



## AN EFFICIENT DETECTION METHOD OF THE PEAK FREQUENCY OF MICROTREMOR H/V SPECTRAL RATIO USING A LOW-PASS FILTER

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### **Abstract**

The present study aims at proposing a method to evaluate the peak frequency in the microtremor H/V spectra efficiently. The proposed method deals with the spectra as the time series data and applies fast Fourier transform (FFT) to the H/V spectra as the first process.

In the next process, both the constant term indicating the average of the time series data and the (m+1)th harmonics and higher are removed and treated as zero. As the results, from the first to the (m)th harmonics in the spectra were remained. The remained spectra were assumed as the smoothed spectra and by applying the inverse fast Fourier transform (IFFT), the smoothed spectra without insignificant ripples were obtained.

The major advantage of the proposed method is the fact that the coherent predominant peak frequency can be evaluated irrespective of the value of the peak frequency.

Authors apply both the proposed method and the ordinal method to 10 observed microtremor H/V spectral ratios with different characteristics in the spectra. Predominant peak frequencies may be evaluated by the cautious evaluation of the skilled engineers without the help of the proposed method, however, it should be noticed that the predominant peak frequencies can be evaluated very easily by the proposed method.

*Keywords: peak frequency, microtremor H/V spectral ratio, low-pass filter*



## 1. Introduction

In recent years, microtremor observation has been widely used as one of effective tools [1] for subsurface explorations. Many researchers have shown an interest in microtremor observation data [2-5] and there has been a great discussion on the microtremor H/V spectral ratios [6]. One of the most important information that can be obtained from microtremor H/V spectral ratio is the first-order peak frequency because that well agrees with the predominant frequency in terms of the amplification of earthquake ground motion.

In general, the microtremor H/V spectral ratio is obtained by smoothing both the horizontal and vertical spectra in order to remove insignificant ripples in the spectra. However, insignificant small ripples in the microtremor H/V spectra cannot be eliminated thoroughly even after the smoothing process and that makes the detection of the significant peak frequency in the spectra difficult in cases.

Accordingly, detected peak frequencies in the microtremor H/V spectra may differ from person to person who interprets the data. The problem is prominent especially in cases with low peak frequencies. If a large band width is used for the window processing, peak in the H/V spectra is very difficult to be detected. However, insignificant ripples cannot be eliminated if the band with for the window processing is small.

In this study, authors propose a method to evaluate the peak frequency in the microtremor H/V spectra efficiently, using filter operation on the spectra of the microtremor H/V spectral ratio. Validity of the proposed method was shown by applying the method to observed microtremor H/V spectral ratios with different characteristics in the spectra.

## 2. Smoothing method and peak frequency detection

### 2.1 Microtremor H/V spectral ratio

Microtremor H/V spectral ratio is the spectral ratio obtained by dividing the horizontal component of microtremor with the vertical component. An example of microtremor H/V spectral ratio which is difficult to detect the peak frequency is shown in Fig.1 (black line). Clear peaks can be recognized in many H/V spectral ratios, while it sometimes is difficult to recognize clear peaks in H/V spectral ratios such as shown in Fig.1.

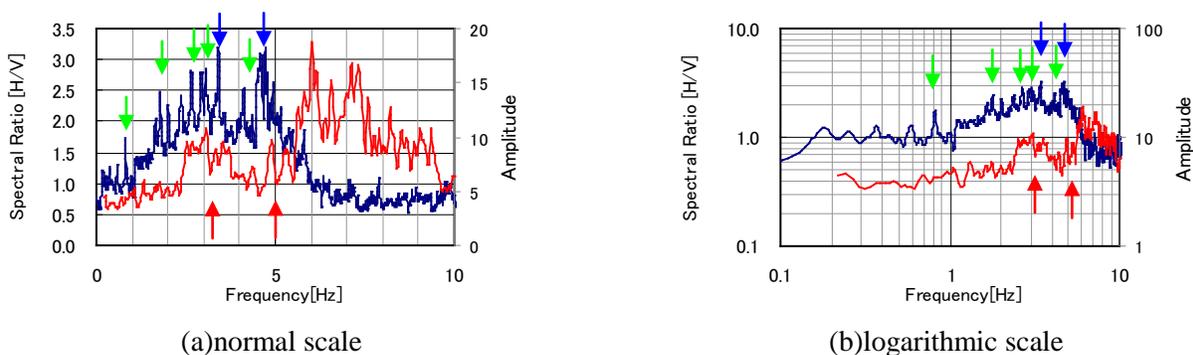


Fig. 1 – Example of microtremor H/V spectral ratio (black line) and the site amplification factor at the same site [8] (red line)

Microtremor H/V spectral ratio in Fig. 1 (black line) was obtained by the average of 3 spectral ratios with 16,384 samples (163.84 seconds) from the observation data with sampling interval 0.01 second (100Hz sampling frequency). Fig.1 (a) shows the microtremor H/V spectral ratio in the normal scale, and (b) shows in the logarithmic scale. In this example, Parzen window with the bandwidth of 0.05Hz was applied in calculation of Fourier spectra.

The peak of the microtremor H/V spectral ratio in Fig. 1 (black line) can be interpreted as around 3.3Hz and 4.5Hz as indicated by blue downward arrows. However, in this example insignificant small ripples (jags) in the



microtremor H/V spectra cannot be eliminated thoroughly. Accordingly, detected peak frequencies in the microtremor H/V spectra may differ from person to person who interprets the data, shown by green downward arrows. This problem is unavoidable especially in cases where microtremor H/V spectral ratios are plotted in the logarithmic scale as shown in Fig.1(b) because the difference in the amplitude of spectral ratio for each peak is not noticeable. As we assume that surface waves is predominant in the microtremors [7], the first-order peak frequency of microtremor H/V spectral ratio corresponds to the predominant frequency in terms of the amplification of earthquake ground motion. In this case, interpreter of the spectra may recognize the peak frequencies as 0.8Hz or 1.8Hz. There might be a room for discussion whether these small ripples (jags) as noise or not, it is expected that the dominant frequency of the ground is likely to be detected by eliminating insignificant small ripples (jags) in the microtremor H/V spectrum.

Fig. 1 also shows the site amplification factor [8] (red line) obtained using spectral inversion on seismic observation records at the same site [9]. Peaks around 3Hz and 5Hz (shown by red upward arrows) on site amplification factor can be found by focusing on the frequency range lower than 6Hz, considering the fact that the peaks do not appear in microtremor H/V spectral ratio in the frequency range larger than 6Hz. These frequencies are close to the peak frequencies interpreted by neglecting ripples in microtremor H/V spectral ratio (shown by blue downward arrows).

## 2.2 Smoothing method using filter operation (proposed method)

The formulae for fast Fourier transform (FFT) and inverse fast Fourier transform (IFFT) are shown below (for example, [10]) :

$$X_k = \frac{1}{N} \sum_{i=0}^{N-1} x_i e^{-j(2\pi ki / N)} \quad (1)$$

$$x_i = \sum_{k=0}^{N-1} X_k e^{+j(2\pi ki / N)} \quad (2)$$

Here,  $x_i$  indicates the time series data,  $X_k$  is complex Fourier coefficient,  $j$  is the imaginary unit and  $N$  is the number of sampling points. The inverse Fourier transform in Equation (2) is modified as follows:

$$x_i = X_0 + \sum_{k=1}^{N-1} X_k e^{+j(2\pi ki / N)} \quad (3)$$

The Fourier coefficient ( $X_0$ ) in the first term on the right side of Equation (3) is a constant term (direct current component) which indicates the average of sample values. Meanwhile, other terms indicate the sine wave (alternating current component), with the second term of alternating current component ( $k=1$ ) being the fundamental wave, with a period corresponding to the number of sampling points ( $N$ ). The third term and later indicate the ( $k$ th order) harmonics whose frequencies are integral multiplication ( $k$ -times) of the fundamental wave.

The proposed method deals with the microtremor H/V spectral ratios as the time series data, and using fast Fourier transform (FFT), transforms this series data into complex Fourier coefficient ( $X_k$ ) as the first process. We next remove both the constant term indicating the average of the time series data and the ( $m+1$ )th harmonics and higher and treat them as zero. As the results, from the first to the ( $m$ )th harmonics in the spectra are remained. We consider the remained spectra as the smoothed spectra and next apply the inverse fast Fourier transform (IFFT). Finally, we obtain the smoothed spectra without insignificant ripples.

## 2.3 Smoothing method using window processing

Smoothing with window processing is popularly used in the calculation of microtremor H/V spectra. The formula for window processing is provided below:



$$\bar{x}(t) = \sum_{k=-b/2}^{b/2} w(k) \cdot x(t-k) \quad (4)$$

Here,  $x$  indicates the original waveform,  $w$  is the window function and  $b$  is the bandwidth. Window processing means to calculate convolution of window function and original waveform.

In this study, The Parzen window, which is generally adopted as the window function, was used. The formula for the Parzen window is shown below:

$$w(f) = \frac{3}{4} u \left( \frac{\sin \frac{\pi u f}{2}}{\frac{\pi u f}{2}} \right)^4, \quad b' = \frac{280}{151u} \quad (5)$$

Here,  $f$  indicates the frequency,  $b'$  is the bandwidth and  $u$  is a constant (truncation width). The bandwidth ( $b'$ ) was set up as the width of rectangular pulse with the same dispersion after calculating the dispersion in the Parzen window.

In general, the attention is paid to spectral leakage in setting bandwidth and so on in the window processing. Since the purpose of this study is to detect the peak frequency (period) by eliminating the ripples (jags), cautious setting the bandwidth is required. In case of the window processing with too narrow bandwidth, ripples (jags) remain in the smoothed spectrum, and it may be difficult to detect the peak frequency. On the other hand, in case of the window processing with wide bandwidth, the spectrum becomes too smooth and the peak becomes obscure. Especially, the detected peak is unreliable in cases with lower peak frequency than the bandwidth.

Furthermore, we decided to use the normal Parzen window as shown by Equation (5) in this study, although Konno and Ohmachi (1995) [11] proposed a logarithmic window with which the bandwidth was constant on the logarithmic scale.

## 2.4 Peak frequency detection

Using the proposed method, the predominant peaks were extracted with the maximum values (local maximums) of 1.0 or larger (as the direct current component was removed) between 0.2Hz and 10.0Hz. The peaks with smaller amplitudes than 1/2 of the largest peak amplitudes were neglected.

On the other hand, when the ordinal smoothing method by window processing was used, the predominant peaks were extracted with the maximum values (local maximums) of 2.0 or larger (as the direct current component was not removed) between 0.2Hz and 10.0Hz. Similarly, the peaks with smaller amplitudes than 1/2 of the largest peak amplitudes were neglected.

## 3. Results of peak frequency detection

### 3.1 Results of the proposed method

In the first process of the proposed method, the number of data used in fast Fourier transform (FFT) was set to 4,096 from 0 to 25Hz with consideration of the necessary number of sampling in FFT (the number of power of 2) and the resolution error. That is, 4,096 points of the microtremor H/V spectral ratio were considered as time series data. Each term (the 0th to 100th order) of the complex Fourier coefficients expanded by fast Fourier transform (FFT) are shown in Fig. 2 (Original). We remove both the constant term indicating the average of the time series data and the 40th harmonics and higher of these complex Fourier coefficients and treat them as zero. As the results, from the first to the 39th harmonics in the spectra were remained (IFFT Points in Fig.2). We consider the remained spectra as the smoothed spectra and next apply the inverse fast Fourier transform (IFFT). In this study, we decided to keep the coefficients up to the 39th harmonic, which is approximately 1/50 of all the complex Fourier coefficients, after checking the amplitudes of the Fourier coefficients of higher harmonic were sufficiently small in Fig. 2. In addition, the number of sampling points for IFFT was set to the same 4,096 points for FFT.

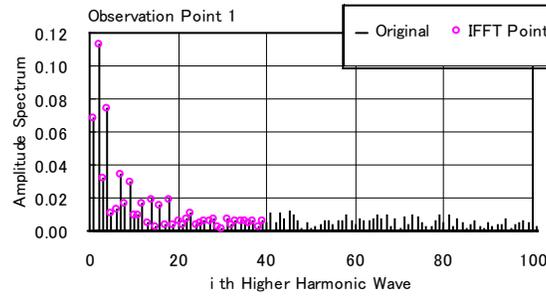


Fig. 2 –Example of complex Fourier coefficient

We have chosen ten sites for the study, considering the variations in the topographical and geological characteristics and scales, characteristics of observation points and characteristics of microtremor H/V spectral ratio, as shown in Table1. Fig. 3 shows the original microtremor H/V spectral ratios (Original), smoothed microtremor H/V spectral ratios (Smoothing) and detected peak frequency (Maximum) with the smoothing method by the proposed method. Fig. 3 shows the results up to 10Hz, and the smoothed microtremor H/V spectral ratio is shifted downward for the entire frequency range because the first term for Fourier coefficient (average value) was treated as zero. It was found that the insignificant ripples (jags) are sufficiently removed in the smoothed microtremor H/V spectral ratios (Smoothing) by the proposed method.

At observation point-1, the peak frequencies (shown by red upward arrows) detected from smoothed microtremor H/V spectral ratio (Smoothing) were 3.2Hz and 4.7Hz. These frequencies correspond well to the dominant frequencies of the ground (around 3Hz and 5Hz) detected from the actual seismic observation records as indicated with red line in Fig. 1. Therefore, dominant frequencies of the ground in terms of earthquake response can be detected by the proposed method at observation point-1.

At observation point-2, the peak frequency (shown by red upward arrow) after smoothing (Smoothing) is slightly shifted to the lower frequency side compared to the one before smoothing (Original) (shown by blue downward arrow). It is assumed that the peaks corresponding to the 1st order or higher order dominant frequencies was detected by taking into consideration the nearby peaks after smoothing.

At observation point-4, the dominant frequencies of the ground in terms of earthquake response (shown by red upward arrow) is thought to be detected after smoothing (Smoothing), while the spectral ratios before smoothing (Original) (shown by blue downward arrows) had two adjacent peaks.

Table 1 –Characteristics of Observation Point

Observation Point	topographical and geological characteristics(scale)	characteristics of observation points	characteristics of microtremor H/V spectral ratio
1	coastal plain(small)	near the mountain with embankment	peak caused by original ground and filling
2		near the mountain without embankment	peak caused by original ground
3		reclaimed land with inclined foundation	2 peaks caused by filling with inclined foundation
4		reclaimed land with slightly inclined foundation	peak caused by filling(separated)
5		peak caused by filling	
6	Sedimentary basin (thickness of 1000m)	edge of basin	peak around 2-3Hz
7		near edge of basin between points 6 and 9	relatively high freq. peak in between points 6 and 9
8		near center of basin between points 6 and 9	relatively low freq. peak in between points 6 and 9
9		center of basin	low frequency peak about 0.25Hz
10	coastal plain(large)	center of plane with horizontal stratification	peak about 1Hz with low frequency noise

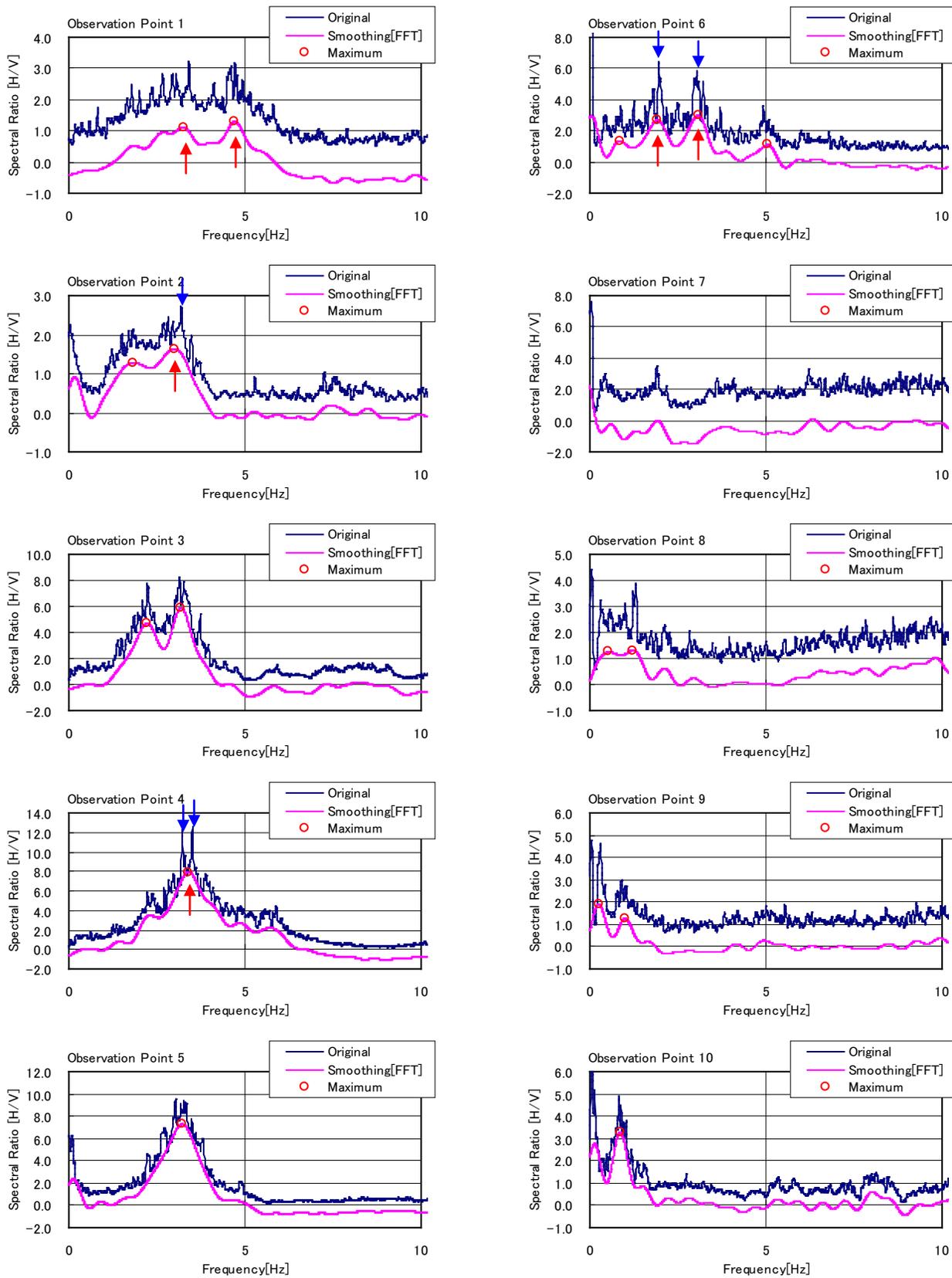


Fig. 3 –Detection of the peak frequency of the microtremor H/V spectral ratio by the proposed method

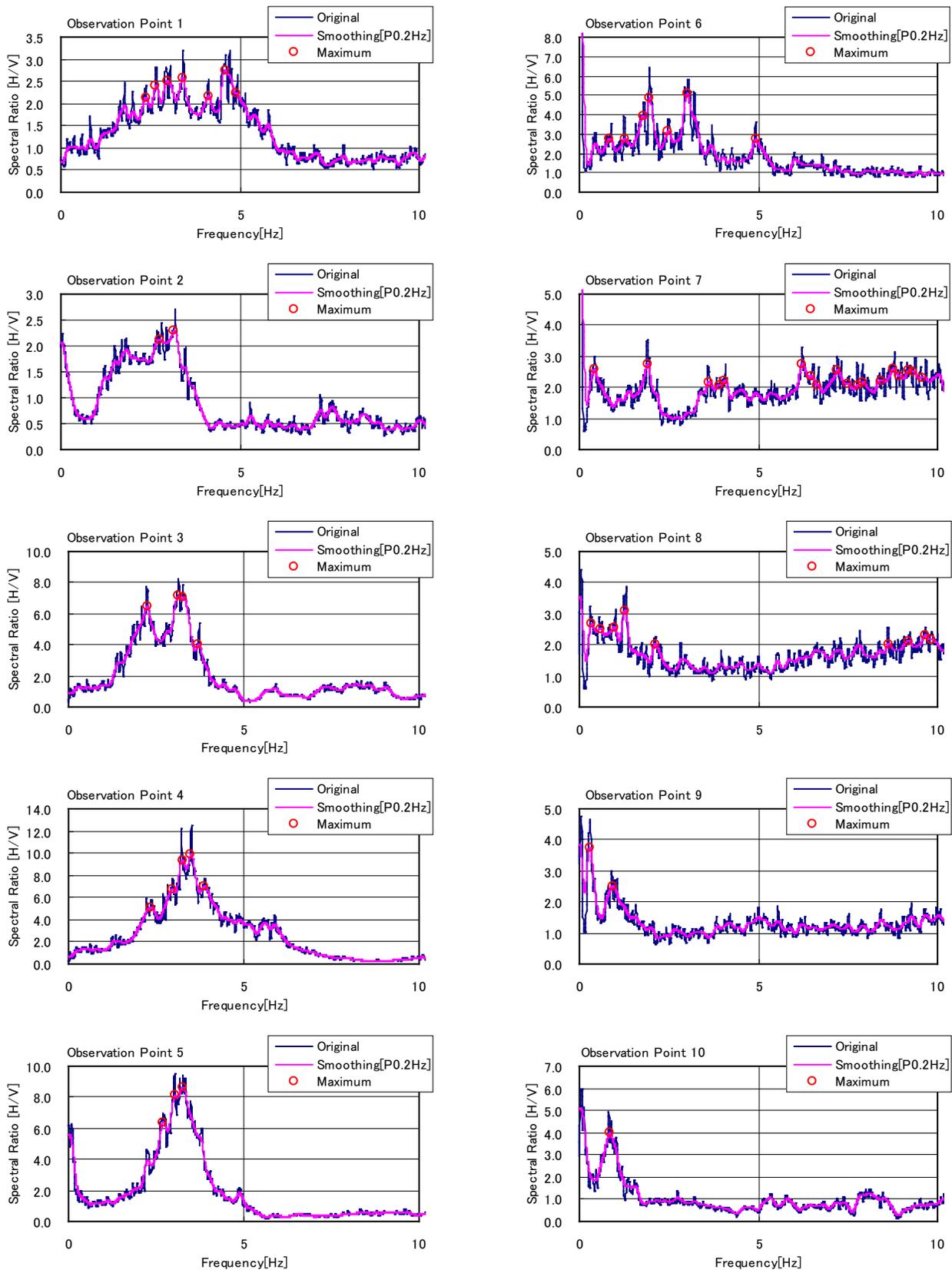


Fig. 4 –Detection of the peak frequency of the microtremor H/V spectral ratio by the window processing with the frequency band width of 0.2Hz

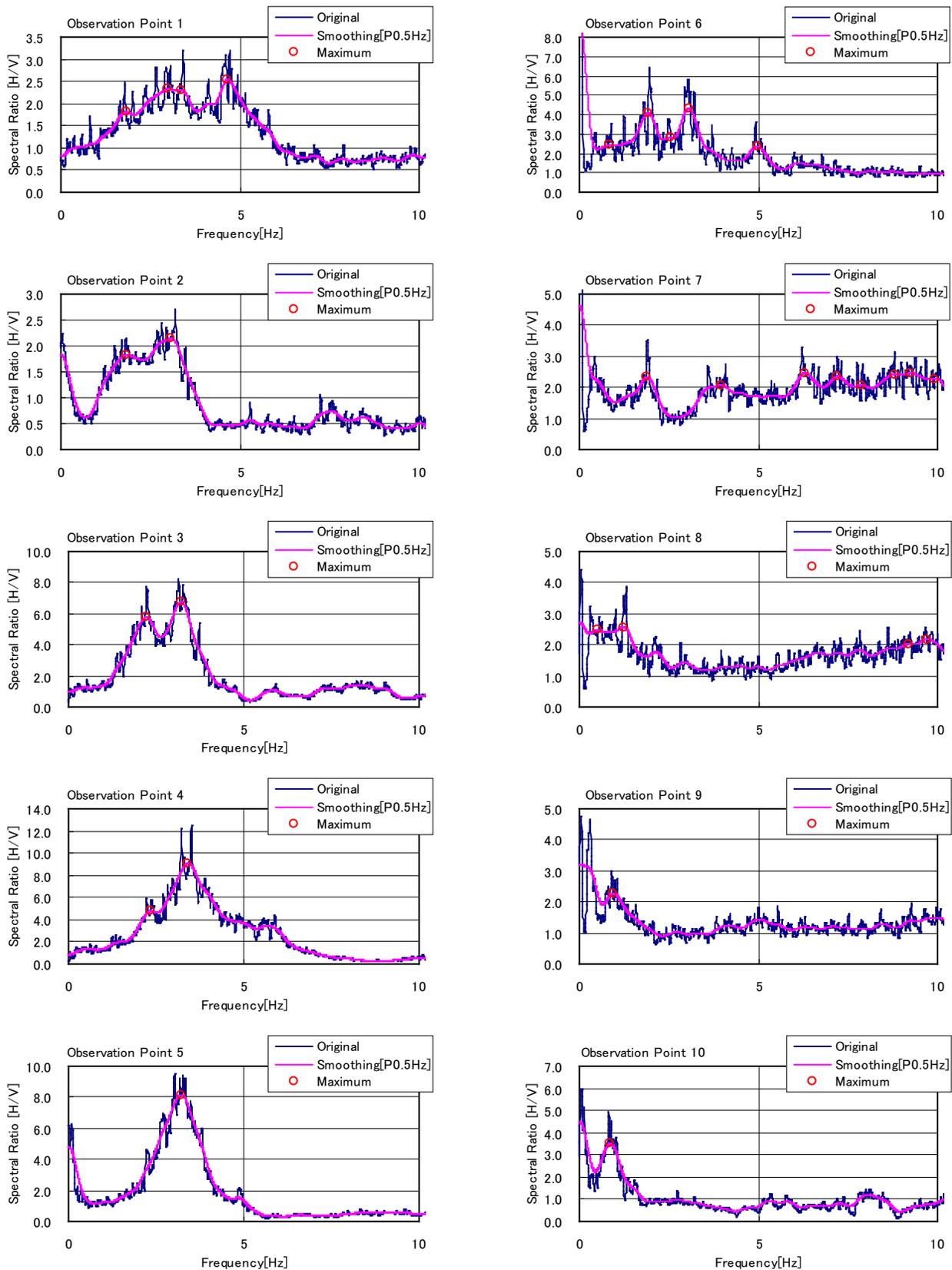


Fig. 5 –Detection of the peak frequency of the microtremor H/V spectral ratio by the window processing with the frequency band width of 0.5Hz



At observation point-6, two peaks were detected in both spectral ratios before smoothing (Original) (shown by blue downward arrows) and after smoothing (Smoothing) (shown by red upward arrows). While the peak on the lower frequency was slightly larger before smoothing, the peak on the higher frequency became larger after smoothing. Peak frequency in accordance with the band width of the large amplitude in the H/V spectral ratio was assumed to be detected by the proposed method.

### 3.2 Results of peak frequency detection with the smoothing method by window processing

Figs. 4 and 5 show the microtremor H/V spectral ratios (Original), smoothed microtremor H/V spectral ratios (Smoothing) and detected peak frequency (Maximum) with the smoothing method by window processing. Figs. 4 and 5 show the results up to 10Hz, and the bandwidth of Parzen window processing for smoothing in Figs. 4 and 5 are 0.2Hz and 0.5Hz respectively.

As shown in the previous section, Parzen window processing involves the operation to average the spectrum values within the frequency range indicated by bandwidth. Consequently, it is assumed not to be inadequate to determine the peaks using smoothed spectrum values in frequencies lower than 0.2Hz in case of the 0.2Hz bandwidth, and in frequencies lower than 0.5Hz in case of the 0.5Hz bandwidth. In this section, we examined two cases to detect the peak frequencies; one with bandwidth 0.2Hz which is expected to have little effect on averaging operation to the range 0.2Hz to 10.0Hz and the other with bandwidth 0.5Hz which is expected to give equivalent results to the filter operation method.

In case of Parzen window with the 0.2Hz bandwidth (Fig. 4), the insignificant ripples (jags) were not completely eliminated, and the local maximums were of large amplitudes greatly affected by the ripples (jags). Therefore, there is a possibility that the dominant frequencies of the ground in terms of earthquake response cannot be detected rationally. Especially, at observation point-1, the peak frequencies (shown by red circles) detected from the smoothed microtremor H/V spectral ratio (Smoothing) do not correspond with the dominant frequencies of the ground detected from the actual seismic observation records indicated with red line in Fig. 1 (around 3Hz and 5Hz). At observation points-2 to 5, several local maximum values were detected near the predominant peak frequency after smoothing (Smoothing). It is assumed that additional factitious judgments would be necessary in order to detect the dominant frequencies of the ground in terms of earthquake response.

On the other hand, it was found that the ripples (jags) were removed to the same degree as the smoothing method with filter operation in case of Parzen window with the 0.5Hz bandwidth (Fig. 5).

At observation points-2 to 6, it is assumed that the ripples (jags) were removed sufficiently after smoothing (Smoothing), and dominant frequencies of the ground in terms of earthquake response were detected. At observation point-1, the peak frequencies (shown by red circles) detected from the smoothed microtremor H/V spectral ratio (Smoothing) correspond well with the dominant frequencies of the ground detected from the actual seismic observation record indicated with red line in Fig. 1 (around 3Hz and 5Hz), compared to the case with 0.2Hz bandwidth. However, the correspondence is worse than that by the proposed method.

At observation point-7, many peaks with small amplitudes that should not be considered as clear peaks were detected because the spectra before smoothing (Original) had increasing amplitude in the high frequency band.

While the peak on the low frequency around 0.5Hz was detected at observation point-8, the peak on the low frequency around 0.25Hz was not detected at observation point-9. This is because of the inability to determine the peak in microtremor H/V spectral ratio in frequency bands lower than the bandwidth.

### 3.3 Comparison of the detected peak frequency

Fig. 6 shows a comparison of the detected peak frequencies between those by the proposed method and by the window processing. It is found that the detected peak frequencies by the two methods coincide with each other in many cases.

However, it is considered that the proposed method is better than the ordinal window processing method for cases such as observation point-7 where peak values were low and unclear and observation point-9 where the peaks were detected in the low frequency band, as shown in the previous section.

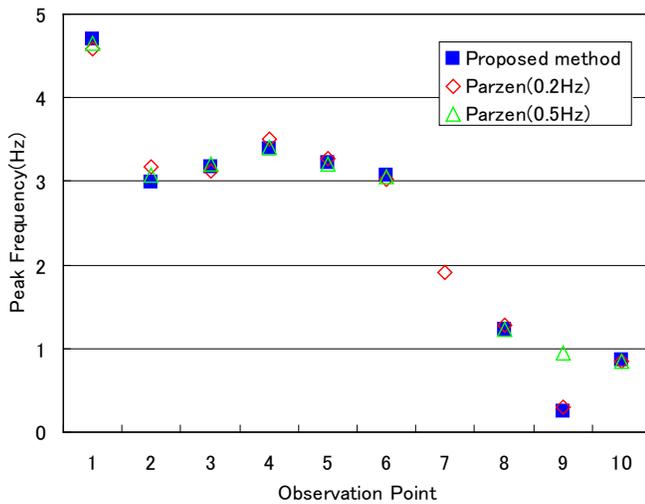


Fig. 6 –Comparison of the detected peak frequencies by the proposed method and by the window processing

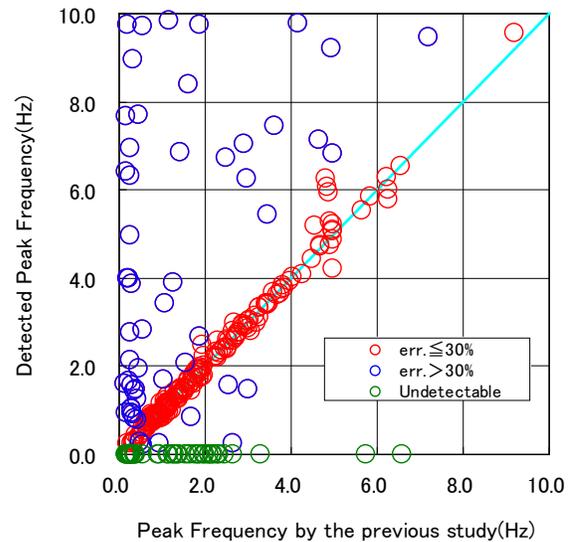


Fig. 7 –Comparison of the detected peak frequencies by the proposed method and by the previous study in Japan

## 4. Application example of the proposed method to microtremor H/V spectral ratios in Japan

### 4.1 Microtremor H/V spectral ratios in Japan

Authors observed microtremor at 283 sites in all parts of Japan where strong motion observation was conducted by Nagao et al. (2010) [2]. For microtremor observation they used servo velocity seismometers with flat frequency response down to 0.2Hz (5 second in period). Microtremor H/V spectral ratios were obtained as the average value of the 3 microtremor H/V spectral ratios of 163.84 seconds duration with little disturbance, so that the effects of long-period wave components could be taken into consideration.

In the previous study [2], peak frequencies of the microtremor H/V spectral ratios were detected by the cautious engineering judgement referring the information on the dominant frequencies of the ground in terms of earthquake response, namely site amplification factors.

### 4.2 Application example of the proposed method to microtremor H/V spectral ratios in Japan

We applied the proposed method to the microtremor H/V spectral ratios at 283 sites observed in the previous study. Here, we defined the detected frequencies as those with the largest amplitude by the proposed method. The detected peak frequencies were compared with the peak frequencies interpreted by Nagao et al. (2010) [2]. The results are shown in Fig. 7. Here, we consider the peak frequencies obtained by Nagao et al. (2010) as the correct frequencies and calculated the estimation error of the peak frequencies.

In Fig. 7, red circle indicates the cases whose estimation error is less than 30%, blue circle indicates the cases whose estimation error exceeds 30% and green circle indicates the cases that peak frequencies are undetectable. The number of cases with estimation error less than 30% was 77% of all observation sites except undetectable sites. The proposed method was shown to be effective for the detection of dominant frequencies of the ground considering the fact that it does not require cautious engineering judgement.

## 5. Conclusion

Authors proposed a method for the detection of dominant frequencies of the ground by using microtremor H/V spectral ratios. The proposed method deals with the microtremor H/V spectral ratios as the time series data and



transforms this series data into complex Fourier coefficient as the first process. The method next removes both the constant term and the  $(m+1)$ th harmonics and higher and treat them as zero. By applying the inverse fast Fourier transform, the smoothed spectra without insignificant ripples are obtained.

It was shown that peak frequencies corresponding to the dominant frequencies of the ground in view of earthquake response can be detected efficiently by the proposed method. The advantage of the proposed method is the fact that it does not require cautious engineering judgement.

A further study on the adequate frequency interval of the microtremor H/V spectral ratios to be used for the proposed method and the order of harmonics to be applied in inverse Fourier transform should be conducted. A further study on the method to determine the predominant peak frequency among the local maximum peaks should also be conducted. When there are several peaks, predominant peak frequency might be the frequency with the largest amplitude or the lowest peak frequency.

## 6. Acknowledgements

We sincerely thank Mr. Koichiro Takezawa, NEWJEC Inc. for providing indispensable advice and information.

## 7. References

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