SEISMIC DESIGN OF STEEL FRAMES
EQUIPPED WITH “FREEDAM” CONNECTIONS

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Abstract

The paper presents a design approach based on the Theory of Plastic Mechanism Control (TPMC) finalized to provide a procedure for the design of MRFs equipped with friction dampers both at beam-to-column connections and first storey base section. The introduction of such dissipative devices in correspondence of the dissipative zones that has to be involved in plastic range, provided that a collapse mechanism of global type is assured, belong to the supplementary energy dissipation strategies. The use of the TPMC as the design tool is able to assure that only the zones equipped with the dissipative devices are involved in plastic range. The devices herein presented have been developed in the framework of the project “FREEDAM” that deals about free from damage connections equipped with pads of frictional materials. In particular, the friction devices of beam-to-column connections are located at the bottom flange level while column-base connections are conceived to transmit the bending moment only. Finally, the proposed design approach is herein presented with reference to a 3bay-4storey moment-resisting frame. The investigation of its seismic performances has been carried out by means of both push-over and IDA analyses. These last ones, refer-ring to a set of 10 recorded ground motions, show the high values of spectral acceleration and peak ground acceleration the structure is able to withstand without yielding.

Keywords: MRFs, steel, dissipative devices, free from damage structures, Theory of Plastic Mechanism Control.
1. Introduction

The optimization of the seismic energy dissipation has ever been one of the main goals to assure in order to design structures able to withstand severe earthquakes. For this reason, modern seismic codes have introduced simplified rules, such as the beam-column hierarchy criterion, promoting the development of plastic hinges at the beam ends constituting the dissipative zones of traditional MRFs [1]. However, this criterion, reported also in the European seismic code [2], is usually able to prevent soft-storey mechanisms, but it is not able to assure a predetermined collapse mode. The optimization of the seismic response of MRFs is achieved when all the beam ends are subjected to yielding, as well as, the base sections of first storey columns. Such mode of collapse is called global type mechanism [3].

The development of such a kind of mechanism is assured by properly applying the Theory of Plastic Mechanism Control (TPMC) [2]. The robustness of the corresponding design method relies on the kinematic theorem of plastic collapse. In addition, a closed form solution has been recently provided and applied to both MRFs [4] and EBF-MRFs dual systems. However, this procedure, has been applied to design all the structural steel typology [5]-[9] and reinforced concrete MRFs [10]. Even though, this approach is able to assure a collapse mechanism of global type, the seismic optimization has to be pursued by assuring that all the dissipative zones engage in yielding as possible as the same time, conforming also to the original hypothesis of a rigid plastic behaviour of members. However, a MRF can be either mainly subjected to seismic-loads, this is the case of MRFs arranged in a direction parallel to the floor warping, or mainly subjected to gravity loads when the beams are orthogonal to the floor warping. In this last situation the promotion of a contemporary engagement of dissipative zones in plastic range become more difficult to obtain because the gravity loads govern the beam dimensioning. In fact, being the seismic shear decreasing as the storey height increase, last storey beams can delay the plastic hinge development at their ends, being designed to withstand severe gravity loads. In other words, beam sections result oversized when subject to the seismic load combination, especially at last storeys. On the contrary, MRFs arranged in a direction parallel to the floor warping have smaller beam sections designed to withstand gravity loads but they could be not able to bear the design seismic actions or to satisfy serviceability requirements. Therefore, beam sections need to be increased. However, in both cases some beam sections can be oversized delaying the involvement in plastic range.

As regards the structural damage, it is essential to dissipate the earthquake input energy but it is the main source of direct and indirect losses. For this reason, many researchers have focused their attention on the strategy of supplementary energy dissipation with the aim to reduce the structural damage under destructive seismic events. This strategy is based on the use of specific energy dissipation devices that have to be inserted in selected points of the structural scheme where the higher relative displacements or velocities are expected under the action of severe ground motions. In this paper, traditional dissipative zones have been substituted by means of friction dampers, whose main advantage is the chance to calibrate the flexural resistance as closely as possible to the bending moments occurring under the seismic load combination, thus promoting the contemporary activation of all the friction dampers. In particular, beam-to-column FREEDAM devices are located at the bottom flange level of beams and are equipped with friction pads whose slippage is the basis of the energy dissipation process while column-base connections are conceived to transmit bending moment only by means of a couple of friction dampers located with a properly selected level arm. It means that the column, only pinned at its base, has to support axial load only while friction dampers works as a double pendulum enduring the bending action. Finally, for the contemporary activation of all the dissipative zones, a design approach inspired to the Performance Based Design approach by Goel and Lee [11] has been adopted. Column sections at each storey, constituting non dissipative zones, are designed to remain in elastic range thanks to the application of TPMC. In this way, the resulting designed structure can be considered as free from damage [12]-[13]. In fact, after a destructive seismic event all the damaged devices can be replaced by new ones.

The accuracy of the proposed design approach, has been investigated by means push-over and non-linear dynamic analyses by applying a properly chosen set of earthquake ground motions.
2. Design Procedure

The “Theory of Plastic Mechanism Control” (TPMC) is able to assure the design of structures failing according to a collapse mechanism of global type. Global mechanism represents the optimum in terms of energy dissipation capacity, because all the dissipative zones are involved in the pattern of yielding, while non dissipative ones remain in elastic range. Dissipative zones of traditional MRFs are the beam ends and the base sections of first storey columns. Therefore, the energy dissipation capacity needed to withstand destructive seismic events is gained at the cost of structural damage which has to be limited to be compatible with the local ductility supply. In this paper, TPMC is applied to innovative MRFs where traditional dissipative zones are substituted by beam-to-column connections, called “FREEDAM connections”, equipped with friction dampers (Figure 1) and, in addition, column-base connections are designed to transmit bending moments by means of a couple of friction dampers located with a properly selected level arm while axial force and shear are transmitted by means of a pin-jointed connection (Figure 2). In particular, TPMC is applied to assure that columns remain in elastic range.

![Figure 1 - Connection equipped with dampers](image1)

![Figure 2 - Column-base connection with dampers](image2)

Regarding the column design, all the possible mechanism typologies have to be considered. In particular, MRFs under seismic horizontal forces fail according to three main collapse typologies (Figure 3) where the rectangles represent the connections whose friction dampers are actively involved in the kinematic mechanism, while the solid circles represent plastic hinges in the columns. The total number of possible mechanisms is $3n_s$, where $n_s$ is the number of storeys. Only one of these mechanisms is the desired one, i.e. the global mechanism, which is a particular case of type-2 mechanism extended to all the storeys. It means that all the other $3n_s - 1$ mechanisms are undesired and must be avoided. According to TPMC, the design conditions are derived by means of the kinematic theorem of plastic collapse extended to the concept of mechanism equilibrium curve [2]-[4] (Figure 4):

$$\alpha_0^{(g)} - \gamma^{(g)} \delta_u \leq \alpha_{0,i_m}^{(t)} - \gamma_{i_m}^{(t)} \delta_u \quad i_m = 1,2,3, \ldots, n_s \quad t = 1,2,3$$

where $\alpha_{0,i_m}^{(t)}$ is the kinematically admissible multiplier of horizontal forces evaluated according to the first order rigid-plastic analysis, $\gamma_{i_m}^{(t)}$ is the slope of the mechanism equilibrium curve, accounting for second-order effects, $i_m$ and $t$ are the mechanism index and the mechanism typology code, respectively. Similarly, $\alpha_0^{(g)}$ and $\gamma^{(g)}$ are the same quantities referred to the global mechanism. The computation of $\alpha_{0,i_m}^{(t)}$ and $\gamma_{i_m}^{(t)}$ is presented and discussed in detail in previous works [2]-[4].
2.1 Design of friction dampers equipping the connections

The first step of the design procedure require the evaluation of the minimum slippage resistances of the friction dampers equipping both beam-to-column connections and column-base connections. Such design can be carried out by imposing the fulfillment of two requirements.

The first requirement is aimed to assure an adequate lateral resistance to withstand the design seismic forces. To this scope it is imposed that the multiplier \( \alpha \) of the seismic forces \( F_k \), acting at the storey level \( h_k \) with respect to the foundation level, has to be equal to 1.0 when the design top sway displacement \( \delta_u \) is achieved:

\[
\alpha = \alpha_0(g) - \gamma(g) \delta_u = 1 \Rightarrow \frac{M_{fb.Ed.n_s} \sum_{k=1}^{n_s} \beta_k + M_{fc.Ed}}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u = 1 \tag{2}
\]

where \( \alpha_0(g) \) is given by:

\[
\alpha_0(g) = \frac{M_{fb.Ed.n_s} \sum_{k=1}^{n_s} \beta_k + M_{fc.Ed}}{\sum_{k=1}^{n_s} F_k h_k} \tag{3}
\]

being \( M_{fc.Ed} \) the sum of the flexural resistances corresponding to the slippage of the friction dampers of the column-base connections and \( M_{fb.Ed.n_s} \) is the sum of the flexural resistances corresponding to the slippage of the friction dampers of the top storey beam-to-column connections. In addition, \( \beta_k = \sum_{i=k}^{n_s} F_i / F_{n_s} \) is the ratio between the storey shear acting at the \( i_m \)-th storey and the top storey shear. It means that aiming to promote the contemporaneous slippage of all the friction dampers of beam-to-column connections of all the storeys, such slippage resistances are distributed along the building height accordingly to storey shear distribution. Moreover, \( \gamma(g) \) is the slope of mechanism equilibrium curve accounting for second order effects.
\[ \gamma(g) = \frac{1}{\frac{\sum_{k=1}^{n_s} F_k h_k}{h_s \sum_{k=1}^{n_s} F_k h_k}} \]  

where \( V_k \) is the sum of all the vertical loads acting at \( k \)-th storey.

The second requirement is aimed to avoid the occurrence of soft-storey mechanism at first storey. To this scope, according to TPMC, it is imposed that the mechanism equilibrium curve corresponding to the global mechanism has to intercept the one corresponding to the first storey soft mechanism at the design displacement \( \delta_u \):

\[ \frac{M_{f_b.Ed,ns} \sum_{k=1}^{n_s} \beta_k + M_{fc.Ed}}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u = \frac{M_{fc.Ed} + \sum_{i=1}^{n_c} M_{c,i1}}{\frac{1}{h_s \sum_{k=1}^{n_s} F_k} - \gamma(3) \delta_u} \]  

(5)  

By assuming, as first attempt, that the sum of plastic moment of columns \( \sum_{i=1}^{n_c} M_{c,i1} \) is equal to \( M_{fc.Ed} \) (i.e. column-base connections able to develop a friction resistance corresponding to full-strength connections) and by solving the equation system (2) and (5), the following relationships are obtained:

\[ M_{fc.Ed} = \frac{h_1}{\frac{1}{2} \frac{\gamma(3) \delta_u}{\sum_{k=1}^{n_s} F_k}} \]  

(6)  

\[ M_{f_b.Ed,ns} = \left[ \left( 1 + \gamma(g) \delta_u \right) \frac{\sum_{k=1}^{n_s} F_k h_k}{h_1 \frac{1}{2} \frac{1 + \gamma(3) \delta_u}{\sum_{k=1}^{n_s} F_k}} \right] \sum_{k=1}^{n_s} \beta_k \]  

(7)  

Eq. (6) provides the sum of the flexural resistances corresponding to the slippage of the column base connections. Eq. (7) provides the sum of the flexural resistances corresponding to the slippage of beam-to-column dampers of the top storey [17], [20].

Starting from the value of the overall flexural resistance of top storey beam-to-column connections, the overall flexural resistance of the connections of all the other storeys are obtained according to the storey shear distribution:

\[ M_{f_b.Ed,k} = \beta_k M_{f_b.Ed,ns} \]  

(8)  

The design slippage bending moment \( M_{f_b.Ed,jk} \) of the beam-to-column connections of \( j \)-th bay at \( k \)-th storey is obtained by properly sharing the overall value (8) between the different bays. Such value is used to design friction dampers, being the slippage resistance equal to the ratio between such bending moment and the lever arm.

As soon as beam-to-column connections are designed, the beams sections can be properly obtained considering that, in this case, they are non dissipative members. In fact, the use of “FREEDAM” beam-to-column connections is aimed to prevent the beam yielding. To this scope, the beam sections have to be designed considering, on one hand, the maximum flexural resistance that “FREEDAM” beam-to-column connections are able to transmit and, on the other hand, the design moment needed to withstand vertical loads (\( q_v = \gamma g G_k + \gamma q Q_k \)).

\[ M_{b.Cd,jk} = \max \left( q_v \frac{L_f^2}{8} ; \gamma_{Rd} M_{f_b.Rd,jk} \right) \]  

(9)  

where \( M_{f_b.Rd,jk} \) is the design resistance of “FREEDAM” beam-to-column connections (selected to assure \( M_{f_b.Ed,jk} \leq M_{f_b.Rd,jk} \)) and \( \gamma_{Rd} \) is an overstrength factor that accounts for the random variability of the connection properties governing their flexural resistance (random variability of the friction coefficient and uncertainties in the control of the bolt preloading).

Regarding the design of the friction dampers equipping the column-base connections, the overall flexural resistance \( M_{fc.Ed} \), has to be shared between all the columns, obtaining internal actions needed for their design, \( M_{fc.Ed,i} \) (being \( i \) the column index). The column-base connections are designed to assure \( M_{fc.Ed,i} \leq M_{fc.Rd,i} \). Their resistance maximum resistance, due to the random variability of the connection properties governing the flexural resistance (random variability of the friction coefficient and uncertainties in the control of the bolt preloading) is expressed as \( M_{fc.Cd,i} \) (i.e. \( M_{fc.Cd,i} = \gamma_{Rd} M_{fc.Rd,i} \)). Therefore, similarly to the case of beams, the
column sections at the first storey are designed according to the maximum internal actions that column-base connections are able to transmit, i.e. \(\sum_{i=1}^{n_f} M_{c,i1}\) has to be greater than \(M_{fc,Cd}\) (being \(M_{fc,Cd}\) the sum of the maximum flexural resistances \(M_{fc,Cd,i1}\) of all the column-base connections).

### 2.2 Design of column members for failure mode control

As already stated in the previous section, the column sections have to be selected in order to assure that they remain in elastic range when the column-base connection transmits its maximum slippage flexural resistance. In addition, aiming to the control of the failure mode, according to TPMC, the column sections have to be selected from standard shapes in order to withstand the bending moment and the axial load acting at the collapse state. To this scope, the sum of the first storey plastic moments (reduced due to the contemporary action of the axial force), \(\sum_{i=1}^{n_c} M'_{c,i1}\), has to be calibrated to avoid first storey soft mechanism according to the following relationship:

\[
\frac{M_{fc,Cd,n_s} \sum_{k=1}^{n_s} \beta_k + M_{fc,Rd}}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u \leq \frac{M_{fc,Rd} + \sum_{i=1}^{n_f} M'_{c,i1}}{h_1 \sum_{k=1}^{n_s} F_k} - \gamma(3) \delta_u
\]

(10)

where \(M_{fc,Cd,n_s}\) is the maximum overall flexural strength that top storey beam-to-column connections are able to transmit, accounting for the random variability of the connection properties governing the flexural resistance (random variability of the friction coefficient and uncertainties in the control of the bolt preloading).

In order to design the column sections of all the other storeys, TPMC is still exploited. The application of Eq. (1) requires the expressions of \(\alpha_{0,lm}^{(t)}\) and \(\gamma_{lm}^{(t)}\). The expressions of \(\alpha_{0,lm}^{(t)}\) are reported in Table 1 with reference to the design slippage resistance of beam-to-column connections. In addition, in Table 2 similar relationships are provided with reference to the maximum slippage resistance that beam-to-column connections are able to transmit (this complies with the second principle of capacity design). Both cases need to be considered because it is undefinable, a priori, which is the most onerous one.

### Table 1 - Multipliers of horizontal forces according to first order rigid-plastic theory \((i_m = 2, 3, ..., n_s)\)

<table>
<thead>
<tr>
<th>Type-1</th>
<th>Type-2</th>
<th>Type-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_{0,lm}^{(1)}) = (\frac{M_{fc,Rd} + M_{fb,Rd,n_s} \sum_{i=1}^{n_f} \beta_k + \sum_{i=1}^{n_f} M_{ci,lm}^{(1)}}{\sum_{k=1}^{n_s} F_k h_k + h_{im} \sum_{k=i_m+1}^{n_s} F_k})</td>
<td>(\alpha_{0,lm}^{(2)} = \frac{\sum_{i=1}^{n_f} M'<em>{c,im} \sum</em>{k=1}^{n_s} \beta_k + M_{fb,Rd,n_s} \sum_{k=1}^{n_s} F_k (h_k - h_{im-1})}{h_1 \sum_{k=1}^{n_s} F_k})</td>
<td>(\alpha_{0,lm}^{(3)} = \frac{2 \sum_{i=1}^{n_f} M_{ci,lm}^{(3)}}{(h_{im} - h_{im-1}) \sum_{k=im}^{n_s} F_k})</td>
</tr>
</tbody>
</table>

### Table 2 - Multipliers of horizontal forces according to first order rigid-plastic theory \((i_m = 2, 3, ..., n_s)\) including overstrength of beam-to-column connections

<table>
<thead>
<tr>
<th>Type-1</th>
<th>Type-2</th>
<th>Type-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\gamma_{lm}^{(1)} = \frac{\sum_{k=1}^{n_s} F_k h_k + h_{im} \sum_{k=i_m+1}^{n_s} F_k}{h_{im} (\sum_{k=1}^{n_s} F_k h_k + h_{im} \sum_{k=i_m+1}^{n_s} F_k)})</td>
<td>(\gamma_{lm}^{(2)} = \frac{\sum_{k=im}^{n_s} F_k (h_k - h_{im-1})}{(h_{im} - h_{im-1}) \sum_{k=im}^{n_s} F_k})</td>
<td>(\gamma_{lm}^{(3)} = \frac{\sum_{k=im}^{n_s} V_k}{(h_{im} - h_{im-1}) \sum_{k=im}^{n_s} F_k})</td>
</tr>
</tbody>
</table>

### Table 3 - Slopes of mechanism equilibrium curves

As already observed with reference to the design of first storey columns, in order to obtain the most onerous design condition, column-base connection overstrength is not accounted for. Therefore, Eq. (1) provides the following design relationships:

\[
\frac{M_{fc,Rd} + M_{fb,Rd,n_s} \sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u \leq \frac{M_{fc,Rd} + M_{fb,Rd,n_s} \sum_{k=1}^{n_s} \beta_k + \sum_{i=1}^{n_f} M_{ci,lm}^{(1,1)}}{\sum_{k=1}^{n_s} F_k h_k + h_{im} \sum_{k=i_m+1}^{n_s} F_k} - \gamma_{lm}^{(1)} \delta_u
\]

(11)
\[
\frac{M_{c,Rd} + M_{f,b,c.d,n_s} \sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u \leq \frac{M_{f,c,Rd} + M_{f,b,c.d,n_s} \sum_{k=1}^{n_s} \beta_k + \sum_{i=1}^{n_c} M_{c,iim}^{(1,2)}}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} - \gamma_{i_m}^{(1)} \delta_u
\]

(12)

for type-1 mechanism;

\[
M_{f,c,Cd} + M_{f,b,c.d,n_s} \sum_{k=1}^{n_s} \beta_k \leq \gamma_{i_m}^{(2)} \delta_u
\]

(13)

\[
M_{f,c,Cd} + M_{f,b,c.d,n_s} \sum_{k=1}^{n_s} \beta_k \leq \gamma_{i_m}^{(2)} \delta_u
\]

(14)

for type-2 mechanism.

Finally, for type-3 mechanism, the most severe design condition is univocally determined and it is achieved when the beam-to-column connection overstrength, due to random variability, is accounted for:

\[
\frac{M_{f,c,Cd} + M_{f,b,c.d,n_s} \sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \gamma(g) \delta_u \leq \frac{2 \sum_{i=1}^{n_c} M_{c,iim}^{(3)}}{(h_{i_m} - h_{i_m-1}) \sum_{k=1}^{n_s} F_k} - \gamma_{i_m}^{(3)} \delta_u
\]

(15)

It is important to observe that, with reference to the design conditions to be satisfied to prevent type-2 and type-3 mechanisms, i.e. Eqs. (15-17), the most severe design condition is obtained when also the overstrength of column-base connections is considered.

From Eqs. (11), (12), (13), (14) and (15), the unknowns of the design procedure, i.e. the sum of the column plastic moment, reduced due to the contemporary action of the axial load, \( M_{c,iim}^{(1,1)} \), \( M_{c,iim}^{(1,2)} \), \( M_{c,iim}^{(2,1)} \), \( M_{c,iim}^{(2,2)} \) and \( M_{c,iim}^{(2,3)} \) are provided for each storey.

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(1,1)} = \left[ M_{f,b,Rd,n_s} \left( \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \right) + \frac{M_{f,c,Rd}}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \right]
\]

(16)

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(2,1)} = \left[ M_{f,b,Cd,n_s} \left( \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k + h_{i_m} \sum_{k=i_m+1}^{n_s} F_k} \right) + \frac{M_{f,c,Cd}}{\sum_{k=1}^{n_s} F_k h_k} \right]
\]

(18)

for type-1 mechanism;

\[
\sum_{i=1}^{n_c} M_{c,iim}^{(1,2)} = \left[ M_{f,b,Rd,n_s} \left( \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} - \frac{\sum_{k=1}^{n_s} \beta_k}{\sum_{k=1}^{n_s} F_k h_k} \right) + \frac{M_{f,c,Rd}}{\sum_{k=1}^{n_s} F_k h_k} \right]
\]

(19)

for type-2 mechanism;
the building deck (Figure 5). With reference to the seismic load combination (respectively. In addition, concentrated loads are applied at the nodes because transmitted by secondary beams of connections. Conversely, Beam-to-column connections (Figure 6) are equipped with bolted friction dampers designed to slip before the yielding of the beam [15] where the energy is dissipated through the slippage between the stem of the bottom angles and the beam flange by means of an interposed friction pad. Adopted bolts are high strength bolts, class 8.8, while the friction pad is constituted by a steel plate coated with thermally sprayed aluminum, exhibiting a friction coefficient of about 0.5 [16].

\[
\sum_{i=1}^{n_c} M_{c,i,m}^{(3)} \geq \left[ M_{f,b,c,d,n} + \sum_{k=1}^{n_b} F_k h_k + \sum_{k=1}^{n_s} F_k h_k + \left( \gamma_1^{(3)} - \gamma(t) \right) \frac{\left(h_{i,m} - h_{i,m-1}\right)}{2} \sum_{k=1}^{n_k} F_k \right] \sum_{c=1}^{n_c} \left( \sum_{i=1}^{n_c} M_{c,i,m}^{(1)} - \sum_{i=1}^{n_c} M_{c,i,m}^{(2)} \right)
\]

(20)

for type-3 mechanism.

Given the sum of reduced plastic moment of columns at each storey (for \(i_m > 1\)) by means of Eqs. (16) to (20), they can be distributed proportionally to the axial loads acting at the collapse state. It is important to underline that \(n_{c,m}^{(1)}\) and \(n_{c,m}^{(2,1)}\) have to be coupled with the axial loads computed starting from the knowledge of the design flexural resistance corresponding to the slippage of the friction dampers of beam-to-column connections. Conversely, \(n_{c,m}^{(1,2)}\), \(n_{c,m}^{(2,2)}\) and \(n_{c,m}^{(3)}\) have to be coupled with the axial loads computed starting from the knowledge of the maximum flexural resistance that beam-to-column connections are able to transmit, including the overstrength due to random variability. In this way, five design conditions are obtained for each storey. Finally, the column sections are designed to satisfy the following requirement:

\[
\sum_{i=1}^{n_c} M_{c,i,m} \geq \left\{ \sum_{i=1}^{n_c} M_{c,i,m}^{(1,1)} ; \sum_{i=1}^{n_c} M_{c,i,m}^{(1,2)} ; \sum_{i=1}^{n_c} M_{c,i,m}^{(2,1)} ; \sum_{i=1}^{n_c} M_{c,i,m}^{(2,1)} ; \sum_{i=1}^{n_c} M_{c,i,m}^{(3)} \right\}
\]

(21)

In addition, the technological condition assuring that section members decrease along the structure height has to be checked.

3. Application and validation

The proposed procedure has been applied to design a 3bays-4storeys MR-Frame whose structural scheme is depicted in Figure 5. The bay span is 5 m while the interstorey height is equal to 3.0 m. The characteristic values of the vertical loads acting on beams are equal to 3.32 kN/m and 2.49 kN/m for permanent and live loads, respectively. In addition, concentrated loads are applied at the nodes because transmitted by secondary beams of the building deck (Figure 5). With reference to the seismic load combination \((G_k + \psi_2 Q_k + E_d)\), the vertical loads acting on the beams of the analysed structure are 4.067 kN/m. The concentrated loads are equal to 40.92 kN and 20.46 kN, for internal and external nodes, respectively. The material adopted for all the structural members is steel grade S275. The design horizontal forces have been determined according to EC8, assuming a peak ground acceleration equal to 0.25g, a seismic response factor equal to 2.5, a behaviour factor equal to 6. The design horizontal forces distributed according to the simplified first vibration mode are reported in Figure 5. Beam-to-column connections (Figure 6) are equipped with bolted friction dampers designed to slip before the yielding of the beam [15] where the energy is dissipated through the slippage between the stem of the bottom angles and the beam flange by means of an interposed friction pad. Adopted bolts are high strength bolts, class 8.8, while the friction pad is constituted by a steel plate coated with thermally sprayed aluminum, exhibiting a friction coefficient of about 0.5 [16].
The selected beam sections are IPE180. In addition, the required slippage moment $M_{fb,Ed,jk}$, the slippage design resistance $M_{fb,Rd,jk}$, the number of bolts $n_b$, the beam height $h_b$ and the preloading force $N_b$, are reported for each beam-to-column connection in Table 4 while the same quantities for the column-base connection are reported in Table 5. In addition, in the same tables the maximum flexural resistance ($M_{fb,Cd,jk}$ and $M_{f,c.Cd,i1}$) which the connection is able to transmit, including random material variability, are given.

### Table 4 - Parameters for the design of beam-to-column connections

<table>
<thead>
<tr>
<th>Storey (k)</th>
<th>$\beta_k$ (-)</th>
<th>$M_{fb,Ed,jk}$ (kNm)</th>
<th>$n_b$ (-)</th>
<th>$h_b$ (mm)</th>
<th>Bolts (-)</th>
<th>$N_b$ (kN)</th>
<th>$M_{fb,Rd,jk}$ (kNm)</th>
<th>$M_{fb,Cd,jk}$ (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.5</td>
<td>28.839</td>
<td>4</td>
<td>180</td>
<td>$\phi 14$</td>
<td>55.00</td>
<td>31.68</td>
<td>38.016</td>
</tr>
<tr>
<td>2</td>
<td>2.25</td>
<td>25.955</td>
<td>4</td>
<td>180</td>
<td>$\phi 14$</td>
<td>50.00</td>
<td>28.8</td>
<td>34.56</td>
</tr>
<tr>
<td>3</td>
<td>1.75</td>
<td>20.187</td>
<td>4</td>
<td>180</td>
<td>$\phi 14$</td>
<td>40.00</td>
<td>23.04</td>
<td>27.648</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>11.535</td>
<td>4</td>
<td>180</td>
<td>$\phi 14$</td>
<td>25.00</td>
<td>14.4</td>
<td>17.28</td>
</tr>
</tbody>
</table>

### Table 5 - Parameters for the design of column-base connections

<table>
<thead>
<tr>
<th>Column (-)</th>
<th>$M_{fc,Ed,i}$ (kNm)</th>
<th>$n_b$ (-)</th>
<th>arm (mm)</th>
<th>Bolts (-)</th>
<th>$N_b$ kN</th>
<th>$M_{fc,Rd,i}$ (kNm)</th>
<th>$M_{fc,Cd,i1}$ (kNm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>49.81</td>
<td>4</td>
<td>500</td>
<td>$\phi 14$</td>
<td>35.00</td>
<td>56.00</td>
<td>67.20</td>
</tr>
<tr>
<td>C2</td>
<td>85.89</td>
<td>4</td>
<td>500</td>
<td>$\phi 14$</td>
<td>55.00</td>
<td>88.00</td>
<td>105.60</td>
</tr>
</tbody>
</table>

### Table 6 - Summary of column sections adopted at each storey

<table>
<thead>
<tr>
<th>STOREY $l_m$</th>
<th>C1</th>
<th>C2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>HE 160 B</td>
<td>HE 180 B</td>
</tr>
<tr>
<td>2</td>
<td>HE 140 B</td>
<td>HE 160 B</td>
</tr>
<tr>
<td>3</td>
<td>HE 140 B</td>
<td>HE 140 B</td>
</tr>
<tr>
<td>4</td>
<td>HE 100 B</td>
<td>HE 120 B</td>
</tr>
</tbody>
</table>

Finally, the selected column sections provided by the procedure are delivered in Table 6. In order to validate the design procedure, a static non-linear analysis (push-over) has been carried out for the designed frame by means of SAP 2000 computer program. This analysis has the primary aim to predict the collapse mechanism typology, testing the accuracy of the proposed design methodology. All the members have been modelled by means of beam-column elements, whose non-linearities have been concentrated in plastic hinges at their ends. In particular, plastic hinges accounting for the interaction between axial force and bending moment have been defined for columns, while dissipative devices at the beam ends and at column-bases have been modelled in pure bending whose resistance threshold is equal to $M_{fb,Cd,ns} \sum_{k=1}^{ns} \beta_k$ for beam-to-column and $M_{f,c.Cd}$ for fist storey column base “FREEDAM” connections. In fact, all hinges have been represented with a rigid plastic curve whose plastic threshold is defined to account for the overstrength. The elastic behaviour is not considered in plastic hinge definition because it is directly taken into account by the beam-column element. The load pattern used for the push-over analyses is in accord with the first mode of vibration of the structure. The analysis has been led under displacement control taking into account both geometrical and mechanical non-linearities. In addition, out of plan stability checks of compressed members have been performed at each step of the non-linear analysis. The results of the push-over analysis are mainly constituted by the capacity curve (Figure 7) where it is possible to observe that the softening branch of the push-over curve is coincident with the global mechanism equilibrium curve. It means that the collapse mechanism exhibited by the structure is in perfect agreement with the global mechanism. Figure 10 testifies this result where the pattern of hinges exhibited by the designed structure at a top sway displacement equal to $\delta_u$ is depicted. In addition, the designed structure has been checked...
also for serviceability requirements under seismic actions evaluated considering that the first period of vibration is equal to 1.67 s. It has been also checked that friction dampers of beam-to-column connections do not slip under the action of the bending moment corresponding to the vertical load combination at the ultimate limit state \((1.3G_k + 1.5Q_k)\).

In addition, in order to provide a more robust validation of the design methodology, non-linear Incremental Dynamic Analyses (IDA) have been carried out with reference to the same structural model used for push-over analysis. Record-to-record variability has been accounted for considering 10 recorded accelerograms selected from PEER database whose main characteristics (name, date, magnitude, ratio between PGA and gravity acceleration, length and step recording) are reported in Table 7. These earthquake records have been selected to approximately match the linear elastic design response spectrum of Eurocode 8 [2], for type A soil. Moreover, in order to perform IDA analyses, each ground motion has been scaled to obtain the same value of the spectral acceleration \(S_a(T_1)\) corresponding to the fundamental period of vibration \(T_1\) of the structure. This is the seismic intensity measure (IM) adopted for IDA analyses where \(S_a(T_1)\) values have been progressively increased until the occurrence of the target condition corresponding to the attainment of the limit value of the rotation of beam-to-column connections and of column-base connections. In addition, 5% damping according to Rayleigh model has been assumed with reference to the first period of vibration, \(T_1=1.67\) s, and third period of vibration, \(T_3=0.32\) s. In addition, for each accelerogram, the obtained pattern of yielding has been monitored as far as the spectral acceleration increases confirming the development of a global mechanism. In Figure 9 and Figure 10 the rotation demand for beam-to-column connections and for column-base connections, respectively, versus spectral acceleration, are reported. The \(S_a/g\) values leading to the connections target rotation (Table 8) are the minimum between those achieved by beam-to-column connections and those achieved by column-base connections. In particular, the average value of \(S_a(T_1)\) leading to the target connection rotation is about 0.890g while the average PGA is about 1.420 g. This is a very high value, compared with the design one (0.25g). Finally, it important to observe that friction dampers do not exceed the damper stroke and are even able to resort to other ductility sources when the spectral acceleration exceeds the value given in Table 8.
Finally, in Figure 11 the distribution of plastic hinges for increasing value of $S_a(T_1)$ with reference to Kobe earthquake record is reported. As it is possible to observe the pattern of yielding is in perfect agreement with the global mechanism testifying the accuracy of the design procedure.

Table 7 - Ground motions adopted for IDA analyses

<table>
<thead>
<tr>
<th>Earthquake (record)</th>
<th>Component</th>
<th>Date</th>
<th>PGA/g</th>
<th>Length (s)</th>
<th>Step recording (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coalinga (Slack Canyon)</td>
<td>H-SCN045</td>
<td>1985/05/02</td>
<td>0.166</td>
<td>29.99</td>
<td>0.01</td>
</tr>
<tr>
<td>Friuli, Italy (Buia)</td>
<td>B-BUI000</td>
<td>1976/09/15</td>
<td>0.110</td>
<td>26.385</td>
<td>0.005</td>
</tr>
<tr>
<td>Imperial Valley (Agrarias)</td>
<td>H-AGR003</td>
<td>1979/10/15</td>
<td>0.370</td>
<td>28.35</td>
<td>0.01</td>
</tr>
<tr>
<td>Kobe (Kakogawa)</td>
<td>KAK000</td>
<td>1995/01/16</td>
<td>0.251</td>
<td>40.95</td>
<td>0.01</td>
</tr>
<tr>
<td>Northridge (Stone Canyon)</td>
<td>SCR000</td>
<td>1994/01/17</td>
<td>0.252</td>
<td>39.99</td>
<td>0.01</td>
</tr>
<tr>
<td>Santa Barbara (Courthouse)</td>
<td>SBA132</td>
<td>1978/08/13</td>
<td>0.102</td>
<td>12.57</td>
<td>0.01</td>
</tr>
<tr>
<td>Spitak Armenia (Gaukarian)</td>
<td>GKS000</td>
<td>1998/12/07</td>
<td>0.199</td>
<td>19.89</td>
<td>0.01</td>
</tr>
<tr>
<td>Friuli, Italy (Tolmezzo)</td>
<td>TMZ000</td>
<td>1976/05/06</td>
<td>0.351</td>
<td>36.345</td>
<td>0.005</td>
</tr>
<tr>
<td>Irpinia (Calitr)</td>
<td>A-CTR000</td>
<td>1980/11/23</td>
<td>0.132</td>
<td>35.79</td>
<td>0.0024</td>
</tr>
<tr>
<td>Victoria, Mexico (Chihuahua)</td>
<td>CHI102</td>
<td>1980/06/09</td>
<td>0.150</td>
<td>26.91</td>
<td>0.01</td>
</tr>
</tbody>
</table>

Table 8 - $S_a(T_1)$ and PGA values corresponding to the attainment of the connection target rotation

<table>
<thead>
<tr>
<th>Earthquake (record)</th>
<th>Sa/g</th>
<th>PGA_c/g</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Coalinga (Slack Canyon) & 0.65 & 0.955 \\
Friuli, Italy (Buia) & 0.85 & 0.937 \\
Imperial Valley (Agrarias) & 1.20 & 1.829 \\
Kobe (Kakogawa) & 1.50 & 1.552 \\
Northridge (Stone Canyon) & 0.65 & 1.620 \\
Santa Barbara (Courthouse) & 0.50 & 1.414 \\
Spitak Armenia (Gaukasian) & 0.65 & 0.997 \\
Friuli, Italy (Tolmezzo) & 0.90 & 3.310 \\
Irpinia (Calitri) & 1.20 & 0.986 \\
Victoria, Mexico (Chihuahua) & 0.80 & 0.609 \\
\textbf{Mean value} & \textbf{0.89} & \textbf{1.420} \\

5. Conclusions

In this paper, a design procedure based on the Theory of Plastic Mechanism Control for design MR-Frames whose connections, either beam-to-column or column-base, are equipped with friction dampers has been presented. The proposed design procedure allows designing structure exhibiting at collapse a global mechanism assuring that all the friction dampers are involved in the dissipation of the earthquake input energy while all the columns remain in elastic range. The mechanism actual developed and the performances achieved by the structure have been investigated by means of both push-over and dynamic analyses. The structure shows high values of both spectral acceleration and PGA needed to achieve the target rotation of friction dampers.

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7. References


