



## APPLICABILITY OF MODAL RESPONSE SPECTRUM ANALYSIS ON ROCKING STRUCTURES

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### Abstract

The design of rocking shear walls or frames can be a difficult task for practicing engineers. Because of the high nonlinearity and in the same time the lack of the adequate energy dissipation, the modal response spectrum analysis is not (or not directly) applicable. Furthermore, the development of a numerically stable and reliable model for nonlinear time-history analysis of these types of buildings is demanding and time consuming.

Response spectrum analysis of ductile structures is based on the classical results of A. S. Veletsos and N. M. Newmark, who in their 1960 paper developed response spectrum for structures with elastic-perfectly plastic, symmetrical force-deformation curves. The basic idea of the design is that the elastic response spectrum is modified by the behavior factor, which depends on the ductility of the structure.

In our paper we wish to develop a general method for the response spectrum analysis of rocking structures. First nonlinear time-history analyses are carried out on single degree of freedom structures with different hysteresis loops: bilinear elastic, flag shaped, full hysteresis, etc.

Based on these calculations a recommendation is given for the application of the modal response spectrum analysis of rocking structures, which can serve as a simple and reliable design tool for design engineers.

*Keywords: self-centering material; modal response spectrum analysis; SDOF*

## 1. Introduction

One of the most common design procedures of elastic structures is the response spectrum analysis, where the design loads can be determined from the period(s) of vibration of the structures ( $T$ ), the modes of vibrations, the masses and the elastic response spectrum, which is given for the location of the structure in national standards.

In 1960 Veletsos and Newmark [1] developed the response spectrum for inelastic structures, where in addition to the above parameters the ductility factor  $\mu$  must also be known:

$$\mu = \frac{u_y}{u_e} \quad , \quad (1)$$

where  $u_e$  is the elastic and  $u_y$  is the plastic deformation of the structure (Fig. 1a). The calculation was carried out for perfectly plastic structures for full hysteretic behavior (Fig 1b). For design purposes in many cases the elastic response spectrum is modified due to plasticity by reduction factors (denoted sometimes by  $R$  or  $q$ ), which are basically the ratios of the elastic and inelastic response spectra, and  $R$  or  $q$  is a function of  $T$ .

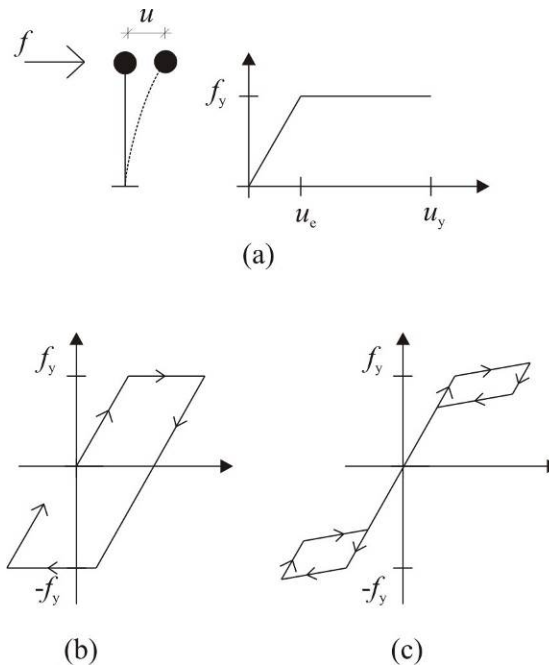


Fig. 1 – Definition of ductility factor (a), fully plastic behavior (b) and flag-shaped behavior (c)

In the last decade there was a substantial increase in the publications of earthquake resistant structures which remain damaged-free even after significant base excitation [2, 3, 4]. The main advantage of these structures, in addition to the lower, or negligible repair cost, is the immediate occupancy after earthquakes. The disadvantage, compared to the conventional structures with high ductility, is the lower energy dissipation. Typical damaged-free structures are the rocking concrete shear walls or rocking concentrically-braced steel frames. Both are strengthened with post tensioned bars to provide self-centering capability. The force-displacement curves of these structure has a “flag shape”, instead of the traditional hysteretic behavior.

Earlier researchers at Lehigh University, Bethlehem PA, USA showed that the above procedure cannot be applied directly for the rocking structures [5]. They found that the effect of the nonlinear behavior depends strongly on the period of vibration, and, most importantly, whether the mode shape belongs to the first, second, or higher modes.

As a consequence, the development of a simple and reliable design method for rocking mechanism is an important task. In this paper we will analyze the possible applicability of the modal response spectrum analysis of rocking structures.

Our main goal is to investigate the possible applicability of the inelastic response spectra for structures with flag-shaped force displacement curves (Fig. 1c). Recently, after submitting our abstract a paper was published on inelastic displacement ratios of self-centering rocking systems [6]. They investigated numerically in detail the effect of flag-shaped hysteretic curves, which is also our task, however we will proceed in three important issues: (i) a new self-centering material model is developed for Opensees [7] since errors were found in the old one [8], (ii) a more complex parameter range was investigated, (iii) a direct simple expression is developed to account for the flag-shaped behavior.

## 2. Problem statement

A single degrees of freedom structure is investigated, where both material hardening ( $b$ ) and the flag-shaped hysteretic curve ( $\beta$ ) are taken into account (Fig 2a). Here  $\beta$  is the fullness of the hysteresis, while  $b \times E$  is the tangent of the curve after yielding. For  $b=0$  there is no hardening (Fig 2b), while for  $\beta=0$  there is no energy dissipation, the behavior is elastic (Fig 2c). For  $\beta=2$  the full hysteresis is considered, Fig 2d.

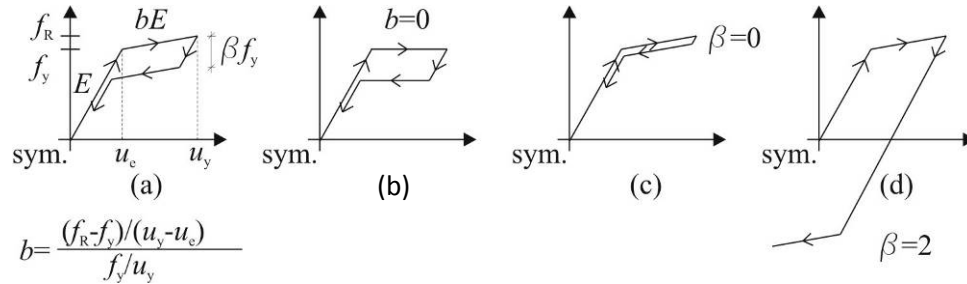


Fig. 2 – Material model (a), no hardening (b), no energy dissipation (c) and full hysteresis (d)

## 3. Approach

To obtain the inelastic response spectra we followed the procedure given in [9]. For a given earthquake, for different periods of vibrations ( $T$ ) and damping ratios ( $\zeta$ ) the following steps must be performed:

Step 1) on the elastic structure the maximum displacements ( $u_e$ ) and the corresponding stress demands ( $f_e$ ) are calculated by time-history analyses and the elastic response spectrum ( $S_e$ ) is constructed (Fig 3a);

Step 2) for prescribed yield stresses

$$f_y = \overline{f_y} \times f_e \quad (\overline{f_y} < 1) \quad (2)$$

the maximum inelastic displacement ( $u_m$ ) and the ductility demand  $\mu = u_m / (\overline{f_y} \times u_e)$  are calculated by time-history analyses;

Step 3) for given ductility factors ( $\mu$ ) the required normalized yield strength  $\overline{f_y}$  was obtained by interpolation. The inverse of  $\overline{f_y}$  is the  $R$  factor (or  $q$ ), with the aid of which the inelastic spectrum can be obtained from the elastic spectrum ( $S_d = S_e / R$ , Fig. 3b).

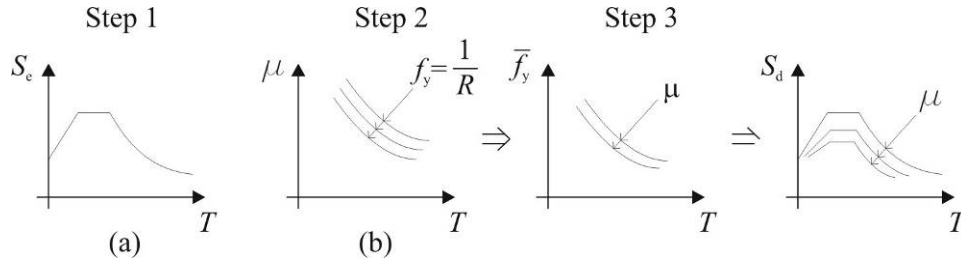


Fig. 3 – Steps in calculating inelastic spectrum

These time-history analyses were carried out by Opensees using the Elastic-Perfectly Plastic [10] and the SelfCentering [8] material models. Unfortunately the two models for the same problem ( $\beta=2$ ) gave different results, the reason was the weakness of the SelfCentering material model which led to the development of an improved SelfCentering2 material model [11], see Section 4 for details.

The calculation was carried out for the FEMA [12] 22 pairs far-field ground motion record sets, and the mean values of the results were presented.

Step 4) The last step of the calculation is to determine the reduction factor  $R_f$  due to the flag-shaped hysteresis.  $R_f$  is the ratio of the normalized yield strengths at  $\beta=2$  (full hysteresis) and at a given  $\beta$ :

$$R_f = \frac{\bar{f}_y(\beta=2)}{\bar{f}_y} = \frac{S_d(\beta=2)}{S_d} \quad (3)$$

which is identical to the ratios of the corresponding inelastic response spectra as shown by the second fraction in Eq. (3). Note that Eq. (3) is evaluated for a given ductility factor,  $\mu$ . The design spectra can be calculated as

$$S_d = \frac{S_e}{R \times R_y} \quad (4)$$

## 4. Improved numerical self centering material model for Opensees

SelfCentering is a one dimensional numerical material model for simulating flag-shaped hysteretic response under cyclic loading in Opensees. It was implemented in 2007 and it is related to work of Tremblay et al [13]. The material model received more attention from us after we noticed the aforementioned problems in the results of time-history analyses. Although some plots of cyclic response and a general description of input parameters are available at the Opensees Wiki, we found no information in the literature or online about the theoretical background of the model. Examination of the source code for SelfCentering revealed the reasons behind its puzzling behavior. Two particular issues were identified as the cause of our problems. We share our findings below to draw attention to the deficiencies of the material model, because they might have led to incorrect results in the work of others during the past decade of its existence. We developed SelfCentering2, an improved, more general version of the original material. The second subsection explains its working mechanism and advantages over the original material. After thorough testing we intend to make SelfCentering2 available to the Opensees community [11].

### 4.1 Application limits of the original SelfCentering model

The structure and logic behind the source code of the original SelfCentering material suggests that it was developed to focus on a specific task with pre-defined, well described boundary conditions. It lacks the potential of general applicability by design, but the special cases when it works properly correspond to a sufficiently broad set of applications for most studies. Nevertheless, we suggest drawing attention to the following limits at its Opensees Wiki page:

- The  $\beta$  parameter has an upper limit of 1.0, thus the area of the flag-shaped hysteresis loop cannot exceed the quarter of the area of a full hysteresis. This problem stems from the assumption that the cyclic

response of the material will always have a central linear elastic part that connects the two flags with each other (e.g. Fig. 2a). If  $\beta > 1.0$ , then there is no central linear part (e.g. Fig. 2d) and this leads to the erroneous response depicted in Fig. 4a.

- The material is sensitive to large strain increments, especially during unloading. The parallel lines of the stress-strain response that correspond to inelastic strain hardening are handled internally by two points that have their coordinates updated at every load step as long as the material response is on the parallel lines. When the material response leaves the parallel lines, the value of the upper and lower points is frozen. The initial positions of these points are at the intersections of the parallel lines and the line corresponding to linear elastic behavior. Because the algorithm does not initialize these points at load reversals (between quadrants), they never return to their initial position, but they are left behind instead on the parallel lines. Provided that the strain increments are small, the error from this issue is not significant. However, application of larger load steps lead to considerable error in the resulting response (Fig. 4b).

#### 4.2 The improved SelfCentering2 material model

The SelfCentering2 material is developed to provide a generally applicable algorithm that is free from the constraints and limitations of the original SelfCentering material. Because the two points on the parallel lines are properly initialized, the response does not depend on the size of load increments (Fig. 4c). The unloading-reloading part of the algorithm has also been extended to allow the use of  $\beta > 1.0$ . The central linear part of the stress-strain response is used only for materials with  $\beta < 1.0$  (Fig. 4d).

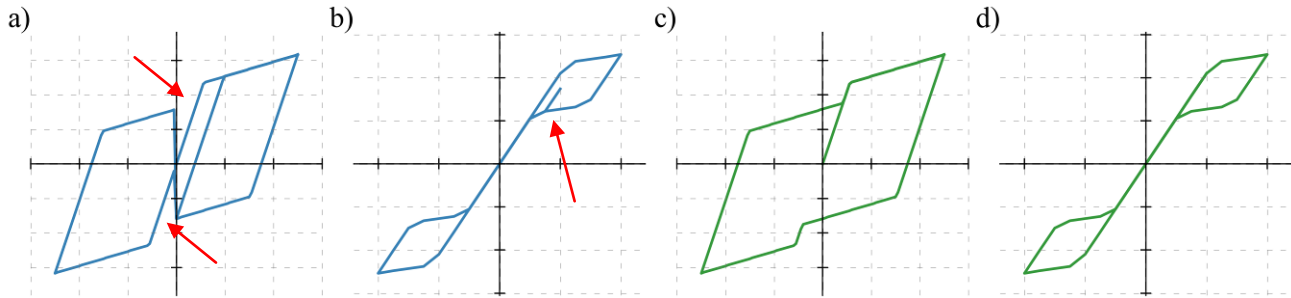


Fig. 4 – Cyclic response of the SelfCentering a), b) (errors marked with red arrows) and the SelfCentering2 material model c), d)

### 5. Parametric study, results

As we stated above 44 earthquake records were evaluated. Furthermore, we varied the following parameters in the calculations: (i) damping ratios  $\zeta=0.02, 0.05$  (2 values); (ii) the fullness of the flag-shaped hysteresis  $\beta=0, 0.25, 0.5, 0.75, 1, 1.25, 1.5, 1.75, 2$  (9 values); (iii) hardening  $b=0, 0.05, 0.1, 0.15, 0.2, 0.25, 0.3$  (7 values); and (v) the period  $T=0.1, 0.11, \dots, 5$  (491 values). Altogether more than 24.5 million nonlinear time-history analyses were carried out.

We present a few illustrative examples below. Step 1 is not given, in Step 2 (Fig. 3b) the calculation of the displacement-force curves for 20 cases (four  $b$ -s and five  $\beta$ -s) are given in Fig. 5 for one earthquake record.

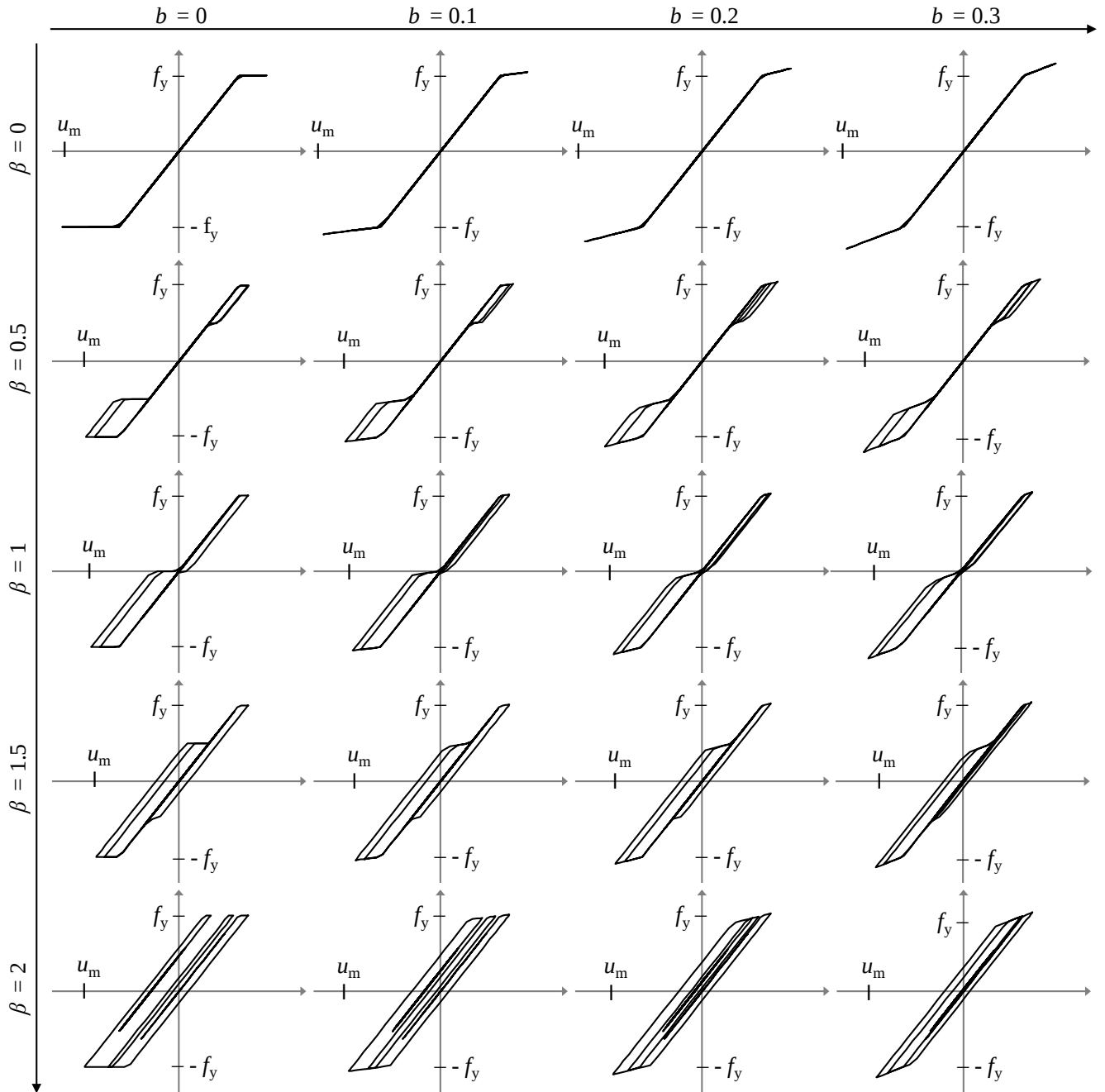


Fig. 5 – Force-displacement relationships of the various  $b$  and  $\beta$  parameters due to El Centro earthquake

In Fig. 6. the ductility demand over  $\overline{f_y}$  is given for four cases, each line belongs to different normalized strength factor  $\overline{f_y}$  ( $1/R$ ). Note, that [6] also presented results for the ductility demand, however, because of the detected errors in the original SelfCentering material model in Opensees their results are different.

As we stated in Section 3 the last two steps are the calculation of the required normalized yield strength as a function of the ductility factor ( $\mu$ ) by interpolation, and calculation of  $R_f$  (Eq. (3)) also for given  $\mu$ -s. Inspecting the results it was decided that  $\mu$  independent expressions are developed for  $R_f$ . In this case, to avoid the inaccuracy of the interpolation in Step 3,  $R_f$  is evaluated (instead of Eq. (3)) by the following expression:

$$R_f = \frac{u_m(\beta = 2)}{u_m} . \quad (5)$$

Note that  $R_f$  defined by Eq. (3) is a function of  $\mu$ , while  $R_f$  defined by Eq. (5) is a function of  $\overline{f_y}$  (or  $R$ ). Four examples are shown in Fig. 7.

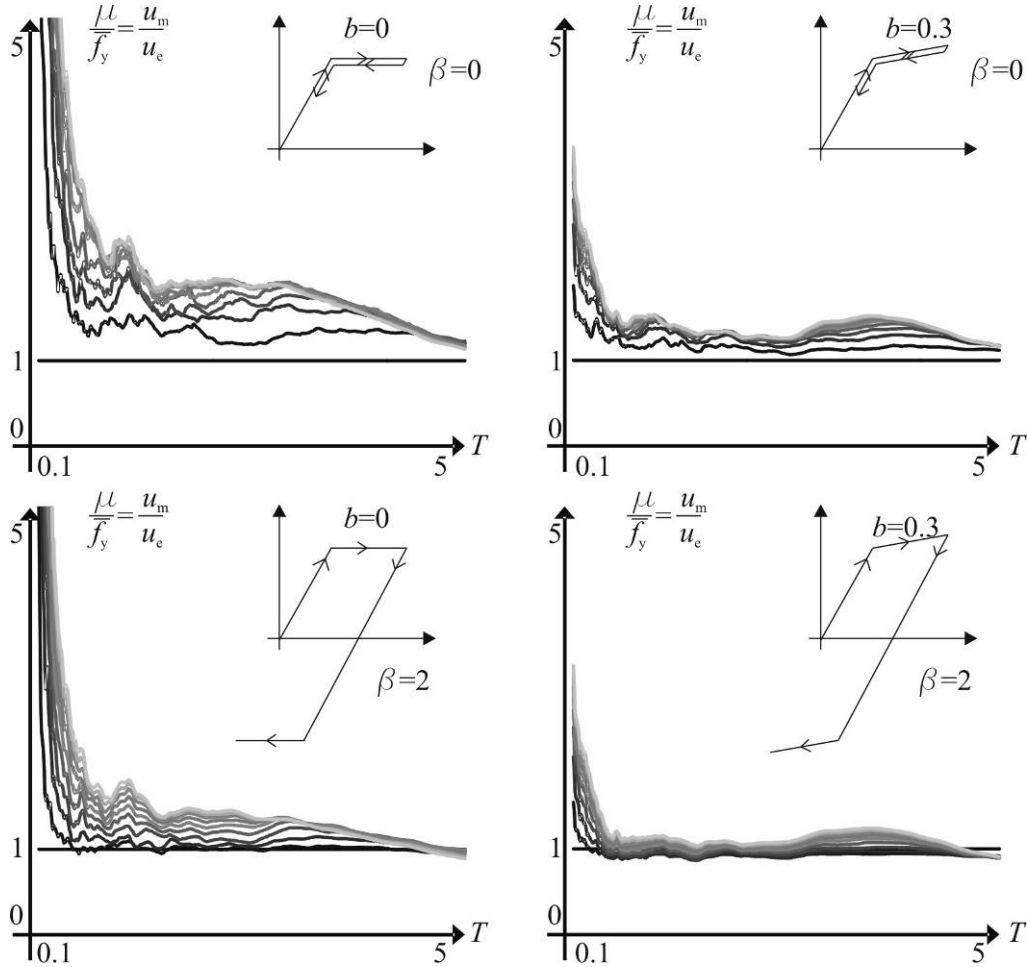


Fig. 6 – Ductility demand over  $\overline{f_y}$  for the mean of the 44 earthquake records ( $\zeta=0.05$ , each curve belongs to different normalized strength factors  $\overline{f_y} = 1/R$ )

We may observe that although the ductility factors (Fig. 7.) depend strongly on the normalized yield strength, the lines of the reduction factor  $R_f$  are closer to each other. It is also important to notice that the lines (Fig. 7.) corresponding to the different  $\overline{f_y}$ -s intersect in an irregular way, hence we will seek a surface fit for these results, which are independent of the  $\overline{f_y}$  . and fit to the lower bound of them (red lines on Fig. 7).

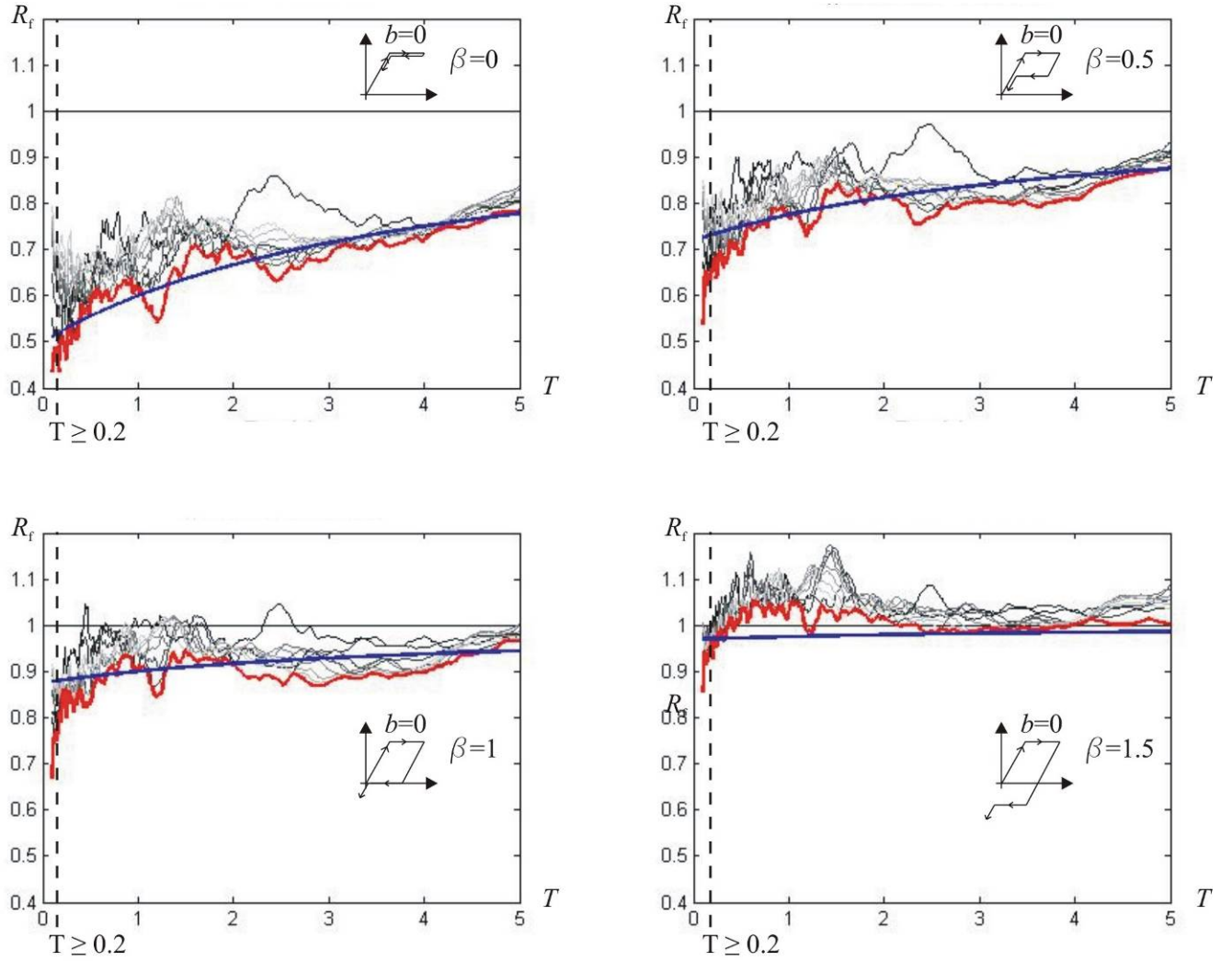


Fig. 7 – Reduction factor due to the fullness ( $\beta$ ) of the hysteretic curve, the mean value of the 44 earthquake records ( $\zeta=0.05$ ,  $b=0$ , each curve belongs to different  $\bar{f}_y$ )

## 6. Approximation for the $R_f$ reduction factor

We fitted the a surface for the lower bound of the  $R_f$  values for case  $b=0$  in the function of period ( $T$ ) and the fullness of the hysteresis curve ( $\beta$ ). The results for the two damping factors are as follows:

$$R_f^{0.02}(\beta, T) = 1 - \frac{\left(1 - \frac{\beta}{2}\right)^2}{1 + 1 \cdot T^{0.5}} \quad \zeta=0.02 \quad (T \geq 0.2) \quad (6)$$

$$R_f^{0.05}(\beta, T) = 1 - \frac{\left(1 - \frac{\beta}{2}\right)^2}{2 + 0.5 \cdot T^1} \quad \zeta=0.05 \quad (T \geq 0.2) \quad (7)$$

We found that – as a conservative estimate – the same expressions can be used for any values of hardening (b). Eq. (7) is shown in in Fig. 7 by solid blue lines.

Fig. 8. also shows the lower boundary of  $R_f$  values and the fitted surfaces for the two damping factors.

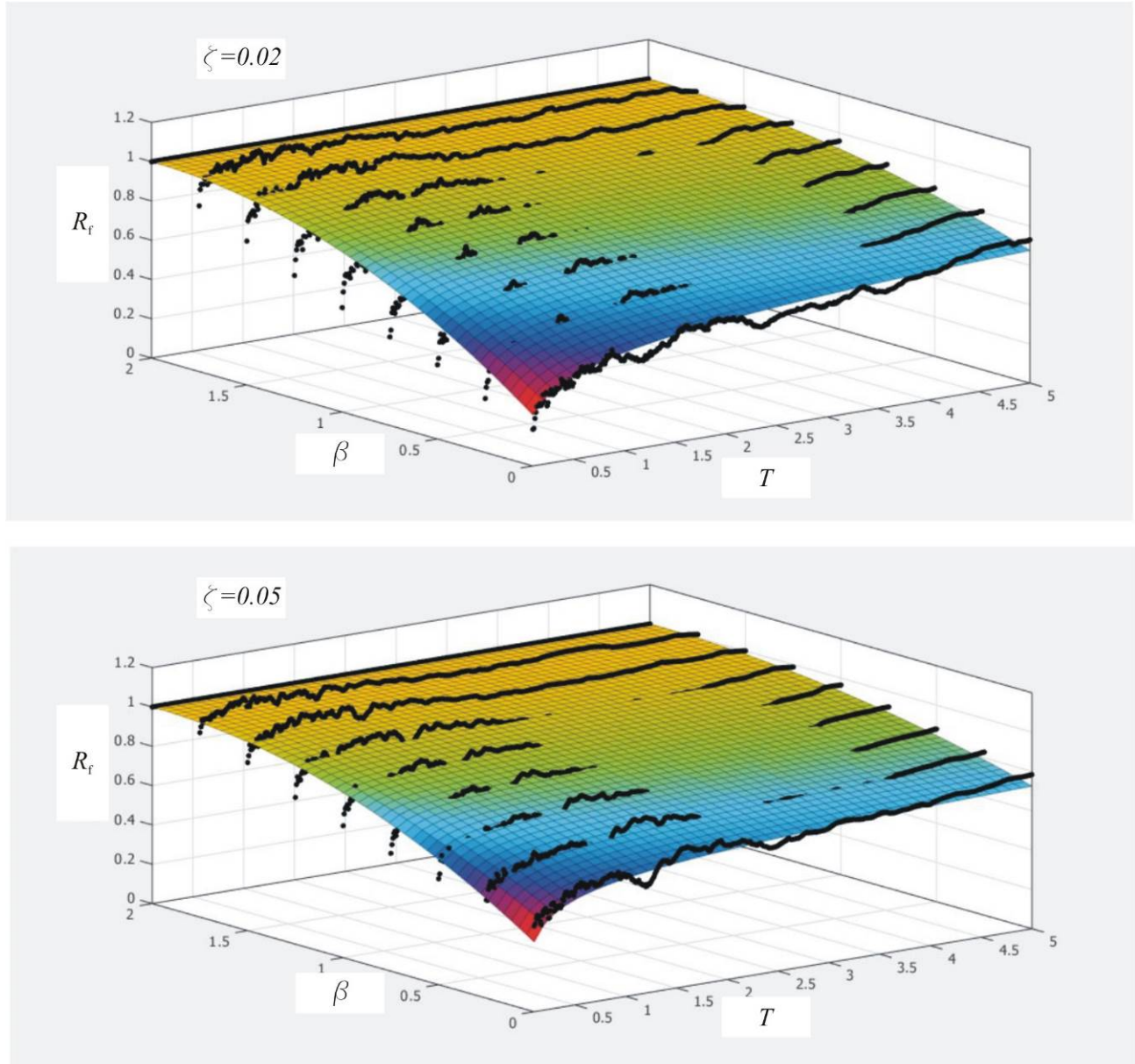


Fig. 8 – Approximation of the  $R_f$  values for damping factors  $\zeta=0.02$  and  $\zeta=0.05$



## 7. Conclusions and further studies

With the aid of a new (improved) material model developed in OpenSees, running over 25 million nonlinear time-history analyses we determined the inelastic response spectra of structures with flag-shaped hysteretic behavior. Since the detected error in the original material model of OpenSees, our results do not agree with those of [6]. For design purposes we propose a simple reduction factor  $R_f$  in Eqs. (6) and (7), which is a function of the fullness of the hysteresis ( $\beta$ ) and period ( $T$ ), to take into account the flag-shape hysteresis. With this parameter the design spectra can be calculated as  $S_d = S_e/(R \times R_f)$  in Eq. (4). The maximum reduction is close to 50%.

For single degree of freedom structures (or structures with dominant rocking mode) this method can be directly used in design for rocking structures, for multi degree of freedom structures further research is required.

We have chosen the mean values of 44 records, however, further possible ground motions should be considered and proper exceedance of probability to be chosen. These are out of the scope of this paper.

## 8. Acknowledgements

The authors gratefully thanks the support of OTKA 115673 project of the Hungarian National Research, Development and Innovation Office and the support of TÁMOP - 4.2.2.B-10/1-2010-0009 project for the usage of Superman, the supercomputer at the Budapest University of Technology and Economics.

## 10. References

- [1] Veletsos, A. S., and Newmark, N. M., "Effect of Inelastic Behavior on the Response of Simple Systems to Earthquake Motions," Proceedings of the 2<sup>nd</sup> World Conference on Earthquake Engineering, Japan, Vol. 2, 1960, pp. 895-912.
- [2] Sause, R.; Ricles, J. M.; Roke, D. A.; Chancellor, N. B., Gonner, N. P., "Seismic Performance of a Self-Centering Rocking Concentrically-Braced Frame," Proceedings of the 9<sup>th</sup> U.S. National Conference on Earthquake Engineering, 2010.
- [3] Eatherton, M., Ma, X., Krawinkler, H., Mar, D., Billington, S., Hajjar, J., and Deierlein, G., "Design Concepts for Controlled Rocking of Self-Centering Steel-Braced Frames," Journal of Structural Engineering, Vol. 140. 2014, pp.
- [4] Kammula, V., Erochko, J., Kwon, O., Christopoulos, C., "Performance Assessment of the Self Centering Energy Dissipative (SCED) Bracing System using Hybrid Simulation," Proceedings of the 15<sup>th</sup> World Conference on Earthquake Engineering, China; 2012.
- [5] Roke D, Sause R, Ricles J.M., Gonner N., "Design Concepts for Damage-free Seismic-resistant Self-centering Steel Concentrically-braced Frames," Proceedings of the 14<sup>th</sup> World Conference on Earthquake Engineering, China; 2008, pp. 1-10.
- [6] Rahgozar, N., Moghadam, A. S., Aziminejad A., "Inelastic Displacement Ratios of Fully Self-centering Controlled Rocking Systems Subjected to Near-source Pulse Like Ground Motions," Engineering Structures, Vol. 108, 2016, pp. 113-133.
- [7] McKenna, F., Scott, M. H., Fenves, G. L., "Nonlinear Finite Element Analysis Software Architecture Using Object Composition," Journal of Computing in Civil Engineering, Vol. 24(1), 2010, pp. 95-107.
- [8] OpenSees SelfCentering Material Wiki page: [http://opensees.berkeley.edu/wiki/index.php/SelfCentering\\_Material](http://opensees.berkeley.edu/wiki/index.php/SelfCentering_Material)
- [9] Chopra, A. K., "Dynamics of Structures: Theory and Applications to Earthquake Engineering," Prentice-Hall, 2001.
- [10] OpenSees Elastic-Perfectly Plastic Material Wiki page: [http://opensees.berkeley.edu/wiki/index.php/Elastic-Perfectly\\_Plastic\\_Material](http://opensees.berkeley.edu/wiki/index.php/Elastic-Perfectly_Plastic_Material)
- [11] OpenSees SelfCentering Material Wiki page: (to be published on OpenSees Wiki)
- [12] FEMA P695, "Quantification of Building Seismic Performance Factors," Applied Technology Council, 2009.
- [13] Tremblay, R., Lacerte, M., Christopoulos, C., "Seismic Response of Multistory Buildings with Self-Centering Energy Dissipative Steel Braces," Journal of Structural Engineering, Vol. 134, 2008, pp. 108-120.