systems tend to expand their spans beyond the original length. As the gap widens, the beams rotate rigidly and bend the column in proportion to the gap. This in turn reduces the effective post tensioning force and severely damages the beam-column-diaphragm interfaces. The use of fully flat end LBs is not encouraged, instead a number of modified link beam systems are suggested Figs. 6(a) and (c). The proposed LBs consist of full length, axially strong elements with provisions for post tensioning cables.
- The ends of the proposed LBs may be beveled Dowden (2011), or truncated Grigorian (2015c), to avoid full contact between the ends of link and the adjoining members. The LB is activated in response to the target drift angle $\phi$ due to external forces. Gap opening and closing need not necessarily occur between contact surfaces. *It can take place between any two adjacent planes at right angles to the axis of the beam*. Gap opening is accompanied with changes in the initial stresses of the post tensioned tendons.
- The response of the LB is sensitive to its layout, the pre-stressing force and the offsets from the center lines of the adjoining columns $D_{left}$ and $D_{right}$. *The effect of such offsets is to increase the gap angle from* $\phi$ *to* $\overline{\phi} = \phi[l + D_{left} + D_{right}]/l = \alpha\phi$. The special cable layouts of Figs. 6 are meant to eliminate loss of stretch due to simultaneous gap opening and closing at opposite ends of the same LB.
- The gap opening property of the LB can be symbolized by equivalent rotational springs at each end. *The equivalent stiffnesses of the* LBs *at the wall and frame side are given as* $K_{Di}$ *and* $\overline{K}_{Di}$ *respectively*.
- Gap movements at the ends of the Ladd natural damping and provide opportunities for self-centering, damage reduction and collapse prevention. *Such devices reduce frame moments and drift ratios.*

## 3 .The theoretical approach

The major components of a typical RCMF consisting of the MF, the RRC, the LB and BRB are shown in Fig.1 (a). The challenge here is to understand the behavior of the major components of the system. This is achieved by looking at the problem from a global point of view and treating the responses of groups of similar elements as the constitutive components of the structure. Here, the drift ratio, stability and failure conditions are imposed rather than investigated. The design of RCMFs with a view to collapse prevention and self-centering involves understanding of the physical phenomena surrounding the following issues;

- The relationship between the global rotational stiffness $K^*$ and any expected drift ratio $\phi$.
- The relationship between the global rotational stiffnesses $K_F$ and $K_M$ of the MF, $K_L$ of the LB, $K_B$ of the BRBs and $K_C$ of the RRC, needed to sustain the drift ratio $\phi$.
- The magnitude of the effective overturning moment, $M_{eff.}$, and
- The ultimate lateral capacity of the system.

Knowing that the entire structure tends to respond as a single degree of freedom system, the answer to the first issue may be found in the simple linear relationship;

$$\phi = \frac{\mathrm{M}_{eff.}}{K^*} \tag{1}$$

Following the same rationale, the answer to the second question can be sought in the moment-rotation relationships of each component, i.e.;

$$\phi_F = \frac{M_F}{K_F},\ \phi_M = \frac{M_M}{K_M},\ \phi_B = \frac{M_B}{K_B} \text{ and } \phi_C = \frac{M_C}{K_C} \tag{2}$$

Subscripts ***F, M, B*** and ***C*** refer to MF, LB, BRB and the RRC respectively. The answer to the third concern is that the total external overturning moment $M_0$ is magnified or accompanied with the global *P*-delta moment, $M_{P\Delta}$ which leads to the effective overturning moment;

$$\mathrm{M}_{eff.} = M_0 + M_{P\Delta} \tag{3}$$

The intuitive answer to the last question may be expressed as;

$$\mathrm{M} = M_0 + M_{P\Delta} - (M_B + 2M_M \ + M_C) \tag{4}$$

Here $\mathrm{M}$ describes the total moment of resistance of the beams of the frame. A poof of statement (4) is presented in section 3 below. The elegance and power of Eq. (1) is in that it allows the external effects as well as the contributions of all resisting elements and supplementary devices to be expressed as global moments. For instance the net effect of the lateral forces of Fig. 1(c) can be replaced with their equivalent overturning moment $M_0$. Conversely, the effect of any global moment can be simulated by equivalent horizontal force acting at roof level.

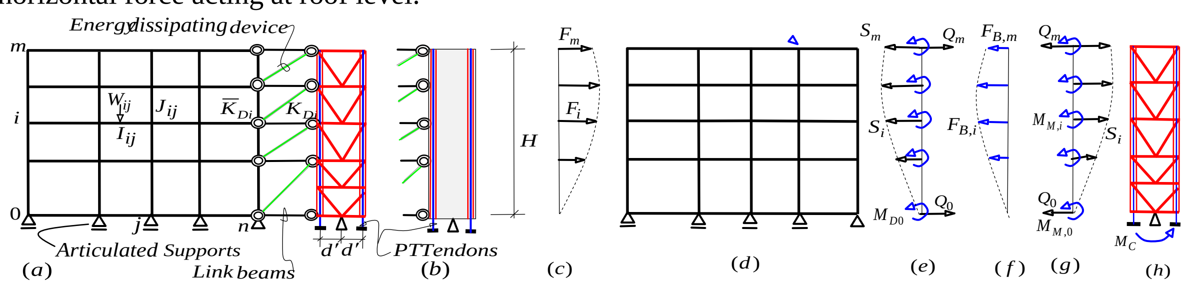

Fig.1 (a) GBSMF with braced RRC and supplementary devices, (b) Concrete RRC, (c) Lateral loading, (d) GBSMF, (e) Interactive forces and gap opening moments acting on the frame, (f) Assumed BRB reactions, (g) Interactive forces and gap opening moments acting on the RRC, (h) RRC base restoring moment.

### 3.1 Global elastic response of the frame

Consider the lateral displacements of the GBSMF of Fig. 1(a), under gravity loads $W_{i,j}$, lateral loading $F_i$, Fig.1(c), opposing brace forces $F_{B,i}$, Fig.1 (f), gap opening moments $M_i$ of Fig. 1(e) and rigid core restoring moments $M_C$ of Fig.1 (h). Figs.1 (e) and (g) depict the distribution of the interactive forces between the frame and the RRC. The influence of gravity loads $W_{i,j}$ on the lateral displacements of the MF has been studied in some detail in section 4 below. The rigid core imposes a uniform drift ratio $\phi$ on the entire structure. The drift ratios of each supplementary device can be studied in terms of the corresponding beam and column rotations, i.e.,

$$\phi_F = \theta_{col,F} + \theta_{beam,F},\ \phi_M = \theta_{col,M} + \theta_{beam,M},\ \phi_B = \theta_{beam,B} + \theta_{col,B} + \varphi \text{ and } \phi_C = \phi \tag{5}$$

Here, $\theta_{col.}$ and $\theta_{beam}$ stand for rotations due to column and beam bending respectively. Since the sum of all end moments due to external moments $M_i$ is zero then $\theta_{col,M} = 0$. Next assuming that the secondary effects due to brace movement are negligible then, $\theta_{beam,B} = \theta_{col,B} = 0$. Hence, the frame rotation may be equated to the rigid body tilt $\varphi$, of the system, in which case Eq. (5) can be simplified as;

$$\phi_F = \theta_{col,F} + \theta_{beam,F},\quad \phi_M = \theta_{beam,M},\quad \phi_B = \varphi \text{ and } \phi_C = \phi \tag{6}$$

The net lateral story level displacements of the combined structure, Fig. 2 (a), can be computed as;

$$\Delta_i = (\phi_F - \phi_M - \phi_B - \phi_C)x_i = \phi x_i \tag{7}$$

### 3.2-Free standing frame under lateral forces $F_i$

Since $\phi$ is constant it would be convenient to compute $\theta_{col.}$ and $\theta_{beam}$ independently, assuming the other is zero. Figs. 2(d) and 2(e) depict the imaginary scenarios where $\theta_{beam} = 0$ or $I_{i,j} = \infty$ and $0 > J_{i,j} > \infty$, and $\theta_{col.} = 0$ or $J_{i,j} = \infty$ and $0 > I_{i,j} > \infty$ respectively. If the resulting end moments of column ***i,j*** due to the former condition is denoted by $M_{col.ij} = 6E\bar{k}_{ij}\theta_{col.F}$, where $\bar{k}_{ij} = J_{ij} / h_i$, then the sum of all column moments should balance the total external overturning moment;

$$M_0 + M_{P\Delta} = 2\sum_{j=0}^{n}\sum_{i=1}^{m} M_{col,ij} = 12E\theta_{col.F}\sum_{j=0}^{n}\sum_{i=1}^{m}\bar{k}_{ij} \tag{8}$$

A method of computing the global *P*-delta moment is presented in section 3.7. Similarly, if the resulting end moments of beam *i,j* due to the latter condition are given by $M_{beam.ij} = 6Ek_{ij}\theta_{beam.F}$, where $k_{ij} = I_{ij} / L_j$, then the sum of all internal beam moments should also balance the total external overturning moment;

$$M_0 + M_{P\Delta} = 2\sum_{j=0}^{n}\sum_{i=1}^{m} M_{beam,ij} = 12E\theta_{beam.F}\sum_{j=1}^{n}\sum_{i=1}^{m} k_{ij} \tag{9}$$

Substitution of Eqs.(8) and (9) into the first of Eqs.(6) gives;

$$\phi_F = \theta_{col,F} + \theta_{beam,F} = \frac{(M_0 + M_{P\Delta})}{12E}\left[\frac{1}{\sum_{j=0}^{n}\sum_{i=1}^{m}\bar{k}_{i,j}} + \frac{1}{\sum_{j=1}^{n}\sum_{i=0}^{m} k_{i,j}}\right] = \frac{(M_0 + M_{P\Delta})}{K_F} \tag{10}$$

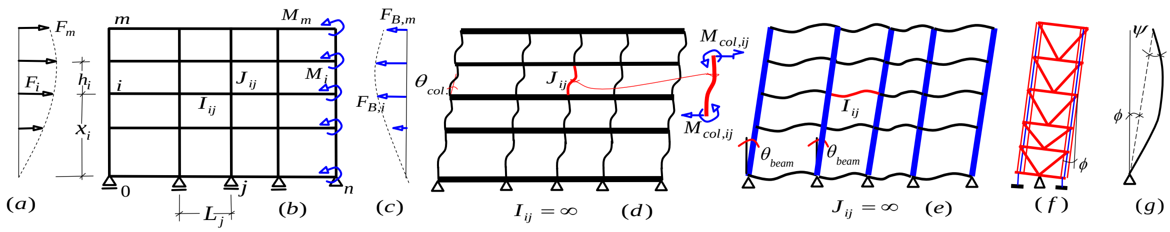


Fig. 2 (a) Loading, (b) GBSMF with gap opening moments, (c), Reactions due to BRBs, (d) GBSMF with rigid beams,(e) GBSMF with rigid columns, (f) Tilted RRC, (g) Deformation of the combined structure.

### 3.3 Frame reacted by link beams (under end moments $M_M$)

Let $M_i$ represent the opposing moments generated at the ends of the link beams. Following the rationale presented in section 3.2, the effects of $M_i$ on the moment frame can be expressed as;

$$\phi_M = \theta_{beam,M} = \frac{\sum_{i=0}^{m} M_i}{12E\sum_{j=1}^{n}\sum_{i=1}^{m} k_{i,j}} = \frac{M_M}{K_M} \tag{11}$$

$M_M$ is the moment of resistance generated by the frame side end of all LBs and as such may be treated as a restoring moment and can be related to the gap opening angle $\bar{\phi}$ and the LB rotational stiffness $k_{L,i}$. i.e.,

$$M_M = \sum_{i=0}^{m} M_i = \sum_{i=0}^{m} k_{L,i}\bar{\phi} = \bar{\phi}\sum_{i=0}^{m} k_{L,i} = \alpha\phi K_L \tag{12}$$

The relationship between the tendon force $T_{ten}$, $\bar{\phi}$ and tendon extension may be expressed as;

$$\bar{\phi} = \alpha\phi = \frac{2T_{ten}L_{ten}}{dA_{ten}E} = \frac{4T_{ten}dL_{ten}}{d^2A_{ten}E} = \frac{M_i}{k_{L,i}} \quad \text{and} \quad k_{L,i} = \frac{d^2A_{ten}E}{4L_{ten}} \tag{13}$$

Moments $M_M$ at the two ends of the LB tend to reduce the drift ratio by bending the MF and tilting the RRC in opposite direction to the applied loading, thus;

$$\phi = \frac{(M_0 + M_{P\Delta} - M_M)}{K_F} - \frac{M_M}{K_M} = \frac{(M_0 + M_{P\Delta} - K_L\phi)}{K_F} - \frac{K_L\phi}{K_M} \quad \text{or} \quad \phi_F = \frac{(M_0 + M_{P\Delta})}{[1+\bar{K}]K_F} \tag{14}$$

$\overline{K} = K_L[(1/K_M)+(1/K_F)]$ and $K_F(1+\overline{K})$ may be regarded as the percentage contribution of the LBs and the equivalent rotational stiffness of the MF respectively. If for any reason $\phi_F$ exceeds $\phi_{Rqd.}$ or $K_F$ is deemed inadequate then Eq. (14) may be utilized to asses the additional stiffnesses needed to satisfy the issue, thus;

$$K_L = \left[\frac{(M_0 + M_{P\Delta})}{K_F \phi_{rqd.}} - 1\right] \frac{K_M K_F}{K_M + K_F} \tag{15}$$

It is instructive to note that as plastic hinges form at the ends of the beams of the frame, the relative stiffness of all beams become zero. $k_{i,j}$ tend toward zero and Eq. (14) reduces to;

$$\phi_F = \frac{(M_0 + M_{P\Delta})}{2K_L} \tag{16}$$

**3.4 Frame supplemented with diagonal braces (**resistive moments $M_B$**)**

The purpose of this section is to assess the contribution of the diagonal BRBs of Fig. 3 (c) to the global stiffness of the structure. This is achieved by assuming that all members of the imaginary braced frame, including the common vertical with the MF are infinitely rigid and constitute an unstable mechanism as shown in Fig. 3(b). All diagonals are pin ended axially hysteretic BRBs. Following Eq. (A2), the axial deformations $\overline{\Delta}_i$ of any such brace can be related to the uniform drift ratio, i.e;

$$\overline{\Delta}_i = \frac{\alpha\phi h_i l}{\overline{L}_i} = \frac{T_i \overline{L}_i}{A_i E_{brc}} \quad , \quad T_i = \frac{\alpha\phi h_i l A_{,i} E_{brc,}}{\overline{L}_i^2} \quad \text{or} \quad A_i = \frac{T_i \overline{L}_i^2}{\alpha\phi h_i l E_{brc}} \tag{17}$$

Considering the rigid body rotation $\phi_B$ of Fig. 3(d), the virtual work concept leads to;

$$\textstyle\sum_{i=1}^m F_{B,i}\phi x_i = \phi M_B = \sum_{i=1}^m T_i \overline{\Delta}_i = \sum_{i=1}^m \left[\dfrac{\alpha^2\phi^2 h_i^2 l^2 A_i E_{brc}}{\overline{L}_i^3}\right] \tag{18}$$

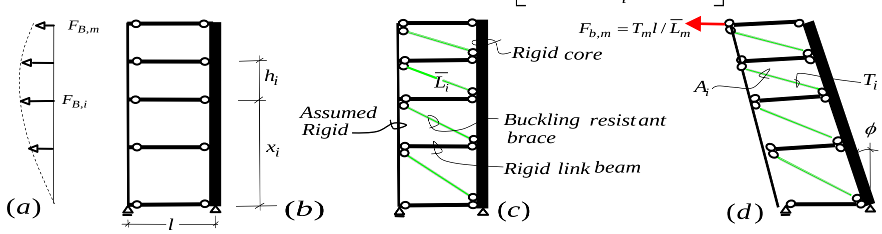


Fig. 3 (a) Notional load, (b) Imaginary mechanism, (c) Braced frame, (d) Rigid body displacement

Or
$$\phi_B = \frac{M_B}{\alpha^2 l^2 E_{brc} \sum_{i=1}^m (h_i^2 A_i / \overline{L}_i^3)} = \frac{M_B}{K_B} \tag{19}$$

Note that $\overline{\phi}$ is a function of the overturning moments caused by forces $F_{B,i}$ of Fig. 3(a). This implies that the effort to generate a rigid body rotation $\overline{\phi}$ would be resisted by an additional internal moment $M_B$ that tends to restore the structure to its original position. Conversely, the use of supplementary devices such as BRBs would tend to reduce the effects of the external overturning moments $M_0$ by as much as $M_B$. If $M_B$ corresponds to $\overline{\phi}$ at first yield, then the total carrying capacity of the system can be estimated as;

$$\textstyle\sum_{i=1}^m F_{B,plastic,i} x_i = M_{B,plastic} = \sum_{i=1}^m T_{plastic,i} \dfrac{\alpha h_i l}{\overline{L}_i} \tag{20}$$

$T_{plastic,i} = \sigma_{yield} A_i$ , is the ultimate axial strength of the brace. The braced frame tends to oppose the external overturning moment by a notional moment of resistance related to the axial resistance of its members, i.e.,

$$\phi = \frac{M_0 + M_{P\Delta} - M_B}{K_F} = \frac{M_0 + M_{P\Delta} - \phi K_B}{K_F} \quad \text{or} \quad \phi_F = \frac{M_0 + M_{P\Delta}}{[K_B + K_F]} \tag{21}$$

Once again, if $\phi$ exceeds $\phi_{Rqd.}$ or $K_F$ is deemed inadequate then Eq. (21) may be utilized to asses the additional stiffnesses of the supplementary braces to satisfy the issue, thus;

$$K_B = \frac{M_0 + M_{P\Delta} - \phi K_F}{\phi} \tag{22}$$

It is expedient to deal with a single force $F_{B,m} = T_m l / L_m$ as shown in Fig. 3(d) that causes constant shear along the height of the imaginary braced frame, rather than the distribution of forces $F_{B,i}$ of Fig. 3(a) that result in a stepwise variation of shear along the frame. The brace force distribution due to the former strategy can be expressed as; $T_m = F_{B,m}\bar{L}_m / l, \ldots$, $T_i = F_{B,m}\bar{L}_i / l, \ldots$, and $T_1 = F_{B,m}\bar{L}_1 / l$. This allows all brace cross sectional areas $A_i$ to be related to any known value such as $A_m$, $A_i = (\bar{L}_i / \bar{L}_m)^3 (h_m / h_i)$. Since all brace forces are functions of the same variable $\phi$ and that internal forces of all members, are in static equilibrium, then the global moment due to brace resistance can be directly assessed as;

$$M_B = \frac{T_m l H}{L_{brc}} \tag{23}$$

### 3.5 RRCs contribution to restoring moments

The RRC is acted upon by the interactive overturning moments ($M_Q - M_S$) corresponding to forces $S_i$ and $Q_m$ and the restoring moments $M_M$ and $M_C$. The interactive force $Q_m$ has been introduced to emphasize the fact that the RRC tends to behave as an upright simply supported beam rather than a fixed base cantilever. The static equilibrium of the RRC requires that, $M_Q - M_S = M_M + M_C$. Assuming the RRC is sufficiently stiff and strong, then the restoring moment, $M_C$ developed at the pivot level due to rigid body rotation $\phi$, can be expressed as;

$$M_C = T_{RRC} d' = K_C \phi \quad \text{and} \quad K_C = d'^2 A_{walltendon,} E / H \tag{24}$$

Since $M_C$ opposes $M_0$ without coupling with other components of the structure, then its contribution to the global deformations of the system can be expressed as;

$$\phi = \frac{M_0 + M_{P\Delta} - M_C}{K_F} = \frac{M_0 + M_{P\Delta} - \phi K_C}{K_F} \quad \text{or} \quad \phi = \frac{M_0 + M_{P\Delta}}{[K_F + K_C]} \tag{25}$$

### 3.6 Global response of the RCMF (under $M_0, M_M$, $M_B$ and $M_C$)

The fully supplemented structure is subjected to external and interactive forces $F_i$ and $(Q_m - S_i)$ respectively, as illustrated in the free body diagrams of Figs. 1(c, d, e and, f). The drift functions (10), (11), (19) and (25) can be utilized to formulate the global drift equation of the subject RCFM, provided that the unknown quantity ($M_Q - M_S$) can either be determined or assigned a value in terms of $(M_M + M_C)$. Following the rationale leading to Eqs. (14) and (21), it may be concluded that;

$$\phi = \frac{M_0 + M_{P\Delta} + M_Q - M_S - M_B}{K_F} - \frac{M_M}{K_M} \quad \text{or} \quad \phi = \frac{M_0 + M_{P\Delta} - (M_B + M_M + M_C)}{K_F} - \frac{M_M}{K_M} \tag{26}$$

where, $(M_M + M_C)$ has been substituted for $(M_Q - M_S)$. Next, substituting for $M_C = K_C\phi, M_M = K_L\phi$ and $M_B = K_B\phi$ into Eq. (26), it gives after simplification and rearrangement;

$$\phi = \left[\frac{K_M(M_0 + M_{P\Delta})}{K_F K_L + K_M(K_F + K_L + K_B + K_C)}\right] = \frac{M_0 + M_{P\Delta}}{K^*} \tag{27}$$

It is instructive to note that as the MF becomes a mechanism, i.e., as $K_F$ and $K_M$ tend toward zero, the structure becomes more flexible. It sustains larger but more stable lateral displacements due to the resistive nature of the supplementary devices. In other words Eq. (27) reduces to;

$$\phi = \left[\frac{1}{2K_L + K_B + K_C}\right](M_0 + M_{P\Delta}) \tag{28}$$

Eq. (28) constitutes a lower bound solution to the failure conditions of the subject RCMF. Denoting $\mathrm{M} = K_F \phi$ as the total moment of resistance of the free standing MF and observing that $(K_F / K_M)$ becomes unity as $k_{i,j}$ approached zero, then the second of Eqs.(25) may be rewritten as;

$$\mathrm{M} = M_0 + M_{P\Delta} - (M_B + 2M_M + M_C) \tag{29}$$

### 3.7 The global P-delta effect

*P*-delta moments adversely influence the performance of all structures during all phases of loading. However, their effects on RCMFs become even more pronounced at incipient collapse. In this section, first the destabilizing effects of the gravity loads on the undamaged MF are studied and expressed in a simple formula, then an attempt is made to estimate the tendon force needed to prevent the catastrophic failure of the RCMF after the frame becomes a mechanism. The *P*-delta moment of any subframe can be expressed as; $P_i \delta_i = \phi h_i \sum_{j=1}^{1} W_{i,j} = \overline{F}_i h_i$, or $\overline{F}_i = \phi \sum_{j=1}^{1} W_{i,j} = P_i \phi$, where $\overline{F}_i$ is the notional equivalent lateral load acting on the subframe. The notional shear force and rocking moment on any subframe can now be estimated as $V_i = \sum_{r=i}^{m} \overline{F}_r = \phi \sum_{r=i}^{m} P_r$ and $\overline{M}_i = \overline{V}_i h_i$ respectively. If $P_i = P$ then $\overline{F}_i = P\phi = \overline{F}$, $V_i = P\phi(m+1-i)$ and $\overline{M}_i = P\phi(m+1-i)h_i$. The global *P*-delta moment acting on the structure can be computed as the sum of the subframe *P*-delta moment;

$$M_{P\Delta} = P\phi \sum_{i=1}^{m} (m+1-i) h_i = \overline{P} \phi \overline{H} \tag{30}$$

where, $\overline{P}$ and $\overline{H}$ may be construed as the sum and of forces $P_i$ and the location of the resultant of story shears $\overline{V}_i$ respectively ( the self weight of the wall can be included in $P_i$). It can easily be shown that for $h_i = h$; $\overline{H} = (m+1)mh/2$. Eq.(30) may be now be adjusted to include the corresponding *P*-delta effects,

$$\phi = \frac{M_0 + \overline{P}\phi\overline{H}}{K^*} \quad \text{or} \quad \phi = \frac{M_0}{f_{cr.} K^*} \tag{31}$$

$f_{cr} = [1 - (\overline{P}\overline{H} / K^*)]$, is defined as the global displacement magnification or load reduction factor.

### 3.8 Determination of interactive forces

If $\theta_{col.F}$, and the contributions of the BRBs and LBs are known, then the magnitude and directions of the interactive forces can be related to total moments acting on any subframe. It is expedient to group the external moments $(M_{0,i} + M_{P\Delta,i})$ and device generated moments $\mathrm{M}_{Dev.i} = (M_{M,i} + M_{B,i} + M_{C,i})$ acting on any subframe *i*. The sum of moments of resistances of columns of any subframe, *r* can be computed as;

$$\mathrm{M}_{col.r} = 2\sum_j^n M_{col.r,j} = \frac{(M_0 + M_{P\Delta} - M_{Dev.})\sum_j^n \overline{k}_{r,j}}{\sum_{j=0}^n \sum_{i=1}^m \overline{k}_{i,j}} \tag{32}$$

The sum of moments of resistances of all columns above level *i* can be computed as;

$$\overline{\mathrm{M}}_{col.i} = 2\sum_{i=r}^m \sum_j^n M_{col.r,j} = \frac{(M_0 + M_{P\Delta} - M_{Dev.})\sum_{i=r}^m \sum_j^n \overline{k}_{r,j}}{\sum_{j=0}^n \sum_{i=1}^m \overline{k}_{i,j}} \tag{33}$$

Consider the racking equilibrium of the columns of the uppermost subframe, *i*=*m*, of Fig. 1(d).i.e.,

$$(F_m + Q_m - S_m) h_m = \mathrm{M}_{col.m} \quad \text{or} \quad S_m - Q_m = F_m - \frac{\mathrm{M}_{col.m}}{h_m} \tag{34}$$

Similarly, the racking equilibrium of the columns of the subframe at *m*-1 can be expresses as; $(F_m + F_{m-1} + Q_m - S_m - S_{m-1})(h_m + h_{m-1}) = \overline{\mathrm{M}}_{col,m-1}$. Substituting for $(Q_m - S_m)$ from Eq. (34) gives;

$$S_{m-1} = F_{m-1} + \frac{\overline{\mathrm{M}}_{col.m}}{h_m} - \frac{\overline{\mathrm{M}}_{col.m-1}}{(h_m + h_{m-1})} \tag{35}$$

The generalized equilibrium equation of the MF, in terms of the interactive can be expressed as;

$$\sum_{r=i}^{m-1} (F_r - S_r) \sum_{s=i}^{r} h_s + (F_m + Q_m - S_m) \sum_{r=i}^{m} h_r = \overline{\mathrm{M}}_{Col,i} \tag{36}$$

Once $(Q_m - S_m)$ and $S_{m-1}$ are known using Eqs. (34) and (35) respectively, $S_{m-2}, \ldots S_1$ can be computed through systematic use of Eq. (36). The limitations of Eqs.(27) and (36) are discussed in section 3.9 below.

### 3.9 Limitations and applications

Two extreme but important cases come to mind, $S_i = 0$, and $S_i = F_i$. They lead to the development of two distinct classes of structures, MFUR and MFUS respectively. Both cases result in MFs of minimum weight. In the first case no interaction takes place between the moment frame and the RRC, i.e., $S_i = 0$, the rigid core and the moment frame tilt compatibly. In conclusion, it would be counterproductive to use un-supplemented RRCs in conjunction with an MFUR. The second extreme scenario, the MFUS, manifests itself if the core absorbs the entire external loading, i.e. $S_i = F_i$.This is the most common condition in practice and happens if the MF is not an MFUR, is augmented with supplementary devices and/or the core is provided with rotational resistance. Under such conditions the RRC behaves as an upright simply supported beam with end reactions $Q_m$ and $Q_0$ as shown in Fig.1.

## 4 .Global plastic response of the frame

Consider the global carrying capacity of the free standing MF of Fig. 4(b) under gravity forces $W_{ij}$, $P$-delta moments $M_{P\Delta}$, lateral loading $F_i$, opposing brace.

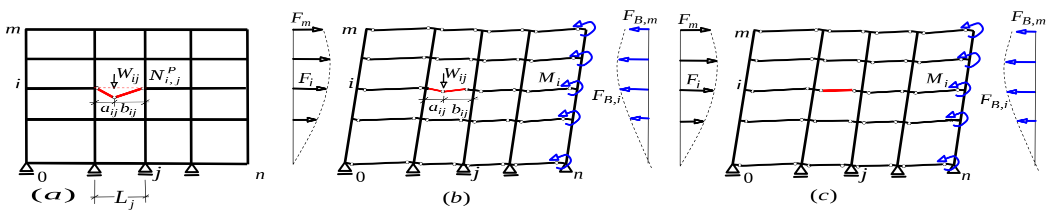


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Fig. 4 (a) Beam mechanisms, (b) Mixed (combined, beam +sway) mechanisms, (c) Purely sway mechanism

If the lateral forces are nil or very small then each loaded span will eventually collapse through a beam failure mechanism, as in Fig. 4(a), at its maximum carrying capacity;

$$W_{i,j}a_{i,j} = 4M^P_{i,j}L_j / b_{i,j} = 4M^P_{i,j}\delta^P_{i,j} \quad (37)$$

where, the Kroneckar's $\delta^P_{i,j} = L_j /(L_j - a_{i,j})$ for $L_j > a_{i,j} \geq 0$ and $\delta^P_{i,j} = 0$ for $a_{i,j} = L_j$. Barring instabilities, the global gravity carrying capacity of the frame can be estimated as $W = \sum_{j=1}^{n}\sum_{i=0}^{m} W_{i,j}$. However, if lateral and gravity loads occur together then two other modes of collapse, depending on the relative magnitudes of the two can also take place. Assuming $W_{i,j}$ is small enough for the frame to fail through a purely sway mode of collapse with plastic hinges forming at beam ends only, as in Fig. 4 (c), and that the LBs generate a total of $2M_M$ moments at their ends, then the lateral carrying capacity of the system can be estimated as;

$$M_0 + M_{P\Delta} - (M_B + 2M_M + M_C) = \sum_{j=1}^{n}\sum_{i=0}^{m} 2M^P_{i,j} \quad (38)$$

Since Eq. (4), (18) and (38) coincide, the solution is exact and in conformity with the requirements of the uniqueness theorem. On the other hand, if some of the gravity forces are large enough to cause combined collapse modes in their own spans, such as that shown in Fig 4(b), while all other beams remain straight with plastic hinges at their ends, then the lateral carrying capacity of the system may be re-evaluated as;

$$\sum_{j=1}^{n}\sum_{i=0}^{m} W_{i,j}a_{i,j} + M_0 + M_{P\Delta} - M_B - 2M_M - M_C = \sum_{j=1}^{n}\sum_{i=0}^{m} 2M^P_{i,j}(1+\delta^P_{i,j}) \quad (39)$$

A comparison of Eqs. (38) and (39) shows that the two lateral failure modes coincide when;

$$\sum_{j=1}^{n}\sum_{i=0}^{m} W_{i,j}a_{i,j} = \sum_{j=1}^{n}\sum_{i=0}^{m} 2M^P_{i,j}\delta^P_{i,j} \quad (40)$$

A satisfactory solution is found if matching terms in either side of (40) are made equal, i.e.,

$$W_{i,j}a_{i,j} = 2M^P_{i,j}L_j / b_{i,j} = 2M^P_{i,j}\delta^P_{i,j} \quad (41)$$

Comparing Eqs.(37) and (41) reveals that small floor loads, $W_{Small} \leq 2M^P L/ab$, have no effect on the lateral carrying capacity of the frame. Furthermore, comparing $W_{Small}$ with $W_{Limit}$ and an intermediate $W = 2M^P c/ab$ where $c$ is an arbitrary quantity defined as; $L \geq c \geq a$, it gives;

$$\left[W_{Limit} = \frac{2M^P L}{ab}\right] \geq \left[W = \frac{2M^P c}{ab}\right] \geq \left[W_{Small} = \frac{2M^P a}{ab}\right] \quad (42)$$

Eq. (42) can be used to assess the plastic limit states of the frame. $W_{Small} \leq (a/L)W_{Limit}$. This gives credence to the notion that; the magnitude of load $W$ at distance ‘‘$a$’’ has little to no effect on the lateral response of the frame if it is less than ($a/L$) of its plastic collapse value acting alone on the same beam. Conversely, if the subject system is designed for increased moments, $(c/a)M^P$, then the lateral carrying capacity of the moment frame will not be affected by the presence of $W$ at ‘‘$a$.’’

## 5. Collapse prevention and self-centering

The fundamental assumption adapted in this section is that at least one group of elements are capable of preventing catastrophic collapse. Permanent deformations and structural collapse are foreseeable phenomena associated with diminishing and exhausted energy absorption capacities respectively. Collapse prevention, in this context, refers to the ability to provide temporary support for the gravity system, after a major seismic event. It also means that there is at least one reliable source of energy absorption that can sustain additional deformations until the seismic event is completely over. Here self-centering is defined as the ability that tends to realign a structural mechanism back to its original form. The purpose therefore is not to prevent the formation of plastic hinges, but rather to prevent collapse due to $P$-delta and similar effects. However, it should be born in mind that residual deformations under seismic loading can significantly affect the re-centering capacity of the structure. However, if complete collapse is to be prevented after formation of a preferred plastic mechanism, then the surviving LBs and the vertical cable system should be strong enough to withstand the entire conditional demand. At incipient failure all moment resisting elements except the RRC tendons (or any one group of supplementary devices) become incapacitated. i.e., $K_F = K_M = K_l = K_B = 0$. Subsequently, the global stiffness of the RCMF reduces to $K^* = K_C$. In other words, if building collapse is to be prevented then the core tendons should be strong enough to withstand the entire seismic demand imposed on the system. With $M_0$, $M_{P\Delta}$ and prescribed $\phi$ known, the basic design parameters for collapse prevention can be computed as;

$$T > \frac{\Omega(M_0 + M_{P\Delta})}{d'} = \frac{\Omega\phi K^*}{d'} \quad (43)$$

where $\Omega$ is the over strength factor and $d'$ is the tendon lever arm. See Fig. 1(a).

## 6. Applications and limitations of RCMFs

The proposed structural system lends itself well to performance control through the use of different types of supplementary devices. Eq. (26) covers a wide spectrum of modes of response for all loading conditions. However, it provides no information regarding the suitability, efficiency and limitations of RCMFs as earthquake resisting systems. A practical way of evaluating the usefulness of such systems is to study the nature of the interactive forces $Q_m$ and $S_i$ with a view to the functional requirements of the main components of the structure. The effects of the supplementary devices on the RRC and the MF have been amply described in the preceding sections.

### 6.1 application and Limitations

The focus of the current section is on the interaction of the RRC and the MF in the absence of auxiliary devices. Two extreme scenarios come to mind. First, no interaction takes place between the MF and the RRC. i.e., $S_i = Q_m = 0$, while the rigid core and the moment frame tilt compatibly. This can happen, if instead of imposing a straight line profile on the frame, the core adapts the linear displacement profile of the frame under the same lateral loading. In conclusion, it would be counterproductive to use a un-supplemented RRCs in conjunction with an MFUR. The second extreme scenario manifests itself if the beams are axially rigid and the core absorbs the entire external loading, i.e. $S_i = F_i$. This is the most commonly occurring condition in practice and happens if the MF is not an MFUR, is augmented with supplementary devices and/or the core is provided with rotational resistance. Under such circumstances the RRC behaves as an upright simply supported beam with end reactions $Q_m$ and $Q_0$ as shown in Fig.1 (h). The materialization of a generic MFUS, in the form of a new earthquake resisting building with self centering and collapse prevention capabilities is presented in section 7 of the current article.

### 6.2 Structural design of the RRC

Whatever the material or the configuration of the RRC, a steel braced frame, reinforced concrete or steel plate shear wall, it should be sufficiently strong and rigid in order to perform its functions as part of the lateral load resisting system, i.e., to withstand the external forces, prevent soft story failure in the MF and provide support for the supplementary devices. As a precautionary measure it would be safe to assume that the RRC alone is capable of withstanding the ultimate earthquake induced and *P*-delta effects. Since the RRC acts as an upright simply supported beam or truss girder, the corresponding distribution of bending moments due to $F_{P,i} = F_P i/m$ and $\overline{F}_i = P\phi = \overline{F}$ at any elevation *i* can be expressed as;

$$M_{core,i} = \frac{F_P ih}{6m}\left[(m^2-1)-(i^2-1)\right] + \frac{\overline{F}ih}{2}\left[(m-1)-(i-1)\right] \tag{44}$$

Ideally speaking the core should be infinitely rigid. However if the rigidity of the RRC, $E_w I_w$, were to be large but finite, its magnitude could be related to its maximum end rotation $\psi_w$, Fig.(2g), or its maximum drift ratio. With $\psi_w$ at hand, the stiffness of the wall can be related to a fraction of the prescribed uniform drift $\phi$ of the system say $5\%\phi$. If this is true then a workable design value for the moment of inertia of the wall can be estimated as $\psi_{\max} = \varepsilon\phi$. The maximum slope of the core due to $F_{P,i} = F_P i/m$ and $\overline{F}_i = P\phi = \overline{F}$ can be expressed as;

$$\psi_{\max} = \frac{F_P h^2(m-1)(2m-1)(2m^2+3m-4)}{180E_w I_w m} + \frac{\overline{F}h^2 m(m-1)(m^2+m-2)}{24E_w I_w} \tag{45}$$

$$I_{w,\min} = \frac{F_P h^2(m-1)(2m-1)(2m^2+3m-4)}{180E_w \varepsilon\phi m} + \frac{\overline{F}h^2(m-1)(m^2+m-2)}{24E_w \varepsilon\phi} \tag{46}$$

## 7. The proposed building system

The subject earthquake resisting system consists of four essential components; The GBSMF, the RRC, the LBs and the PT tendons. The system is capable of accommodating BRBs and similar devices.

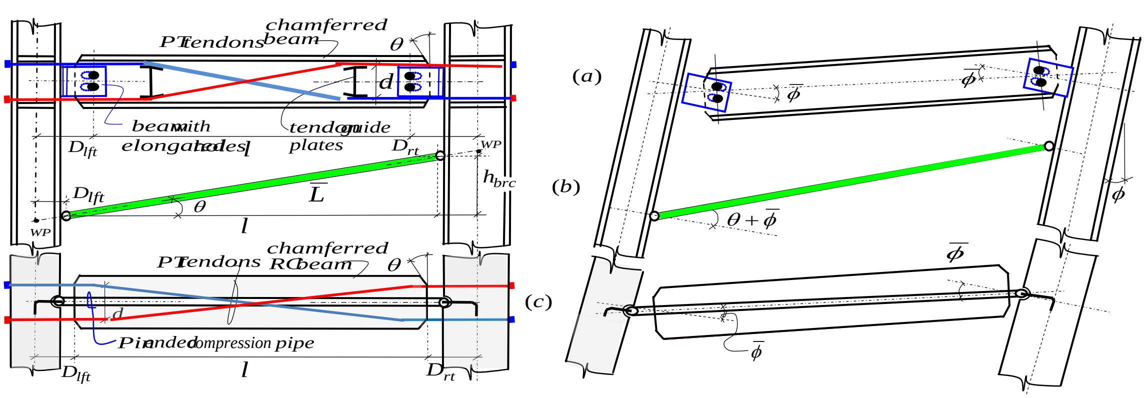


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Fig.6 (a) Chamfered steel LB, (b) BRB (9c) Chamfered reinforced concrete LB before and after rotation

- The gravity system and the earthquake resisting MF are designed in accordance with the rules and regulations of the codes of practice with special attention to the week beam–strong column requirement.

The RRC should be free to pivot about its base and rotate freely at all slab wall junctions. Fig.(7) show one such detail that allows horizontal shear transfer from the slab to core without inhibiting the vertical component of the rocking movement at the same junction.

- The most commonly utilized post-tensioned gap opening LB system with butting flat ends against column sides tends to expand the frame beyond its original span length. As the gap widens at the beam –column interface, the LB rotates upwards, and bends the column inwards. This in turn opposes the post- tensioning force. In order to alleviate or reduce these effects, the senior author has proposed the use of a truncated version of the same LB as shown in Figs. 6(a) and (c). The proposed LB consists of a coaxial rigid compression elements surrounded by a reinforced concrete cage that holds the PT tendons together and provides stability for the axial core. Instead of using axial springs equivalent rotational schemes have been utilized to capture the restraining effects of the post tensioned tendons.
- While it is good practice to extend the tendons from end to end, their length, layout cross sectional areas and the pre-stressing forces should be assessed in terms of the required drift angle, self centering and collapse prevention requirements.
- The slab acts as a rigid diaphragm. Seismic shear is transferred to the RCMF system through stressed cables, compression of the LB, as well as direct shear connectors between the slab and the wall, Fig. (7).

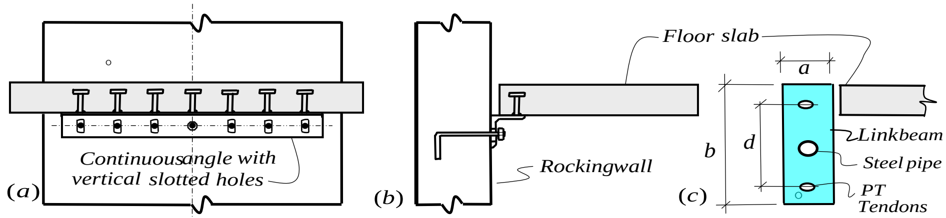


Figure 7(a) Symbolic wall-slab shear connection, (b) Section showing same, (c) Link beam section

## 8. Conclusions

Global Design Led analysis which is a new analytic approach has been used to develop exact, closed form solutions for RCMFs under gravity and lateral loading conditions. The proposed mathematical model is ideally suited for manual preliminary design of such systems. The proposed configuration is both construction friendly and satisfies the theoretical conditions of minimum weight design. Two new gap opening link mechanisms, one for steel and the other for concrete structures that do not induce unwanted moments in the columns have also been introduced. The success of the proposed formulations, as compared with classical methods, lies in the provision of the necessary means that lead to the prescribed objectives rather than investigating the validity of randomly developed initial schemes. For instance, the adaption of grade beam-supported, pinned base columns, as opposed to fixed boundaries, allow the entire structure to respond as a truly controllable rocking systems.

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