Stochastic dynamic response analysis of bridge piers in deep water under combined earthquake and wave actions

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Abstract

Earthquake and wave actions may be simultaneously applied to bridge piers in deep water. It is necessary to calculate the dynamic response of bridge piers under combined earthquake and wave actions. In order to consider the randomness of earthquake and wave action, stochastic dynamic response analysis is adopted. The interaction between bridge piers and water can be considered into two parts: the hydrodynamic pressure and wave action. Since most bridge piers belong to large-scale cylinder, the hydrodynamic pressure is established using radiation wave theory, and the wave action is considered by the diffraction theory. Based on the pseudo excitation method, a stochastic dynamic analysis method for bridge piers in deep water under combined actions of earthquake and wave is developed. A circle solid pier is calculated with a sectional diameter of 5 m and a height of 24.7 m. The Clough-Penzien spectrum is used as the earthquake acceleration spectrum and the Bretschneider-Mitsuyasu spectrum is used as the wave height spectrum. Stochastic dynamic responses of bridge piers under the earthquake action, wave action and combined actions of earthquake and wave, are analyzed respectively. Different relative water depths and earthquake acceleration amplitudes are considered for the analysis of the numerical model under earthquake action. While the model under wave action is calculated, different wave heights and relative water depths are analyzed and compared. Moreover, in order to compare with the response under combined earthquake and wave actions, the response under the direct superposition of two individual actions should be achieved. The results show that the stochastic dynamic response of the pier in deep water is increasing because of the influence of the hydrodynamic pressure. For a pier with a fixed water depth under different earthquake acceleration amplitudes, the increased percentage of the internal force power spectrums at the bottom of piers is identical, considering the effect of the hydrodynamic pressure. With relative water depth becoming deeper, the effect of the hydrodynamic pressure to the internal force power spectrums at the bottom of the pier becomes more significant. Because the attached mass generated by the pier’s elastic motion in the water has little effect on the frequency response function of the pier, stochastic dynamic responses under combined earthquake and wave actions are identical with these under the direct superposition of two individual actions.

Keywords: bridge piers in deep water, earthquake action, wave action, pseudo excitation method
1. Introduction

With the development of economy, the number of bridges, especially sea-crossing and river-crossing bridges, increases faster. There are already a large number of studies about the dynamic responses of multiform bridges under earthquakes [1, 2]. As the sea-crossing or river-crossing bridges are in a water environment, it needs to consider the interaction between piers and water under earthquakes. Generally, the interaction can be considered into two parts: the hydrodynamic pressure generated by moving piers in static water and the wave action generated by static piers in moving water. Earthquake ground motion and wave height are both random processes, therefore it is necessary to analyze the response of bridge piers under combined earthquake and wave actions using the stochastic vibration method.

The studies on analysis method of the interaction between piers and water mainly include numerical method and semi-numerical method. Ozdemir Z [3] proposed a fully nonlinear liquid-solid coupling finite elements approach based on the numerical method. Yeung RW [4] studied on the hydrodynamic attached mass and damping of a vertical column in finitely deep water. Keming S [5] deduced the hydrodynamic pressure formula of an axisymmetric structure based on the Trefftz-complete function. Lai W [6] deduced the exterior attached hydrodynamic pressure of circle piers based on the linear radiation wave theory. There are two methods normally adopted for calculation of the wave force, including the diffraction wave theory for large-diameter columns and Morison function for small-diameter columns [7-9]. The studies shown above mainly analyzed the effects of the interaction of piers and water for piers or bridges in time domain, and the effect is related to the earthquake wave [10]. To date, there are only limited number of studies on stochastic dynamic responses analysis of bridge piers in deep water under the earthquake action or combined earthquake and wave actions.

Stochastic vibration methods applied on engineering structures mainly include the pseudo excitation method and Monte Carlo method. Lu F [11] proposed an efficient algorithm for non-stationary random vibration of vehicle-bridge systems basing on the pseudo excitation method. Fan LC [12] investigated the response characteristics of long-span cable-stayed bridges under seismic action with spatial variation by using the pseudo excitation method. Considering the uncertainties in the ground motion characteristics through the Monte Carlo method, Moghaddasi M [13] evaluated the influence of foundation flexibility on the structural seismic response. However, these studies only analyzed the responses under an single load. In this paper, based on the pseudo excitation method, this paper establishes a stochastic dynamic responses analysis method for bridge piers in deep water under combined earthquake and wave actions in linear scope. Stochastic dynamic responses of bridge piers under the earthquake action, the wave action and combined earthquake and wave actions are analyzed respectively.

2. Stochastic dynamic analysis method of bridge piers in deep water under combined earthquake and wave actions

2.1 Hydrodynamic pressure based on radiation wave theory

Generally, the radiation wave theory uses the separation variable method or Green function to obtain general solutions of the velocity potential of the Laplace equation, and solve the constant of the velocity potential by water’s boundary condition. After that, the expression of the hydrodynamic pressure from the Bernoulli equation can be obtained, and the deduced process is provided in reference [5] in detail.

The structure-fluid-ground system is illustrated in Fig.1. The water is assumed to be irrotational, inviscid, and incompressible; and the motion of the water is limited to small amplitudes during the earthquake action.
In the cylinder coordinate, when the ground undergoes a harmonic motion with frequency $\omega$, the velocity potential of the water can be expressed by the complex form $\Phi(r,\theta,z,t)=\phi(r,\theta,z)e^{i\omega t}$ according to the radiation wave theory. Substitute $\phi(r,\theta,z)$ into the Laplace equation, the flow control equation is

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r}\frac{\partial \phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \phi}{\partial \theta^2} + \frac{\partial^2 \phi}{\partial z^2} = 0$$

(1)

$\phi(r,\theta,z)$ can be expressed by introducing the Trefftz-complete function and meets boundary conditions as shown below

$$\frac{\partial \phi}{\partial z} = 0, z = h$$

(2)

$$\frac{\partial \phi}{\partial z} = 0, z = 0$$

(3)

$$\frac{\partial \phi}{\partial r} = \frac{\partial U}{\partial t}\cos \theta, z = a, 0 \leq z \leq h$$

(4)

$$\lim_{r\to\infty}\sqrt{r}\left(\frac{\partial \phi}{\partial r} - ik\phi\right) = 0$$

(5)

Eqs. (2)-(5) are the free surface boundary condition, the bottom boundary condition, the interface surface boundary condition and Sommerfeld radiation boundary condition, respectively; $i=\sqrt{-1}$; $a$ is the pier’s radius; $h$ is water depth; $k=\omega/c$, where $c$ denotes the sound velocity in the water; and $U$ is the absolute displacement of the pier, including the rigid displacement and elastic displacement. The full velocity potential can be taken into two parts: the velocity potential $\Phi_1$ generated by the pier’s rigid motion and velocity potential $\Phi_2$ generated by the pier’s elastic motion. They can be derived by Eqs. (1)-(5). Note that the effect of free surface wave is ignored in the analysis.

According to the the small amplitudes wave theory, the hydrodynamic pressure $F$ of the pier’s surface along a unit height is

$$F = \rho \int_0^{2\pi} \frac{\partial \Phi(r,\theta,z,t)}{\partial t} a \cos \theta \, d\theta$$

(6)

where $\rho$ is the water density. Substitute the rigid velocity potential $\Phi_1$ and elastic velocity potential $\Phi_2$ into Eq. (6), the hydrodynamic pressure can be expressed as
where \( F_1 \) and \( F_2 \) is the hydrodynamic pressure generated by the ground motion and the elastic vibration of the structure respectively; \( \ddot{x}_g \) and \( \ddot{x} \) are the harmonic ground motion acceleration and elastic vibration acceleration of the structure respectively. \( M_1 \) and \( M_2 \) are the rigid attached mass matrix and elastic attached mass matrix respectively. \( M_1 \) is a diagonal matrix, and the \( i \)th diagonal element is

\[
m_{ii} = -2\pi a \rho \int_{\Gamma_i} \sin(\lambda_m z) K_1(\beta_m z) \frac{\cos(\lambda_m z)}{\lambda_m \beta_m h} \, dz
\]

The elements in the mass matrix \( M_2 \) are

\[
m_{xy} = -2\pi a \rho \int_{\Gamma_i} \cos(\lambda_m z) L_i K_1(\beta_m z) \frac{\cos(\lambda_m z)}{\beta_m h} \, dz
\]

in which

\[
K_1(\beta_m a) = \frac{K_1(\beta_m a)}{K_1(\beta_m a)} , \quad K_0(\beta_m a) = \frac{\partial K_1(\beta_m r)}{\partial \beta_m r} \bigg|_{r=a}
\]

\[
\lambda_m = \beta_m = \frac{(2m-1)\pi}{2h}
\]

where \( \Gamma_i \) denotes the integral interval of the \( i \)th node; \( L_i \) is the \( i \)th element’s length; \( Z_i \) is the \( i \)th node’s coordinate of axis \( Z \); and \( K_1(\beta_m r) \) is the modified Bessel function of the second kind and first order.

2.2 Wave force based on diffraction theory

Generally, the velocity potential of the diffraction problem can be considered into two parts: the incident potential \( \phi_i \) and diffraction potential \( \phi_d \). Substitute both potentials into the Bernoulli equation, the wave force acting on a circle pier can be solved [10]. In the cylinder coordinate, the incident potential \( \phi_i(r,\theta,z) \) should be expressed as

\[
\phi_i(r,\theta,z) = -\frac{igA \cosh kz}{\omega \cosh kh} \sum_{m=0}^{\infty} \varepsilon_m i^m J_m(\alpha r) \cos m\theta
\]

where \( i = \sqrt{-1} \); \( \varepsilon_m = \begin{cases} 1 & (m = 0) \\ 2 & (m \geq 1) \end{cases} \); \( A \) is the wave amplitude; \( J_m(\alpha r) \) is the Bessel function of the first kind; and \( k \) is the wave number.

The diffraction potential \( \phi_d(r,\theta,z) \) can be expressed as

\[
\phi_d(r,\theta,z) = -\frac{igA \cosh kz}{\omega \cosh kh} \sum_{m=0}^{\infty} \varepsilon_m i^m A_m H_m(\alpha r) \cos m\theta
\]

where \( H_m(\alpha r) \) denotes the Hankel function of the first kind. The incident potential and diffraction potential should meet the boundary conditions below
Substitute the incident potential and diffraction potential into the boundary conditions and the full velocity potential space factor of the whole wave field can be solved as

\[
\phi(r, \theta, z) = -\frac{igA \cosh kz}{\omega \cosh kh} \sum_{m=0}^{\infty} E_m i^m \left[ J_m(ka) - J'_m(ka) H_m(ka) \right] \cos m\theta
\] (19)

Using the orthogonality of the cosine function and Wornkcy identical equation, the wave force acting on the pier structure can be expressed in Eq. (20) by substituting Eq. (19) into the Bernoulli equation.

\[
P(z) = -\frac{4A \rho g}{kh} \cosh kz \cosh kh e^{ikz - i\omega t}
\] (20)

where \(x_0\) is the space position with the incident wave spreading direction on the pier’s central axis; and \(H_1^{(1)}(ka)\) is the Hankel function of the first kind and first order. The Wornkcy identical equation is

\[
J_m(z) H'_m(z) - J'_m(z) H_m(z) = \frac{2}{\pi z}
\] (21)

Considering the phase difference of incident wave motion in different horizontal location at the same time, the wave force acting on the circle pier is the real part of \(P(z)\) in Eq. (20).

2.3 Establishment of stochastic dynamic analysis method

Substitute the hydrodynamic pressure obtained using the radiation wave theory and the wave force obtained using the diffraction theory into the dynamic equation of the pier, it gives

\[
\left( M + M_z \right) \ddot{x} + C \dot{x} + K x = -\left( M + M_1 \right) \ddot{k}_g(t) + P(t)
\] (22)

where \(M, C\) and \(K\) are the mass, damping and stiffness matrices of the pier; and \(P(t)\) is the wave force. Only the wave action is considered when \(P(t)\) is zero, and only the earthquake action is considered when \(M_1\) and \(M_2\) are zero. There is no correlation between earthquake ground motions and wave heights, thus the correlation coefficient is zero. Assuming that \(Z(t) = -(M + M_1) \ddot{k}_g(t) + P(t)\), the auto-correlation function \(R_{zz}(\tau)\) of it which is used for the Fourier transform is

\[
R_{zz}(\tau) = (M + M_1) I R_s(k_\tau) I^T (M + M_1) + R_{pp}(\tau)
\] (23)

where \(R_{s(k_\tau)}(\tau)\) and \(R_{pp}(\tau)\) are the auto-correlation functions of \(\ddot{k}_g\) and \(P(t)\) respectively. The power spectrum \(S_{zz}(\omega)\) of \(Z(t)\) is

\[
S_{zz}(\omega) = (M + M_1) I S_{s(k_\omega)}(\omega) I^T (M + M_1) + S_{pp}(\omega)
\] (24)
where $S_{v_s}(\omega)$ is the earthquake acceleration spectrum; and the wave force spectrum $S_{pp}(\omega)$ can be written as

$$S_{pp}(\omega) = \left( \frac{4A \rho g}{k_g H_1(\omega)} \cosh kz \right)^2 S_{iii}(\omega)$$

(25)

where $S_{HH}(\omega)$ is the wave height spectrum.

Generally, $S_{ZZ}(\omega)$ is a positive definite or positive semi-definite symmetric matrix. Assuming that the matrix’s rank is $r_0$, it can be decomposed into the product of the $n \times r_0$ matrix $L$ and its transposition matrix $L^T$ according to the LDL$^T$ decomposition as

$$S_{ZZ}(\omega) = LL^T$$

(26)

By means of the method established by Lin JH [14], a pesudo excitation is created as

$$p = Le^{j\omega t}$$

(27)

Substitute the pesudo excitation above into the dynamic equation, the pesudo displacement response $\ddot{x}$ is

$$\ddot{x} = H(\omega)Le^{j\omega t}$$

(28)

where the frequency response function $H(\omega)$ is

$$H(\omega) = \frac{1}{-\omega^2 (M + M_z) + K + i\omega C}$$

(29)

The displacement response spectrum $S_{xx}(\omega)$, the internal force response spectrum $S_{FF}(\omega)$ and their square-variances $\sigma_s$ and $\sigma_F$ can be expressed respectively as

$$S_{xx}(\omega) = \ddot{x}\ddot{x}^T$$

(30)

$$S_{FF}(\omega) = K(\ddot{x}\ddot{x}^T)K^T$$

(31)

$$\sigma_s = \sqrt{\int_0^\infty S_{xx}(\omega)\,d\omega}$$

(32)

$$\sigma_F = \sqrt{\int_0^\infty S_{FF}(\omega)\,d\omega}$$

(33)

In order to apply stochastic dynamic results in engineering designs, the extreme value distributions, means and variances of responses can be calculated by random extreme value theory [15, 16].

### 3. Numerical example

#### 3.1 Calculation model

A circle solid pier is calculated with a sectional diameter of 5 m and a height of 24.7 m. The calculated span, height, density and elastic modulus are 51.1 m, 4.25 m, 2500 kg/m$^3$ and $3.0 \times 10^4$ Mpa, respectively. The girder of 525000 kg is considered as a lumped mass acting on the top of the pier. In the numerical model, the pier is divided into 13 elements. The height is 0.7 m for the bottom element and 2 m for the others. The finite element model adopts smeared model to consider the reinforced concrete. The fundamental frequency of the model is 14.9 rad/s.
The Clough-Penzien spectrum is used as the earthquake acceleration spectrum $S_{kg}(\omega)$ and the Bretschneider-Mitsuyasu spectrum is used as the wave height spectrum $S_{\text{H}H}(\omega)$. The expressions are given as below.

$$S_{kg}(\omega) = \frac{\omega^4 + 4\xi^2 \omega^2 \omega^2 + \omega^4}{(\omega^2 - \omega^2)^2 + 4\xi^2 \omega^2 \omega^2 (\omega^2 - \omega^2) + 4\xi^2 \omega^2 \omega^2} S_0$$  \hspace{1cm} (34)$$

$$S_{\text{H}H}(\omega) = 400.5 \left( \frac{H_s}{T_{H1/3}} \right)^2 \frac{1}{\omega^2} \exp \left( -1605 \left( \frac{1}{T_{H1/3}} \right) \right)$$  \hspace{1cm} (35)$$

For both spectrums above, $\omega_g$ presents the predominant frequency and $\zeta_g$ is the damping ratio of the first order highpass filter respectively; $\omega_f$ and $\zeta_f$ are the parameters of the second order highpass filter; $S_0$ is the constant spectral density of the input white noise process; besides, $H_s$ is the significant wave height; and $T_{H1/3}$ is the significant wave period. The frequency ranges, frequency spacings, durations and time intervals are [0, 20] rad/s, 0.05 rad/s, 4 s and 0.02 s, respectively. In order to analyze the responses results, two parameters are defined: the relative water depth $h_0=h/H$ where $H$ is the pier’s height; and the increased percentage $\Delta=(S_2-S_1)/S_1$ where $S_1$ is the power spectrum of internal forces with no water at a specific frequency $\omega$; and $S_2$ is the power spectrum of internal forces with a relative water depth $h_0$ at the same frequency $\omega$. In this paper, the nonlinear solitary wave generated by strong seismic attack is not considered.

3.2 Stochastic dynamic analysis of the pier in deep water under earthquake action

Stochastic dynamic analysis of the pier in deep water under earthquake action is calculated with six conditions. The relative water depth $h_0$ of 0.60 is chosen, and different earthquake acceleration amplitudes $a_g$ of 0.1 g, 0.2 g and 0.4 g are adopted. Zero water depth is also considered for comparision. The shear force power spectrum and moment power spectrum of the pier’s bottom at different conditions are shown in Figs.2 and 3 respectively. The figures show that the peak values of the shear force and moment power spectrums appear at the fundamental frequency of the pier. With the increase of earthquake acceleration amplitudes, the internal force power spectrums at the pier’s bottom are increasing. Fig. 4 graphically shows the increased percentage $\Delta$. As seen from the figure, the internal force power spectrums at the pier’s bottom are affected by the hydrodynamic pressure with the same increased percentage $\Delta$ at different frequencies. The red circles in Fig.4 represent the increased percentage $\Delta$ at the peak value of the internal force power spectrums. Due to the hydrodynamic pressure, the peak value of the shear force power spectrum at the pier’s bottom increases 13% ,while the peak value of the moment power spectrum at the pier’s bottom increases only 4%.

![Fig. 2 -The shear force power spectrum at the pier’s bottom with no water and $h_0=0.60$](image-url)
Fig. 3 – The moment power spectrum at the pier’s bottom with no water and $h_0=0.60$

(a) $a_g=0.1$ g
(b) $a_g=0.2$ g
(c) $a_g=0.4$ g

Fig. 4 – The increased percentages of internal force power spectrum with earthquake acceleration amplitudes

In order to consider the influence of different relative water depths to the hydrodynamic pressure, four conditions are calculated with the relative water depth of 0, 0.45, 0.60 and 0.75. 0.1 g is adopted for the earthquake acceleration amplitude. The shear force power spectrum and moment power spectrum at the bottom of the pier at different conditions are shown in Fig.5. As shown from the figure, the influence of the response of the pier by the hydrodynamic pressure is increasing when $h_0$ becomes deeper. The red circles in Fig.6 represent the increased percentage $\Delta$ at the peak value of the internal force power spectrums. Fig.6 shows that the peak values of the shear force power spectrums at the pier’s bottom increase by 4%, 15% and 31% respectively, while the peak values of the moment power spectrums at the pier’s bottom increase by only 1%, 4% and 9%, respectively. It shows that the hydrodynamic pressure increases the pier’s response under earthquake action. Moreover, the relationship between the hydrodynamic pressure and relative water depth $h_0$ is nonlinear and the influence of hydrodynamic pressure to the internal forces becomes more significant with the increase of relative water depth $h_0$.

Fig. 5 – The internal force power spectrum at the pier’s bottom with different relative water depths

(a) The shear force power spectrum
(b) The moment power spectrum
3.3 Stochastic dynamic analysis of the pier in deep water under wave action

Stochastic dynamic analysis of the pier in deep water under wave action is calculated, considering the wave height $H_0$ and relative water depth $h_0$ as two variables. When $h_0=0.60$, with $H_0=2$ m, 3 m and 4 m, stochastic dynamic responses of the pier are plotted in Fig.7. In addition, When $H_0=2$ m, with $h_0=0.45$, 0.60 and 0.75, the stochastic dynamic responses of the pier are shown in Fig.8.

Fig.7 shows that the internal force power spectrums increase as the wave becomes higher. However, the results shown in Fig.8 indicate that if the wave height $H_0$ is the same, the internal force power spectrum increase when $h_0$ tends to decrease. The reason is that the velocity potential decreases with the increase of $h_0$. Different from the responses under earthquake action, Figs.7 and 8 show that the peak values of the shear force and moment power spectrums appear at the frequency of the peak value of the wave height spectrum $S_{int}(\omega)$.

Fig. 6 – The internal force power spectrum increased percentage with different relative water depths

Fig. 7 – The internal force power spectrums at the pier’s bottom with different wave heights

Fig. 8 – The internal force power spectrums at the pier’s bottom with different water depths
3.4 Stochastic dynamic analysis of the pier in deep water under combined earthquake and wave actions

Stochastic dynamic analysis of the pier in deep water under combined earthquake and wave actions is calculated with $h_0=0.60$ and 0.1 g earthquake acceleration amplitude. The responses of the pier under earthquake action and wave action are also added for comparison. The internal force power spectrums of two conditions are provided in Fig.9.

![Shear force power spectrum](image1)
![Moment power spectrum](image2)

Fig. 9 – Internal force spectrums at the pier’s bottom

From Fig.9, there are two peak values of the internal force power spectrums. It illustrates that the internal force power spectrums under combined earthquake and wave actions are nearly identical with these under the direct superposition of two individual actions. As it is shown in Eq. (22), the difference between combined actions and direct superposition is that the former considers the attached mass $M_2$ under wave action. In this example, because the pier has high stiffness, the difference between the frequency response function $H(\omega)$ with the mass matrix $M$ and $M+M_2$ at every frequency is negligible. Thus, the effect of the attached mass $M_2$ on the frequency response function $H(\omega)$ can be ignored and the responses of two conditions are nearly identical.

4. Conclusions

1. The stochastic dynamic response of piers in deep water is increasing because of the influence of the hydrodynamic pressure. For a pier with a fixed water depth under different earthquake acceleration amplitudes, the increased percentage of internal force power spectrums at the bottom of piers is identical, considering the effect of the hydrodynamic pressure.

2. With relative water depth becoming deeper, the effect of the hydrodynamic pressure to the internal force power spectrums at the bottom of the pier becomes more significant. The effect of the hydrodynamic pressure under earthquake action should be considered.

3. Because the attached mass generated by the pier’s elastic motion has little effect on the frequency response function of the pier, stochastic dynamic responses under combined earthquake and wave actions are identical with these under the direct superposition of two individual actions.

5. References


