GEOGRAPHICALLY VARYING GROUND MOTION PREDICTIONS: CASE STUDY FOR TWO HISTORICAL EVENTS

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Abstract

Ground motions induced by earthquake vary geographically. The use of geographically weighted regression technique to estimate geographically varying ground motion measures has been considered in the literature to predict the peak ground acceleration, spectral accelerations and Arias intensity (AI). It has been shown that its use provides equivalent or better performance as compared to the geostatistical interpolation techniques. An advantage of geographically varying ground motion prediction model (GVGPM) is that it could take into account the underlying physics by adopting a functional form that is similar to that of a conventional ground motion prediction equation (GMPE). In this study, we develop GVGPM to predict the cumulative absolute velocity (CAV) for the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake. The CAV can be used as an indicator of structural damage that is more predictable than other ground motion intensity measures and has recently been considered as an alternative to AI in earthquake engineering and geotechnical applications. We show that the developed GVGPM out-performs the preferred geostatistical interpolation technique and GMPE.

Keywords: ground motion prediction; Geographically Weighted Regression; cumulative absolute velocity.
1. Introduction

Seismic ground motions vary geographically, due to the source geometry, wave propagation path and/or local site conditions. Maps of contours of constant macroseismic intensity and ground motion measures for historical events are often irregular [1, 2]. For seismic hazard and risk analysis, ground motions from an earthquake that is similar or identical to a historical event at the sites of the buildings or spatially distributed infrastructure are needed to perform the procedures. The co-location of the sites of interest and the ground motion records is ideal but rare. In this case ground motions need to be estimated by predictive models. Ground motion prediction equations (GMPE) are widely used in seismic hazard and risk analysis. Often the adopted mathematical forms for the GMPE are based on implicit assumptions that the ground motion measures are independent of wave propagation path and/or focal mechanism and geometries [3, 4]. In other words, empirical GMPE does not distinguish the stations located at different azimuths, leading to very similar predictions at the same source-to-site distance [5].

One of the approaches to predict geographically varying ground motions is to use spatial interpolation techniques to interpolate the ground motion measures at the sites of interest. An example of this is the application in the ShakeMap, which is interpolated based on instrumental recordings of ground motions and empirical ground motion functions to represent the shaking intensity shortly after the occurrence of an earthquake [6]. Geostatistical interpolation techniques including Kriging and Co-Kriging have also been used in the literature [7, 8]. However, it is noted that the use of the spatial interpolation techniques for the ground motion measures is aimed at replicating some spatially varying characteristics of the recorded motions [8]; it does not seek to incorporate or model the underlying physics of the ground motion measures.

To predict ground motion measures for an event identical or similar to a historical earthquake, it is proposed to use geographically weighted regression technique to estimate site-dependent (or geographically varying) ground motions based on physics-based GMPE [5]. Using the proposed method, geographically varying ground motion prediction models (GVGMPM) for peak ground acceleration (PGA), spectral acceleration (SA) at different vibration periods and Arias intensity (AI) were developed for two historical events: the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake by [5]. They showed that the intraevent standard deviation of the GVGMPM is lower than that of the ground motion prediction equation developed based on the same dataset; and that GVGMPM is advantageous comparing to geostatistical spatial interpolation techniques because it takes into account the underlying physics while providing equivalent performance for the two historical earthquakes. Ground motion intensity measures (GMIMs), including PGA, SA, AI and cumulative absolute velocity (CAV), are of interest since different GMIM represents different characteristics of the ground shaking. The CAV is defined as the integral of the absolute value of the acceleration time series:

\[
\text{CAV} = \int_0^{t_{\text{max}}} |a(t)| \, dt,
\]

where \(|a(t)|\) is the absolute value of the acceleration time history at time \(t\) and \(t_{\text{max}}\) is the total duration of the record. The CAV was originally developed and proposed as an index to indicate the onset of structural damage to engineered structures by EPRI [9]; it has recently been considered as an alternative to AI in earthquake engineering and geotechnical applications where the latter intensity measure is traditionally used [10]. This is based on the observation that the standard deviation of CAV is smaller and less sensitive to amplitude than AI.

This study is focusing on the prediction of the CAV. For the prediction, we extend the previous study [5] by developing GVGMPM for CAV for the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake. The objectives are to: i) examine the performance of the GVGMPM in predicting the CAV in relation to the GMPE as well as the geostatistical interpolation techniques for two particular historical seismic events developed based on the same dataset; ii) compare the standard deviation of CAV with that of other IMs for the GVGMPM. In the following, we first describe the approach used to develop GVGMPM, followed by its application to records from two historical seismic events, and finally summarize the key findings.
2. Approach to develop geographically varying ground motion prediction model

To model the geographically varying ground motions induced by an earthquake event, as proposed in [5], the GVGMPM can be developed based on geographically weighted regression (GWR) modeling [11, 12]. The application of GWR requires the model to be a linear regression equation, which is satisfied by the physics-based GMPE that is used in this paper, as shown in the following. The basic idea of the GWR is that the regression for a site of interest is carried out by borrowing the observations from other sites; all the borrowed information is weighted based on the distances from the site of interest to the sites where the information is gathered, where the weight is calculated according to an adopted weighting or kernel function. The regression coefficients are obtained by minimizing the sum of the weighted squared error (i.e., difference between the observed and predicted value) at each site.

The simple functional form below was considered by [13] for developing GMPE for the 1999 Chi-Chi earthquake:

\[
\ln Y = c_0 + c_1 \ln R + c_2 R + c_3 \ln \left( \frac{V_{s30}}{V_{ref}} \right) + \varepsilon
\]  

(2)

where \( c_i \) (i = 0,...,3), are the model parameters; \( R = \sqrt{R_{up}^2 + h^2} \); \( R_{up} \) (km) is the closest distance from the recording site to the fault rupture plane; \( h \) (km) represents a fictitious depth within [0, 10] km and is determined by searching the value that minimizes the intra-event variability; \( V_{s30} \) is the average shear-wave velocity of the top 30 m soil; the shear-wave velocity for the reference soil condition \( V_{ref} = 760 \) m/s; and \( \varepsilon \) denotes the intra-event residual. Eq. (2) can be viewed as a simplified version of the GMPE given in [14] for a specific historical event. A similar model was considered by [2] for the 2008 Wenchuan earthquake. Note that while CAV is not considered in these studies, previous studies uses same functional form when developing GMPE for PGA, SA, AI and CAV [10]. Therefore, Eq. (2) is adopted for both events in this paper.

If the GMIM are available at \( m \) stations, the model parameters for Eq. (2) can be obtained by minimizing the sum of the square of the errors for all the considered stations, \( \varepsilon_T = \sum_{i=1}^{m} \varepsilon_i^2 \). To illustrate the differences between ordinary linear regression and GWR, we note that for a given \( h \) value, Eq. (2) is a linear model, and the application of linear regression leads to the estimated coefficients \( \hat{C} = [c_0, c_1, c_2, c_3]^T \), denoted by \( \hat{C} \), that is given by,

\[
\hat{C} = (X^T X)^{-1} X^T Z
\]  

(3)

where the superscript T denotes the transpose of the matrix; \( Z = [\ln Y_1, \ldots, \ln Y_m]^T \); \( X \) is an \( m \times 4 \) matrix with element \( x_{i,1} = 1, x_{i,2} = \ln R_i, x_{i,3} = R_i, x_{i,4} = \ln (V_{s30,i}/V_{ref}) \), in which \( Y_i, R_i \) and \( V_{s30,i} \) denote the values of \( Y \), \( R \) and \( V_{s30} \) for the \( i \)-th station. This analysis can be carried out for a range of \( h \) values, and the best model is the one that minimizes \( \varepsilon_T \).

By considering the geographically varying relations, Eq. (2) is re-written as,

\[
\ln Y_i = c_{0,i} + c_{1,i} \ln R + c_{2,i} R + c_{3,i} \ln \left( \frac{V_{s30,i}}{V_{ref}} \right) + \varepsilon_i
\]  

(4)

where the symbols have the same meaning as before except that the symbols with an additional subscript \( i \) indicate that they depend on the \( i \)-th location. In this GVGMPM, the regression coefficients for a given value of \( h \) are given by [11, 12],

\[
\hat{C}_i = (X_i^T W_i X_i)^{-1} X_i^T W_i Z
\]  

(5)
where \( \hat{C}_i \) denotes the estimate of the vector of coefficients \( C_i = [c_{0,i}, c_{1,i}, c_{2,i}, c_{3,i}]^T \) for the \( i \)-th station, and \( W_i \) is a diagonal matrix with diagonal elements \( w_{ij}, j = 1, \ldots, m \), representing the weight calculated using an adopted kernel function. One of the popular kernel functions is the Gaussian kernel function,

\[
W_{ij} = \exp \left(-\frac{(\Delta_{ij} / b)^2}{2} \right)
\]

where \( b \) is referred to as the bandwidth, and \( \Delta_{ij} \) is the distance between the \( i \)-th and the \( j \)-th stations. The selection of the best \( b \) value for the GWR can be carried out based on the Akaike information criterion or on a cross-validation statistical analysis [15, 12]. For a given value of \( b \), the differences and similarities in the analysis procedures for the models shown in Eqs. (2) and (4) are apparent by comparing Eqs. (3) and (5).

An alternative to the ground motion prediction models shown in Eqs. (2) and (4) is using geostatistical interpolation techniques to predict geographically varying ground motions. These techniques that are commonly used include ordinary kriging (KO), simple kriging (KS), universal kriging (KU), ordinary co-kriging (Co-KO), simple co-kriging (Co-KS), universal co-kriging (Co-KU) [16, 17]. Kriging uses a linear combination of weighted measured values to estimate the value of the surface at a point without measurement, and the surface is treated as a random field. Co-kriging is similar to kriging, except that it incorporates additional covariates and the correlations among different variables. Co-kriging could be effective for data with significant inter-variable correlation. For Co-kriging interpolations, \( V_{s30} \) is used as the co-variate, since it represents the local site condition that may amplify the ground motions at the site. To select the preferred geostatistical interpolation technique and compare interpolation models with GMPE and GVGMPM, the so-called cross validation analysis is carried out. In the (leave-one-out) cross-validation analysis, a sample at one measurement location is withheld and a prediction is made using a selected spatial interpolation technique with the remaining samples [16, 17]; statistics of the differences between the measured and predicted values obtained for the measurement locations are evaluated and used as performance indicators.

It is emphasized that the developed GVGMPM is conceptually more attractive than geostatistical interpolation techniques because the underlying physics of the effects of earthquake source, wave propagation and local site conditions on the induced ground motions are taking into account by adopting a functional form similar to the conventional GMPE. As will be shown in the next section, the performance of GVGMPM is equivalent to (or better than) that of the preferred geostatistical interpolation techniques, at least for the two historical events we considered.

### 3. Application to historical events

In this section, we first summarize the key findings through comparisons between GMPE, GVGMPM, Kriging and Co-kriging that are developed for PGA, SA and AI based on the ground motion records from two well-recorded damaging earthquakes: the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake. Based on the approach presented in the previous section, we then show the GVGMPM for CAV for these two events. The results obtained by using the developed GVGMPM is compared with those obtained based on the conventional GMPE and geostatistical interpolation techniques developed based on the same dataset.

A set of 389 ground motion records from the NGA database were selected to develop GMPEs and to investigate spatial correlation characteristics of the excitations [13], and were adopted in this study. The stations with \( R_{rup} \) up to about 180 km covering the azimuth angle from 0 to 360 degrees, provide a well-defined setting to test the potential of the GVGMPM. The \( V_{s30} \) for Taiwan is spatially interpolated based on a total of 663 stations including the \( V_{s30} \) values in the NGA database and the measured \( V_{s30} \) values reported by [18]. For the 2008 Wenchuan earthquake, 272 records are selected from China Strong Motion Networks Center, based on those considered by [2]. To ensure data quality, the records are processed by applying a zeroth-order correction, a baseline correction and a fourth-order low-cut Butterworth filter with corner frequency equal to 0.05 Hz [19]. The \( V_{s30} \) values for 225 of the considered stations are obtained from the NGA West2 database; the Vs30 values for the rest stations are spatially interpolated based on all the stations available in the NGA West2 database. The
GMIMs including PGA and SA for a range of vibration periods are reported in terms of the 50th percentile of a set of geometric means for the as-recorded orthogonal horizontal motions rotated through all possible non-redundant rotation angles (i.e., GMRotI50, [20]). For consistency, AI and CAV for the 1999 Chi-Chi earthquake and GMIMs for the 2008 Wenchuan earthquake based on GMRotI50 are calculated; the GMIMs based on GMRotI50 are considered throughout this study although other measures for the excitations in the orthogonal horizontal plane could be considered [21].

3.1 GVGMPM for PGA, SA and AI

Based on the selected ground motion records for the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake, GVGMPM were developed for PGA, SA at 0.1 s, 0.5 s and 2.0 s and AI [5]. The developed GVGMPM are compared with GMPE and geostatistical interpolation methods including KO, KS, KU, Co-KO, Co-KS, and Co-KU. As expected, the ground motions predicted by GVGMPM vary geographically whereas those predicted by GMPE follow more regular oval pattern surrounding the finite fault. It was also shown that the intraevent standard deviation of the GVGMPM is within 50% to 65% of that of the GMPE. The predictions of GVGMPM is practically similar with that predicted by the preferred interpolation technique, KO. Comparison of the mean of the prediction error (ME) and the root-mean-square-error (RMSE) from the cross-validation for KO and GVGMPM indicates that GVGMPM is generally equivalent to, but some cases out-performs the preferred interpolation technique (i.e., KO). Similar observations can be made based on the analysis of the 2008 Wenchuan earthquake.

![Figure 1](image1.png)

**Fig. 1 –** Comparison of ME and RMSE for the preferred geostatistical interpolation technique, ordinary kriging (KO), and the geographically varying ground motion prediction model (GVGMPM).

3.2 GVGMPM of CAV for the 1999 Chi-Chi earthquake

Using the GMRotI50 of CAV for the selected 389 ground motion records, GVGMPM for the 1999 Chi-Chi earthquake is developed and compared with geostatistical interpolation techniques as well as GMPE. We first carry out the cross-validation analysis for the six interpolation techniques (i.e., KO, KS, KU, Co-KO, Co-KS and Co-KU), the ME and RMSE are shown in Table 1. Based on the results, KO is the preferred interpolation technique since it has the lowest RMSE. This is consistent with the observation in [5]. The contour map for CAV based on KO is shown in Fig. 2a, where the geographically varying characteristics of the GMIM is illustrated. There is a bull’s eye centered near [latitude, longitude] = [23.8°N, 120.9°E], and the attenuation of the ground motions is faster on the eastern than the western part of the island.
Table 1. Statistics from cross-validation analysis for different geostatistical interpolation techniques and GVGMPM.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>Statistics</th>
<th>KO</th>
<th>KS</th>
<th>KU</th>
<th>Co-KO</th>
<th>Co-KS</th>
<th>Co-KU</th>
<th>GVGMPM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 Chi-Chi</td>
<td>ME</td>
<td>7.02E-03</td>
<td>-2.28E-03</td>
<td>-5.47E-03</td>
<td>-4.88E-03</td>
<td>-2.16E-03</td>
<td>-4.88E-03</td>
<td>-2.41E-03</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.335</td>
<td>0.339</td>
<td>0.343</td>
<td>0.341</td>
<td>0.339</td>
<td>0.341</td>
<td>0.305</td>
</tr>
<tr>
<td>2008 Wenchuan</td>
<td>ME</td>
<td>-2.80E-03</td>
<td>2.29E-02</td>
<td>6.99E-04</td>
<td>-1.47E-03</td>
<td>2.51E-02</td>
<td>-5.68E-02</td>
<td>-6.70E-03</td>
</tr>
<tr>
<td></td>
<td>RMSE</td>
<td>0.593</td>
<td>0.623</td>
<td>0.597</td>
<td>0.597</td>
<td>0.631</td>
<td>0.610</td>
<td>0.573</td>
</tr>
</tbody>
</table>

Fig. 2 – Contour maps of CAV for the 1999 Chi-Chi earthquake: (a) interpolated based on the preferred geostatistical interpolation technique, ordinary kriging; (b) predicted using GMPE; (c) predicted using GVGMPM.

Next, a GMPE using the functional form as shown in Eq. (2) for CAV is developed based on the same dataset. The regression coefficients are shown in Table 2. The contour map for CAV that is predicted based on the developed GMPE is plotted in Fig. 2b using the same color scale as in Fig. 2a. Fig. 2b shows more regular contours that exhibit oval shapes with their major axis about 5° with respect to the north (with clockwise rotation defined as positive), which is consistent with the strike angle of the finite fault solution of this event. We also note that the predictions from GMPE does not reproduce the bull’s eye seen in Fig. 2a, neither its location nor the amplitude of ground motion.

Table 2. Regression coefficients of GMPE (Eq. (2)) for CAV.

<table>
<thead>
<tr>
<th>Earthquake</th>
<th>$\hat{h}$</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$\sigma_\varepsilon$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999 Chi-Chi</td>
<td>10</td>
<td>8.429</td>
<td>-0.584</td>
<td>-0.005</td>
<td>-0.659</td>
<td>0.376</td>
</tr>
<tr>
<td>2008 Wenchuan</td>
<td>10</td>
<td>11.226</td>
<td>-1.015</td>
<td>2.11E-04</td>
<td>-1.020</td>
<td>0.736</td>
</tr>
</tbody>
</table>
Finally, we show the results for the GVGMPM for the 1999 Chi-Chi earthquake. The predicted contour map for CAV is shown in Fig 2c. For any practical purposes, the geographical distribution of the GMIM in Fig 2c is very similar to that developed based on the preferred interpolation technique (KO) and shown Fig. 2a. However, we notice that the maximum value predicted by the GVGMPM is lower than that from the actual records. To further quantify the performance of GVGMPM, the cross-validation results are also shown in Table 1. As expected, comparison of RMSE values indicates the GVGMPM out-performs the preferred interpolation technique (i.e., KO). Again this observation is consistent with [5]. The intraevent standard deviation of GVGMPM (i.e., RMSE in Table 1) is about 80% of $\sigma_e$ shown in Table 2. We also note that the standard deviation for CAV is lower than that for PGA, SA at 0.1 s, 0.5 s, and 2.0 s and AI for a given model (regardless of interpolation, GMPE or GVGMPM). This observation is also made by [10] when comparing the uncertainties of the GMPE for CAV with that for other GMIM.

3.3 GVGMPM of CAV for the 2008 Wenchuan earthquake

For the 2008 Wenchuan earthquake, we repeat the analysis that was employed for the 1999 Chi-Chi earthquake. The results for cross-validation for both interpolation techniques and GVGMPM are shown in Table 1, and the regression coefficients for the GMPE are shown in Table 2. Similar to Fig. 2, the contour maps for CAV are shown in Fig. 3 for the interpolation based on KO, predictions using GMPE and GVGMPM. Some observations can be made based on these results:

- The preferred geostatistical interpolation technique is KO. Contour map based on the KO (Fig. 3a) illustrates the geographical distribution of the CAV.
- The contours based on GMPE (Fig. 3b) exhibit oval shapes and differ from those shown in Fig. 3a. This again indicates that for a particular event, the use of the GMPE could lead to unsatisfactory results.
- The GVGMPM out-performs KO with similar ME and lower RMSE.
- The contour map based on GVGMPM (Fig. 3c) shows improved predictions than Fig. 3b, although the contours still show more regular oval shape than that shown in Fig. 3a. The slower attenuation of the GMIM on the southeastern side of the source (comparing to the northwestern side) seen in Fig. 3a is somewhat reproduced in Fig 3c. The relatively high intensity area in the north of Xi’an and southeast of Yinchuan that is missing from the predictions of GMPE is also reproduced in Fig. 3c.
Fig. 3 – Contour maps of CAV for the 2008 Wenchuan earthquake: (a) interpolated based on the preferred geostatistical interpolation technique, ordinary kriging. (To be continued.)
Fig. 3 – (continued) Contour maps of CAV for the 2008 Wenchuan earthquake: (b) predicted using GMPE and (c) predicted using GVGMPM.

- We note that there is some abnormal prediction on the southeast corner on the contour map based on GVGMPM (Fig. 3c), near Changsha and Wuhan. This is because the distribution of the recording stations is uneven (see Fig. 6 in [5]), scarce recording coverage provides very little constrain on the model for this area.
- The observations based on the comparison of standard deviations from the 1999 Chi-Chi earthquake are equally applicable for the 2008 Wenchuan earthquakes.

4. Conclusions
In this study, the use of the geographically weighted regression technique to predict the cumulative absolute velocity (CAV) for an event identical or similar to a historical earthquake is considered. Two historical event, the 1999 Chi-Chi earthquake and the 2008 Wenchuan earthquake, are used in the case study. CAV is used as the intensity measure because it is related to the structural damage and has recently been proposed to be used as an alternative to AI. The developed GVGPM for CAV perform better in the cross-validation analysis comparing to the preferred geostatistical interpolation technique and GMPE. The (intraevent) standard deviation is reduced to about 80% of that for the GMPE that is developed based on the same dataset.

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7. References


