16th World Conference on Earthquake Engineering, 16WCEE 2017
Santiago Chile, January 9th to 13th 2017
Paper N° 468
Registration Code: S-M1462778256

# COLLAPSE ASSESSMENT OF HIGH-RISE RC STRUCTURES UNDER STOCHASTIC EARTHQUAKE EXCITATIONS

J. Li(1), H. Zhou(2)

(1) *Distinguished Professor, School of Civil Engineering, Tongji University, lijie@tongji.edu.cn*
(2) *Ph.D. Candidate, School of Civil Engineering, Tongji University, zhouhao76@126.com*

## Abstract

Reinforced concrete (RC) structures induced by extreme earthquakes will experience a strong nonlinear state which may cause partial or overall collapse. Meanwhile, the corresponding nonlinear process might be conspicuously influenced by the randomness stemmed from the ground motions. Although rapid progresses have been made in the numerical simulation techniques during the past decades, the addressing of randomness propagation in the collapse analyses up till now remains challenging. To provide knowledge beyond the conventional engineering insights, attention is focused on a comprehensive approach towards the stochastic seismic collapse analysis and the reliability assessment for large complex RC structures in this paper. Of which three key notions are emphasized: the refined finite element modeling and analysis aspects, a physical random ground motion model, and an energy-based structural collapse criterion. The softening of concrete material which substantially contributes to the collapses of the RC structures is modeled by a concrete stochastic damage constitutive model with a physical damage evolution law. And the dynamic equations of motion in the finite element analysis are solved by an explicit integration algorithm. The physical random ground motion model is introduced to describe the stochastic properties of the earthquake ground motions. Then the collapse-resistance performance of a certain RC structure can be quantitatively evaluated based on the probability density evolution method (PDEM) combining with the proposed energy-based collapse criterion. Numerical results in terms of a prototype RC frame-shear wall structure indicate that the randomness from ground motions dramatically affects the collapse behaviors of the structure, and even leads to entirely different collapse modes. Thus, it is of great significance to investigate the coupling effect of the damage evolution and the randomness propagation in collapse analysis and assessment of structures. The above integrated methodology gives rise to a new perspective to gain better understanding of the anti-collapse design and collapse prediction of large complex RC buildings.

*Keywords: collapse simulation; random ground motion; collapse criterion; randomness propagation; reliability assessment*

# 1. Introduction

Civil engineering structures subjected to strong earthquakes will inevitably undergo damage, destruction and even collapse. Due to the randomness rooted from seismic actions some structures may not survive even under the excitation of earthquakes lower than the intended level. Actually, many of the observed collapses have been the result of deficiencies in our knowledge of the regional seismic hazard and the structural behaviors under dynamic loadings. Structural collapse is a complicated process on account of the damage growth within various scales and stochastic evolutions. Up till now, it is still hard to quantitatively evaluate the reliability of structures that to be or have been built under diverse performance targets. Though the collapse modes of a structure under seismic excitations could be predicted via numerical analysis currently [1, 2], there isn't yet an effective criterion to tell when the structure will collapse under seismic actions.

To determinate such a structural critical ultimate state, over the past few decades, several methods have been developed from various visions, e.g., the ultimate deformation, stiffness reduction, energy supply-demand, deformation with energy, and dynamic instability, etc. However, considering the redistribution and the variation of each individual component damage within the system level, it is not easy to objectively assess the structural integral resistance performance. Since energy parameters at structural level are aggregated quantities, a number of researchers have shifted their attention to energy as the key to understanding the dynamics of structural collapse in recent years [3, 4, 5, 6]. For instance, the incidence of gravity energy exceeding dynamic energy with a sudden increase can be considered as an indicator of structural collapse under seismic actions [5]. From the viewpoint of structural intrinsic energy, which relates to the energy-dissipating capacity of a structure, a novel criterion regarding the first occurrence of structural intrinsic energy exceeding the total external input energy for dynamic instability of hardening structural systems has also been suggested [6]. Nevertheless, it is the strength destruction that firstly leads to dynamic instability and then to collapse for deteriorating structural systems which much distinguishes from the collapse directly caused by dynamic instability of the hardening structural systems. Thus the above energy-based methods cannot be used to predict the collapse of a deteriorating structural system.

The present research is focused on a new energy-based criterion for collapse identification of deteriorating structural systems. From the perspective of energy changing during the full-range evolution of the structural nonlinearities when suffering a strong earthquake, the criterion takes into consideration of both the structural properties and the nature of external excitations. Starting from the entire process description of the system's dynamic stability, the new criterion can predict the specific occurrence time of structural collapse. With the criterion, a stochastic seismic collapse analysis framework towards the assessment of reliability against collapse for large complex RC structures can be established if integrated with the refined finite element analysis and the probability density evolution method (PDEM). To illustrate the overall procedure of the developed methodology, the stochastic collapse analysis and reliability assessment on a prototype high-rise building structure is carried out. Results show that the randomness of seismic excitations affects notably on the behaviours of RC structures.

# 2. Stochastic Seismic Collapse Analysis of RC Structures

## 2.1 Stochastic damage constitutive model of concrete

For concrete under uniaxial loading condition, a representative volume element (RVE) can be idealized as a series of micro-elements jointed in parallel [7]. The individual element represents the micro properties of the material and the bundle describes the response of the RVE. The stress-strain relationship of each micro-element is considered as the perfect elasto-brittle type with random fracture strain $\Delta_i$. By introducing the random distribution of fracture strain of micro-elements and utilizing the stochastic integral strategy, the damage variable for the parallel element model can be defined as

$$D(\varepsilon^e) = \int_0^1 H[\varepsilon^e - \Delta(x)]\mathrm{d}x \tag{1}$$

where $\Delta(x)$ is the one-dimensional micro-fracture strain random field; $x$ denotes the spatial coordinate of the micro-element; $H(\cdot)$ is the Heaviside function.

Suppose that $\Delta(x)$ is a homogenous random field and follows the lognormal distribution. It can be proved that the expectation of the stochastic damage evolution law, Eq. (1), happens to be the cumulative distribution function (CDF) of $\Delta(x)$ which can be given by

$$F(\varepsilon^e, \varepsilon^e; \eta) = \varPhi\left[\frac{\ln \varepsilon^e - \lambda}{\zeta}, \frac{\ln \varepsilon^e - \lambda}{\zeta} | \rho_Z\right] \tag{2}$$

where $\varPhi(\cdot)$ is the CDF of the standard normal distribution; $\zeta = |x_1 - x_2|$; $\lambda$ and $\zeta$ are the mean and the standard deviation parameters of the lognormal distribution; $\rho_Z(\eta) = \exp(-\omega \cdot \eta)$ is the auto-correlation coefficient function for $Z(x) = \ln \Delta(x)$, in which, $\omega$ is a correlation parameter.

Within the framework of continuum damage mechanics [8], the concept of the energy equivalent strain is proposed to bridge the gap between uniaxial and multiaxial damage constitutive models [7]. Based on the damage energy release rates and the damage consistent condition, the energy equivalent strain is expressed as

$$\varepsilon_{eq}^{e+} = \sqrt{\frac{2Y^+}{E_0}} \;\; ; \;\; \varepsilon_{eq}^{e-} = \frac{1}{(1-\alpha)E_0}\sqrt{\frac{Y^-}{b_0}} \tag{3}$$

where $E_0$ is the Young's modulus of the initial undamaged material and $b_0$ is a material parameter; $\alpha$ is the biaxial strength increase factor; $Y^+, Y^-$ are the tensile and shear damage energy release rates [7, 8].

Substituting Eq. (3) into Eq. (1), the multi-dimensional damage evolution functions can be established as

$$D^{\pm}(\varepsilon_{eq}^{e\pm}) = \int_0^1 H[\varepsilon_{eq}^{e\pm} - \Delta^{\pm}(x)]\mathrm{d}x \tag{4}$$

Then the nonlinear constitutive relationship of concrete can be described as [9]

$$\boldsymbol{\sigma} = (\mathbb{I} - \mathbb{D}) : \bar{\boldsymbol{\sigma}} = (\mathbb{I} - \mathbb{D}) : \mathbb{E}_0 : (\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}^p) \tag{5}$$

where $\bar{\boldsymbol{\sigma}}$ denotes the effective stress tensor; $\mathbb{E}_0$ denotes the initial undamaged elastic stiffness; $\mathbb{I}$ is a fourth order unit tensor and $\mathbb{D}$ is a fourth order damage tensor. The evolution of plastic strain can be calculated by [10]

$$\dot{\boldsymbol{\varepsilon}}^{p\pm} = f_p^{\pm}\dot{\boldsymbol{\varepsilon}}^{e\pm} = H[\dot{D}^{\pm}]\xi_p^{\pm}(D^{\pm})^{n_p^{\pm}}\dot{\boldsymbol{\varepsilon}}^{e\pm} \tag{6}$$

where scalars $f_p^{\pm}$ are considered as the function of the corresponding damage variable $D^{\pm}$; $\xi_p^{\pm}$ and $n_p^{\pm}$ are the material parameters fitted by the experimental data.

To obtain the stress-strain curves in the sense of random samples, we need to model the random field of microscopic fracture strain by numerical strategy. As one of the most effective ways, the stochastic harmonic function (SHF) method [11] can be used to generate the random field of the fracture strain $\Delta(x)$ as below

$$\Delta(x) = \sum_{i=1}^{N} A(\omega_i)\cos(\omega_i x + \phi_i) \tag{7}$$

where the $\omega_i$'s and $\varphi_i$'s are the random frequencies and random phase angles, respectively. The amplitudes $A(\omega_i)$ are functions of $\omega_i$, and $N$ is the number of the components.

## 2.2 Modeling of random ground motions

Combining the Fourier transfer form with the special source, path and site models, the ground motions can be modeled via the physical random function model as follows [12]

$$a_R(t) = -\frac{1}{2\pi}\int_{-\infty}^{+\infty} \omega^2 A_R(\boldsymbol{\Theta}_\alpha, \omega) \times \cos[\omega t + \Phi_R(\boldsymbol{\Theta}_\alpha, \omega)]\mathrm{d}\omega \tag{8}$$

where, the amplitude spectrum $A_R(\boldsymbol{\Theta}_\alpha, \omega)$ and the phase spectrum $\Phi_R(\boldsymbol{\Theta}_\alpha, \omega)$ respectively reads

$$A_R(\boldsymbol{\Theta}_\alpha,\omega)=\frac{A_0\omega e^{-K\omega R}}{\sqrt{\omega^2+(1/\tau)^2}}\times\sqrt{\frac{1+4\xi_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2+4\xi_g^2(\omega/\omega_g)^2}} \tag{9}$$

$$\Phi_R(\boldsymbol{\Theta}_\alpha,\omega)=\arctan\left(\frac{1}{\tau\omega}\right)-R\cdot d\cdot\ln\left[(a+0.5)\omega+b+\frac{1}{4c}\sin(2c\omega)\right] \tag{10}$$

where, $R$ is the distance between the seismic source and the local site, which is a deterministic variable for a given engineering site; $K$ is a parameter measuring the friction attenuation and usually reads the value of $10^{-5}$ s/km; $A_0$ is a random variable reflecting the source amplitude intensity; $\tau$ is a attribute parameter for the source model of the Brune's dislocation model and is considered as a random variable; $a$, $b$, $c$ and $d$ are empirical parameters that are determined by the realistic wave number-frequency relationship; $\xi_g$ is the equivalent damping ratio and $\omega_g$ is the equivalent predominant circular frequency, and both of them are considered as random variables; $\boldsymbol{\Theta}_\alpha=(A_0,\tau,\xi_g,\omega_g)$, is the basic random variable set of the above random function model.

This model can be applied to simulate the realistic earthquake ground motions and to synthesize the non-stationary ground motion samples using the superposition method of narrow-band harmonic wave groups [12]. Also it is of great significance to the stochastic seismic analysis and the reliability evaluation of engineering structures. Some representative simulated samples of acceleration time histories of earthquake ground motions are plotted in Fig. 1.

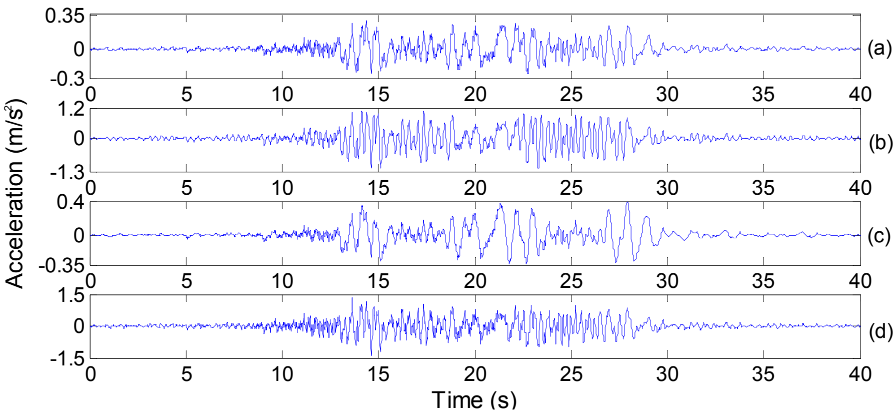



Fig. 1 – Typical simulated time histories of ground motion accelerations

## 2.3 Stochastic seismic response analysis

For a general MDOF nonlinear dynamical system considering the randomness from both the structural properties and the external excitations, the equtions of motion can be uniformly expressed as [13]

$$\mathbf{M}(\boldsymbol{\Theta})\ddot{\mathbf{u}}(t)+\mathbf{C}(\boldsymbol{\Theta})\dot{\mathbf{u}}(t)+\mathbf{f}(\boldsymbol{\Theta},\mathbf{u}(t))=\mathbf{F}(\boldsymbol{\Theta},t) \tag{11}$$

where $\mathbf{M}$, $\mathbf{C}$ are the mass and damping matrices, respectively; $\mathbf{f}(\mathbf{u},t),\mathbf{F}(t)$ are the restoring force and external dynamic excitation vector, respectively; $\ddot{\mathbf{u}}(t),\dot{\mathbf{u}}(t),\mathbf{u}(t)$ are the vector of nodal accelerations, velocities and displacements, respectively; $\boldsymbol{\Theta}=(\Theta_1, \Theta_2, \ldots, \Theta_s)$ are all the random parameters involved in the system, and $s$ is the total number of basic random variables.

Suppose that $\mathbf{Z}=(Z_1, Z_2, \ldots, Z_m)^T$ denotes all the physical quantities interested in the system, then considering Eq. (11) the augmented system ($\mathbf{Z}$, $\boldsymbol{\Theta}$) is probability preserved because all the random factors are involved. According to the random event description of the principle of preservation of probability, the joint PDF of the augmented state vector ($\mathbf{Z}$, $\boldsymbol{\Theta}$) satisfies the governing partial differential equation [14]

$$\frac{\partial p_{Z\Theta}(z,\boldsymbol{\theta},t)}{\partial t}+\dot{Z}(\boldsymbol{\theta},t)\frac{\partial p_{Z\Theta}(z,\boldsymbol{\theta},t)}{\partial z}=0 \tag{12}$$

where $\dot{Z}(\boldsymbol{\theta},t)$ is the velocity of the response for a prescribed $\boldsymbol{\theta}$.

It is easy to specify the initial condition of Eq. (12), i.e.

$$p_{Z\Theta}(z,\boldsymbol{\theta},t)\big|_{t=0} = \delta(z-z_0)p_{\Theta}(\boldsymbol{\theta}) \tag{13}$$

where $z_0$ is the deterministic initial value.

Eq. (12) is referred to as a generalized density evolution equation (GDEE) which reveals the intrinsic connections between a stochastic dynamical system and its deterministic counterpart. Using the GDEE and the physical equation (11), the PDF of an interested physical quantity can be obtained once its velocity is known since varying of the PDF results from varying of the state of the quantity.

Actually, solving the initial-value problem of Eqs. (12) and (13) with Eq. (11), the instantaneous PDF of Z($t$) can be obtained by

$$p_Z(z,t) = \int_{\Omega_\Theta} p_{Z\Theta}(z,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \tag{14}$$

where $\Omega_{\boldsymbol{\Theta}}$ is the distribution domain of $\boldsymbol{\Theta}$.

### 2.3 Numerical implementation for stochastic collapses of structures

As is known, the framework from constitutive model to structural analysis constitutes the foundation of modern finite element formulation. With the finite element method the whole structural responses can be easily obtained. During the process of structural collapse, the whole structure changes from a continuum system into discrete parts due to structural fracturing and element crushing. This process can be simulated by elemental deactivation technique, where the failed elements are deactivated when a specified elemental-failure criterion is reached [1]. Specifically, the failure of concrete and steel are respectively detected in terms of material damage threshold and ultimate strain. To possibly avoid the issues on stability and convergence, the explicit central difference method [15] is adopted herein to solve the dynamical equations. Combining the explicit integral algorithm with the element removal strategy, the process of structural collapse could be simulated with ease. For a given structure, if subjected to the random ground motions, then the stochastic collapse responses can be investigated. Additional finite difference method with the total variation diminishing (TVD) scheme [14] is employed to obtain the probability density evolution information of structural responses as regards the solving of Eqs. (11)-(13).

## 3. Energy-Based Structural Collapse Criterion

In this study, the concept of energy balance is utilized to represent the severe structural damage history that eventually leads to structural collapses. On that basis, a new dynamic-instability based collapse criterion is developed in accordance with the energetic criterion for structural dynamic instability [6]. Consequently, the structural reliability against collapse can be assessed integrating with the stochastic seismic response analysis.

### 3.1 Energy flow during structural collapse

Given a deterministic MDOF nonlinear dynamic system, the equations of motion Eq. (11) can be revised as

$$\mathbf{M}\ddot{\mathbf{u}}(t) + \mathbf{C}\dot{\mathbf{u}}(t) + \mathbf{f}(\mathbf{u},t) = \mathbf{F}(t) \tag{15}$$

Integrate Eq. (15) with respect to the differential displacements, we have the energy balance equation [16]

$$E_\mathrm{k} + E_\mathrm{d} + E_\mathrm{e} + E_\mathrm{p} = E_\mathrm{input} \tag{16}$$

where $E_\mathrm{k}$, $E_\mathrm{d}$, $E_\mathrm{e}$, $E_\mathrm{p}$ and $E_\mathrm{input}$ are the kinetic energy, dissipated damping energy, elastic strain energy, accumulated hysteretic energy and total external input energy of the system, respectively.

For a conventional structure subject to dynamic loads, most of the external input energy is finally absorbed or dissipated by irreversible deformation energy and damping energy. While the system's kinetic energy due to structural vibration is commonly dissipated within structural members by the transformation into their deformation energy in the end. During a collapse, however, the system's kinetic energy results not merely from the structural vibration but also from the rigid motions of fractured fragments. And simultaneously the work done by gravity associated with rigid motions is incorporated in the total external input energy.

In the early stages of an earthquake excitation, as the global structure is basically in the elastic state or only some very localized areas are damaged, both the plastic energy and the damping energy are of quite low levels. Before collapse, the amount of kinetic energy and elastic strain energy also maintains at relatively low level. While once collapse begins, the kinetic energy as well as external work will increase dramatically due to rigid motions. And that's why some researchers have chosen the system's kinetic energy as a main indicator of structural collapse [4]. During a collapse propagation event, the energy balance equation (16) is constant. And this kind of equilibrium will become unstable once collapse due to dynamic instability occurs.

### 3.2 Criterion for dynamic stability of hardening structures

In structural dynamics, except for the structural kinetic energy and dissipative energy by damping, the nonlinear restoring force will dissipate or absorb a considerably large amount of energy when the structure vibrates. If the energy imported into the structure is equal to the sum of the above energy, the structure can keep the dynamic equilibrium states and is dynamically stable. Otherwise, if the input energy is less than the energy needed for dissipation, some other energy-absorbing processes must exist in the structural dynamic system to compensate the difference. Based on the above idea, an energetic criterion to identify the structural dynamic instability has been developed recently [6].

Define the system's intrinsic energy at any time $t$ as

$$E_{\mathrm{intr}}(t) = \left| \mathbf{f}^{\mathrm{T}}(\mathbf{u},t)\mathbf{u}(t) - \int_0^{\mathbf{u}} \mathbf{f}^{\mathrm{T}}(\mathbf{u},t)\mathrm{d}\mathbf{u} \right| \tag{17}$$

Here, the intrinsic energy is a certain kind of energetic index associated with the consumed energy by structural vibration. Then the structural dynamic stability can be identified by

$$\begin{aligned} &E_{\mathrm{intr}}(t) \le E_{\mathrm{input}}(t) \quad \text{Dynamic Stability} \\ &E_{\mathrm{intr}}(t) > E_{\mathrm{input}}(t) \quad \text{Dynamic Instability} \end{aligned} \tag{18}$$

Compared with the conventional concept of dynamic stability, the above criterion has obvious advantages in addressing the issues of dynamic stability for the general elastic, or elastic-plastic non-degenerate structural systems. Specifically, the exact instant of dynamic instability of a hardening structure can be predicted via the proposed criterion. Further, both the accuracy and the efficiency of identification are notably enhanced.

However, the criterion cannot be used to predict the collapse critical state of a deteriorating structural system which usually distinguishes from the instability critical state of a hardening structural system. In general, at critical phases of instability, there haven't yet been significant plastic deformations, extensive fractures of components, or rigid body motions involved in a hardening structure. Also the damping dissipation energy is of very limited amount. Nevertheless, the strength softening and stiffness degradation of structural components commonly go ahead of partial buckling or global instability resulting in the collapse of a deteriorating structure. When the collapse critical state is reached, massive permanent deformation, and even rigid motions of some destruction areas have occurred. On this occasion, the proportion of damping and plastic energy dissipation will increase remarkably as a result of the sharp growth of structural energy-dissipating capacity. In view of this, the structural collapse criterion should be established with a careful consideration of the above differences.

### 3.3 Energy-based criterion for structural collapse

With regard to the above discussions, the notions of the structural intrinsic energy and the external work in Eq. (18) are renovated to extend the stability criterion of a hardening system to a collapse criterion for a generalized deteriorating system. All that is needed is to subtract the cumulative plastic deformation energy from both sides of Eq. (18), and simultaneously to subtract the accumulated damping energy from the right side considering its irrecoverability. Then we have the definition of "effective intrinsic energy" on the left side, and the definition of "effective instantaneous external work" on the right side. Thus, both sides of Eq. (18) become quantities of state. What's more, the differences between instability of hardening structures and collapse of deteriorating structures are subtly taken into account in the new criterion. In the above process, it seems that the dynamic stability of a elastoplastic deteriorating structural system is equivalent to that of the "remaining" elastic part of the structure. As it turns out, this idea indeed works in identifying the collapse of deteriorating structural systems.

The above methodology also stems from an important fact that both the plastic deformation energy and the damping dissipation energy are irrecoverable. At any time *t*, the energy effectively imported to the structure does not include the plastic deformation energy and the damping energy that have been dissipated before this moment's gone. In other words, in conformity to the principle of energy balance, only the system's elastic strain energy and kinetic energy could be freely released or transformed from one moment to the next.

Accordingly, referring to the definition of "intrinsic energy" in the identification criterion for dynamic instability [6], redefine the following structural effective intrinsic energy

$$E_{\text{eff\_intr}}(t) = \left| \mathbf{f}^{\mathrm{T}}(\mathbf{u},t)\mathbf{u}(t) - \int_V \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}_{\mathrm{e}}\, \mathrm{d}V \right| \tag{19}$$

where $\boldsymbol{\sigma}$ is the stress tensor; $\dot{\boldsymbol{\varepsilon}}_{\mathrm{e}}$ is the elastic strain rate tensor; and $V$ denotes the solution domain. In contrast to the definition of intrinsic energy in Eq. (17), subtracting the plastic dissipation energy from that intrinsic energy exactly constitutes the effective intrinsic energy herein.

Considering that both the damping dissipation energy $E_{\mathrm{d}}$ and the plastic deformation energy $E_{\mathrm{p}}$ are irrecoverable, the effective external work to the system at any time *t* yields

$$\Omega(t) = E_{\text{input}}(t) - E_{\mathrm{d}}(t) - E_{\mathrm{p}}(t) = \int_0^t \mathbf{F}^{\mathrm{T}}(t)\mathrm{d}\mathbf{u}(t) - \int_0^t \dot{\mathbf{u}}^{\mathrm{T}}(t)\mathbf{C}\dot{\mathbf{u}}(t)\mathrm{d}t - \int_0^t \left(\int_V \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}_{\mathrm{p}}\, \mathrm{d}V\right)\mathrm{d}t \tag{20}$$

where $\dot{\boldsymbol{\varepsilon}}_{\mathrm{p}}$ is the plastic strain rate tensor. By substituting Eq. (16) into Eq. (20), $\Omega(t)$ can be rewritten as

$$\Omega(t) = E_{\mathrm{k}}(t) + E_{\mathrm{e}}(t) = \frac{1}{2}\dot{\mathbf{u}}^{\mathrm{T}}(t)\mathbf{M}\dot{\mathbf{u}}(t) + \int_V \boldsymbol{\sigma} : \dot{\boldsymbol{\varepsilon}}_{\mathrm{e}}\, \mathrm{d}V \tag{21}$$

The seismic collapse process of an arbitrary large complex structure is substantially a process of dynamic instability. Hence, a new seismic collapse criterion for general deteriorating systems is put forward as below

$$\begin{aligned} &E_{\text{eff\_intr}}(t) \le \Omega(t) \qquad \text{Dynamic Stability} \\ &E_{\text{eff\_intr}}(t) > \Omega(t) \qquad \text{Dynamic Instability (Collapse)} \end{aligned} \tag{22}$$

With the collapse criterion, the dynamic stability state for a given structure under arbitrary seismic actions can be quantitatively evaluated. Specifically, if the structural effective intrinsic energy ("intrinsic energy" for short) is less than the effective input energy ("input energy" for short), the structure will keep a stable dynamic equilibrium state. Otherwise, the structure will go in an unstable equilibrium state seeking another way to dissipate energy and collapse. Meanwhile, the moment that the structural intrinsic energy exceeds the effective external work for the first time happens to be the occurrence time of collapse. Without prejudice to the principle of energy balance, the proposed criterion is just a critical condition that the system vibrates from a stable equilibrium state to an unstable one. Starting from the entire process description of the system's dynamic stability, the new criterion can tell when a structure will collapse if subjected to a certain earthquake excitation.

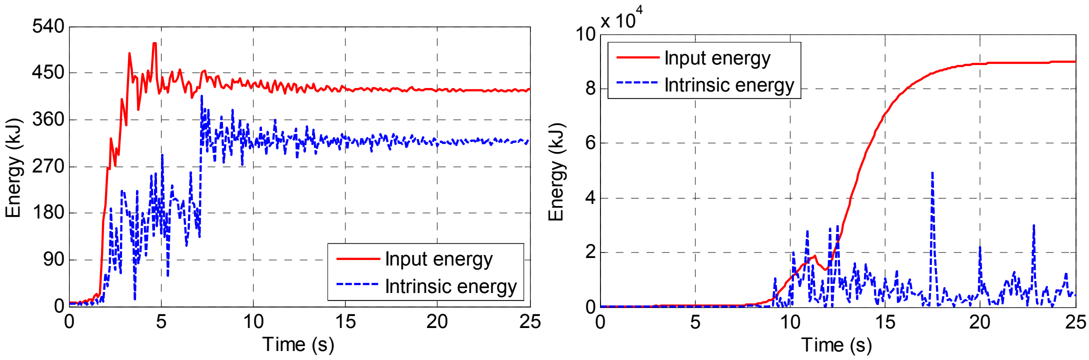
### 3.4 Example verification of the collapse criterion

To verify the proposed collapse criterion, the collapse analysis with respect to a RC frame-shear wall structure is carried out in the following. The structural configuration, material properties, and further information about test details refer to the literature [17]. In the structural finite element model by ABAQUS, the fiber beam-column element is adopted to model the beam and column members. And the multi-layer shell element is adopted to model the floor slab, roof, wing wall and shear wall components. Totally 19896 nodes and 26400 elements are developed. The stochastic damage constitutive model [7, 9] is introduced to model the concrete. And the elastic-plastic with hardening and progressive damage constitutive model [18] is employed to model the reinforcement bars. The ground motion recorded at the KJMA station during the 1995-01-17 Kobe earthquake is selected as the input base motion. All three components are applied simultaneously. The directions of seismic inputs are the same with the original test configurations [19]. The overall process of structural collapse is simulated using the explicit integration algorithm [15] with the aforementioned energy values computed during the analysis.

The collapse simulations are classified into two cases with different input peak ground motions, as listed in Table 1. Fig. 2 depicts the time histories of effective input energy and structural intrinsic energy. It is found that the intrinsic energy does not exceed the input energy under the entire process of seismic excitation in case 1. Whereas the intrinsic energy exceeds the input energy at about 9.2 s for the first time in case 2.

Table 1 – Peak ground accelerations input to the frame-wall structure

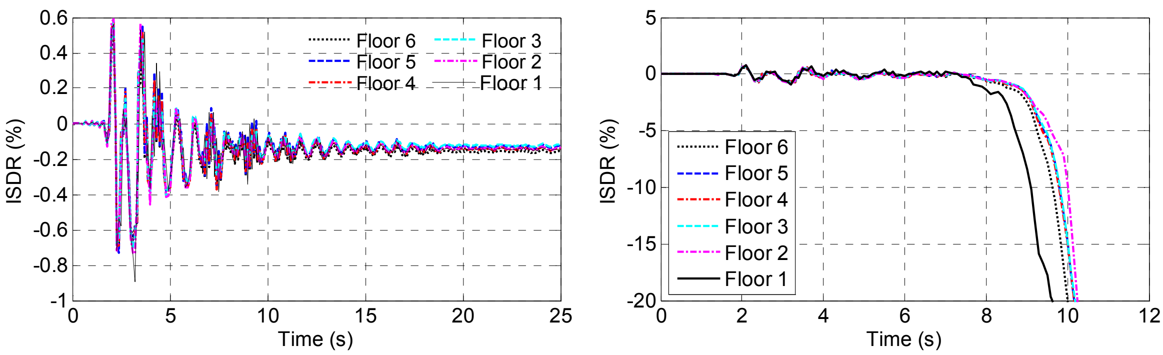
| Case NO. | Amplification Factor | Peak Ground Acceleration (m/s$^2$) | | |
|---|---|---|---|---|
| | | Local X-axis | Local Y-axis | Local Z-axis |
| Original Wave | 1.00 | 6.17 (0.63 g) | 8.13 (0.83 g) | 3.33 (0.34 g) |
| Case 1 | 0.96 | 5.88 (0.60 g) | 7.84 (0.80 g) | 2.94 (0.30 g) |
| Case 2 | 1.08 | 6.69 (0.68 g) | 8.82 (0.90 g) | 3.61 (0.37 g) |




(a) Case 1: dynamic stability (b) Case 2: dynamic instability

Fig. 2 – Comparison of effective input energy and structural intrinsic energy

The inter-story drift ratios (ISDRs) changing over time of each floor of the structure with two cases are plotted in Fig. 3. It is seen that the structure indeed does not collapse in case 1, while in case 2 the structure collapses due to dynamic instability induced by the material softening and structural deterioration. As shown in Fig. 3(b), the critical instant of collapse happening is around 9.2 s, which shows good conformity with the results identified by the proposed criterion in Fig. 2(b).




(a) Case 1 (b) Case 2

Fig. 3 – The inter-story drift ratio time histories of the structure

The above results indicate that the structure could maintain its stable dynamic equilibrium state as long as the intrinsic energy is always less that the valid external work during a seismic event. On the contrary, the structure could not survive its stability and collapses once the intrinsic energy exceeds the effective input energy. And the time of such first passage exactly indicates the onset of collapse induced by dynamic instability. The proposed criterion can predict the collapse of deteriorating structures with fair accuracy and efficiency.

### 3.5 Reliability against collapse

Based on the theory of dynamic reliability for the first passage problems [13], an absorbing boundary condition associated with the structural collapse criterion (22) should be imposed on Eq. (12), i.e.

$$p_{Z\Theta}(z,\boldsymbol{\theta},t)=0,\quad z\in\Omega_{\mathrm{f}} \tag{23}$$

where $\Omega_{\mathrm{f}}$ is the failure domain; suppose that $\Omega$ is the response space of $Z$ and $\Omega_s$ is the safe domain, then $\Omega_{\mathrm{f}}\cap\Omega_{\mathrm{s}}=\varnothing$, $\Omega_{\mathrm{f}}\cup\Omega_{\mathrm{s}}=\Omega$.

Solving the simultaneous Eqs. (12), (13) and (23), we can obtain the "remaining" joint PDF $\breve{p}_{Z\Theta}(z,\boldsymbol{\theta},t)$. Then the "remaining" PDF reads

$$\breve{p}_Z(z,t)=\int_{\Omega_\Theta}\breve{p}_{Z\Theta}(z,\boldsymbol{\theta},t)\mathrm{d}\boldsymbol{\theta} \tag{24}$$

and the structural reliability against collapse yields

$$R(t)=\int_{\Omega_{\mathrm{s}}}\breve{p}_Z(z,t)\mathrm{d}z \tag{25}$$

## 4. Numerical Example

To illustrate the above integrated methodology for the stochastic seismic collapses as well as the reliability assessments, the stochastic collapse analyses on a prototype building structure is carried out. The deterministic numerical simulations are implemented by the finite element package ABAQUS, in which the constitutive models for concrete and steel materials are embedded with VUMAT and Python script.

### 4.1 Description of the model structure

The Pudong Broadcast & TV Center (BTC for short) is an 18-story large high-rise building located in Pudong New Area, Shanghai. The lateral-force-resisting system of BTC is a RC frame-shear wall structure. There is a shear wall core tube with an elevator shaft located close to either side of the structure, respectively. Structural configuration, material properties, and other details see [20]. In the finite element model of the BTC structure, the fiber beam-column element and the multi-layer shell element are respectively adopted to model the frame members and the floor slab and shear wall members. Totally 53372 elements are developed.

### 4.2 Random seismic input

By employing the physical random function model [12], the random ground motions can be generated for the seismic input. The distribution types and their statistical parameters associated with the basic random variables therein are given in Table 2.

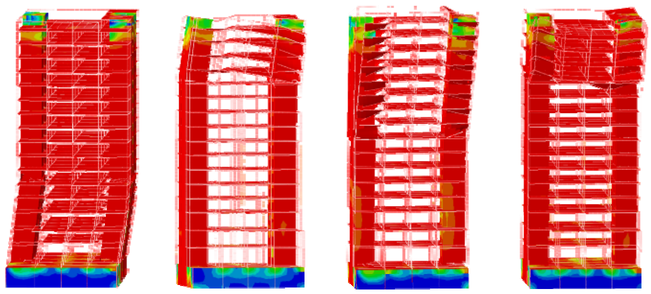
Table 2 – Model parameters of random ground motions

| Variable | Distribution Type | Probability Distribution Function (PDF) | Parameters of PDF | |
|---|---|---|---|---|
| $A_0$ | Lognormal | $\frac{1}{x\sigma\sqrt{2\pi}}e^{-\frac{(\ln x-\mu)^2}{2\sigma^2}},x\in(0,\infty)$ | $\mu=-1.1047$ | $\sigma=0.7388$ |
| $\tau$ | Lognormal | | $\mu=-1.1574$ | $\sigma=1.1341$ |
| $\xi_g$ | Gamma | $\frac{1}{\Gamma(k)\theta^k}x^{k-1}e^{-\frac{x}{\theta}},x\in(0,\infty)$ | $k=6.1838$ | $1/\theta=0.0689$ |
| $\omega_g$ | Gamma | | $k=2.0866$ | $1/\theta=5.6598$ |

Accordingly, 200 representative points in the sample space are intelligently selected during the stochastic seismic collapse analysis of the BTC structure and the corresponding assigned probabilities are computed in accordance with the number theoretical method based algorithm [21]. To investigate the structural collapse potential, all 200 ground notion samples are scaled with a mean value of 1.0 g (one unit of gravity) of their peak accelerations, while without changing the assigned probabilities.

### 4.3 Stochastic seismic collapses and the reliability against collapse

After a series of deterministic seismic collapse analyses of the BTC structure, various concerned physical quantities such as stress, strain, damage, reaction forces, displacements, velocities and accelerations, etc., can be obtained. Accordingly we can get the probability density information and further the reliability of the structure via the PDEM. Four typical structural collapse modes are depicted in Fig. 4. As is shown, the locations and occurrence time of initial destructions, and the subsequent damage evolutions of structures, i.e., the structural collapse modes, are quite variant induced by different earthquake ground motions even at the same site. This phenomenon lies in an important fact that the randomness could transfer from the excitation to the structure and affect the damage evolution in both material level and structural level. Hence the influence of random ground motions on the overall performance of RC structures should be carefully considered. Besides, what's noteworthy is that the essence of structural collapses results from the local or global instability, which leads to continuous and rapid developments of structural damage evolution.



t=17.0 s t=16.0 s t=11.0 s t=19.0 s

(a) Sample 1 (b) Sample 2 (c) Sample 3 (d) Sample 4

Fig. 4 – Typical collapse modes of BTC structure under different ground motions

Since the inter-story drift ratio (ISDR) is an important index in structural aseismic design specifications, here the ISDR of 16th floor of the BTC structure is selected as an concerned physical response quantity. Then using the PDEM, the probability density evolution information of the response variable can be easily obtained. Fig. 5 pictures the PDF surface varying with time and the corresponding contour to it is shown in Fig. 6, which seem like a mountain and a river stretching to the distance, respectively. Visibly, the evolution of the PDFs involves plentiful information valuable to capture the fluctuation, reliability and global performance of the structure. These results indicate that the structural response process is a complex stochastic damage evolution process and should be investigated from the development process of the nonlinearity.

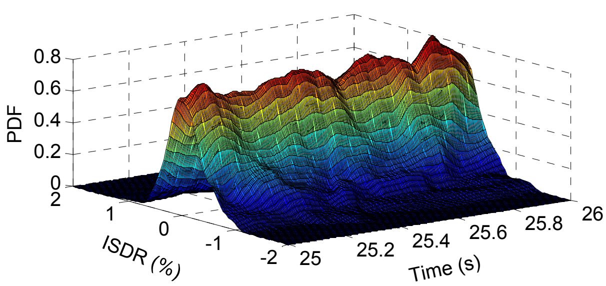



Fig. 5 – The PDF surface evolving against time

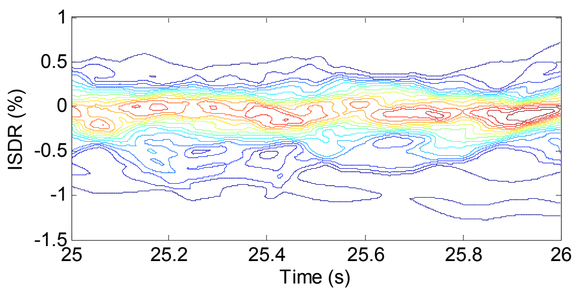



Fig. 6 – The contour to the PDF surface

Combining with the proposed structural collapse criterion in Section 3.3, the reliability against collapse of the BTC structure is evaluated utilizing the PDEM. The final anti-collapse reliability of BTC structure excited by a set of ground motion samples with an average acceleration amplitude of 1.0 g is 72.1%, i.e., the likelihood of collapse occurrence of the BTC structure is 27.9%.

## 5. Conclusions

This paper presents a new energy-based criterion for collapse identifications of deteriorating structures induced by earthquakes. Upon which a stochastic seismic analysis framework towards the reliability against collapse is recommended for the large complex RC structures subjected to random strong ground motions. The parametric uncertainties of seismic inputs are quantified based on the physical random function model with a superposition method of narrow-band harmonic wave groups. While the propagation of randomness from excitations to structures are well addressed through the probability density evolution method (PDEM). The structural collapse probability can be evaluated integrating with the proposed collapse criterion, which can be particularly useful for quantitative collapse detection during stochastic collapse analyses. Results of an illustrative example indicate that the randomness rooted from the ground motion records affect remarkably on the seismic responses of RC structures. The present paper provides a new perspective regarding the assessment of integrated structural capacity again collapse, which should be involved in future structural design and analysis to ensure safety.

## 6. Acknowledgements

Financial supports from the National Natural Science Foundation of China (Grant Nos. 51261120374 and 51538010) are gratefully appreciated.

## 7. Copyrights



## 8. References

[1] Lu X, Lu XZ, Guan H, Ye LP (2013): Collapse simulation of reinforced concrete high-rise building induced by extreme earthquakes. *Earthquake Engineering & Structure Dynamics*, **42**(5), 705-723.

[2] Huang TC, Ren XD, Li J (2015): Numerical investigation of seismic collapse for super-stall structures. *7th International Conference on Computational Stochastic Mechanics (CSM-7)*, Santorini, Greece.

[3] Smyth AW, Gjelsvik A (2006): Energy capacity criterion for the design of columns against collapse. *Journal of Engineering Mechanics*, **132**(6), 594-599.

[4] Szyniszewski S, Krauthammer T (2012): Energy flow in progressive collapse of steel framed buildings. *Engineering Structures*, **42**, 142-153.

[5] Deniz D (2014): Stochastic prediction of collapse of building structures under seismic excitations. *Ph.D. Dissertation*, University of Illinois at Urbana-Champaign, Urbana, USA.

[6] Xu J, Li J (2015): An energetic criterion for dynamic instability of structures under arbitrary excitations. *International Journal of Structural Stability and Dynamics*, **15**(02), 1450043.

[7] Li J, Ren XD (2009): Stochastic damage model for concrete based on energy equivalent strain. *International Journal of Solids and Structures*, **46**(11/12), 2407-2419.

[8] Wu JY, Li J, Faria R (2006): An energy release rate-based plastic-damage model for concrete. *International Journal of Solids and Structures*, **43**(3/4), 583-612.

[9] Li J, Wu JY, Chen JB (2014): *Stochastic Damage Mechanics of Concrete Structures* (in Chinese). Science Press, Beijing, China.

[10] Ren XD, Zeng SJ, Li J (2015): A rate-dependent stochastic damage-plasticity model for quasi-brittle materials. *Computational Mechanics*, **55**(2), 267-285.

[11] Chen JB, Sun WL, Li J, Xu J (2013): Stochastic harmonic function representation of stochastic processes. *Journal of Applied Mechanics*, **80**(1), 011001.

[12] Wang D, Li J (2011): Physical random function model of ground motions for engineering purposes. *Science China Technological Sciences*, **54**(1), 175-182.

[13] Chen JB, Li J (2005): Dynamic response and reliability analysis of nonlinear stochastic structures. *Probabilistic Engineering Mechanics*, **20**(1), 33-44.

[14] Li J, Chen JB (2009): *Stochastic Dynamics of Structures*. John Wiley & Sons, Singapore.

[15] Belytschko T, Liu WK, Moran B (2000): *Nonlinear Finite Elements for Continua and Structures*. John Wiley & Sons, New York, USA.

[16] Uang CM, Bertero VV (1990): Evaluation of seismic energy in structures. *Earthquake Engineering & Structural Dynamics*, **19**(1), 77-90.

[17] Kabeyasawa T, Matsumori T, Katsumata H, Shirai K (2005): Design of the full-scale six-story reinforced concrete wall-frame building for testing at E-Defense. *1st NEES/E-Defense US-Japan Workshop on Collapse Simulation of Reinforced Concrete Building Structures*, Berkeley, USA.

[18] Johnson GR, Cook WH (1985): Fracture characteristics of three metals subjected to various strains, strain rates, temperatures and pressures. *Engineering Fracture Mechanics*, **21**(1), 31-48.

[19] Matsumori T, Shirai K, Kabeyasawa T (2006): Simulated earthquake test on a full-scale six-story reinforced concrete building at E-Defense, Part 1: Outline of test program. *2nd NEES/E-Defense US-Japan Workshop on Collapse Simulation of Reinforced Concrete Building Structures*, Miki, Japan.

[20] Zhou H, Li J, Ren XD (2016): Multi-scale stochastic structural analysis towards reliability assessment for large complex RC structures. *International Journal for Multiscale Computational Engineering*, in press. DOI: 10.1615/IntJMultCompEng.2016015745.

[21] Li J, Chen JB (2007): The number theoretical method in response analysis of nonlinear stochastic structures. *Computational Mechanics*, **39**(6): 693-708.