*16th World Conference on Earthquake Engineering, 16WCEE 2017*

*Santiago Chile, January 9th to 13th 2017*

*Paper N° 0516 (Abstract ID)*

*Registration Code: S-L1464751448*

# SIMULATION OF A BASAL SOLICITATION, GENERATED FROM SAWTOOTH FUNCTION AND ITS EVOLUTION OVER THE STRUCTURE.

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## *Abstract*

To date, earthquakes have been represented trough functions based on random vibrations without representative results of the phenomena. To that purpose, a mathematical procedure is proposed, which is the result of a series of "sawtooth" functions that allows to simulate any given earthquake ground acceleration record that represents the phenomena, and allows us to determine and modify the characteristic parameters and evaluate future events.

On the other hand, the displacements and ground acceleration are spread through the structure from the ground to the top, generating displacements and accelerations in the intermediate levels. This is, if the structure stiffness were infinite, the acceleration on top would be de same as the bottom, but considering a different stiffness, the acceleration on any upper floor is affected. All this makes it necessary to consider, the variations of the ground acceleration and stiffness, through the ground to the top in the dynamic motion equilibrium equations.

*Keywords: Simulation; Basal solicitation; Evolution over the structure.*

## 1. Introduction to Basal Movement

Considering a structure initially at rest and then subjected to a basal pulse, at the beginning t = to the structure tends to follow at rest but then the masses move with the acceleration imposed, and the structure reacts with a force proportional to the deformation between time $t_0$ and $t_i$.

The end of basal shift is due to a new pulse in the opposite direction, but the structure and the masses continue its displacement due to inertia, which sharply changes due to new stresses generated by the new basal pulse. The equilibrium equations are given by:

$$\left.\begin{array}{l} m_1\ddot{y}_1+k_1(y_s-y_1)-k_2(y_2-y_1)=0 \\ m_2\ddot{y}_2+k_2(y_1-y_2)-k_3(y_3-y_2)=0 \\ \vdots \\ m_n\ddot{y}_n+k_n(y_n-y_{(n-1)})=0 \end{array}\right\} \text{ with } u_i=\pm(y_s-y_i)$$

Where it is get Eq. (1)

$$[M]\{\ddot{u}\}+[K]\{u\}=[M]\{\ddot{y}_s\}=\{F(t)\} \tag{1}$$

To decouple the equations is necessary a linear transformation given by Eq. (2):

$$\{u\}=[\Phi]\{z\} \tag{2}$$

Where $[\Phi]$ is the eigenvectors matrix obtained by solving the eigenvalues problem given by Eq. (3).

$$([K]-\omega^2[M])\{\Phi\}=0 \tag{3}$$

The solution of this equation in terms of $\omega^2$ is given by a family of eigenvectors, which makes it necessary to impose the orthonormal condition given by, $\phi_i^T\phi_i=\delta_{ij} \Rightarrow \begin{cases}\delta_{ij}=1, i=j \\ \delta_{ij}=0, i\neq j\end{cases}$ in Eq. (4),

$$\Phi_{ij}=\frac{\phi_{ij}}{\sqrt{\sum_{i=1}^{n}\phi_{ij}^2}} \tag{4}$$

The $[\Phi]$matrix is given by Eq. (5).

$$[\Phi]=\langle\Phi_1 \quad \Phi_2 \quad \dots \quad \Phi_n\rangle=\begin{bmatrix}\Phi_{11} & \Phi_{12} & \cdots & \Phi_{1n} \\ \Phi_{21} & \Phi_{22} & \cdots & \Phi_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \Phi_{n1} & \Phi_{n2} & \cdots & \Phi_{nn}\end{bmatrix} \tag{5}$$

Replacing Eq. (2) in Eq. (1), Eq. (6) is obtained.

$$[M][\Phi]\{\ddot{z}\}+[K][\Phi]\{z\}=[M]\{\ddot{y}_s\} \tag{6}$$

Premultiplying by $[M]^{-1}$ and then by $[\Phi]^T$ it gets.

$$[\Phi]^T[M]^{-1}[M][\Phi]\{\ddot{z}\}+[\Phi]^T[M]^{-1}[K][\Phi]\{z\}=[\Phi]^T[M]^{-1}[M]\{\ddot{y}_s\}$$

$$[\Phi]^T[\Phi]\{\ddot{z}\}+\underbrace{[\Phi]^T[M]^{-1}[K][\Phi]}\{z\}=[\Phi]^T\{\ddot{y}_s\} \tag{7}$$

$$\left[\Phi_j{}^2W_i{}^2\right]=[\Phi]^T[M]^{-1}[K][\Phi]=\begin{bmatrix} \left(\frac{\phi_{11}{}^2k_{11}}{m_1}+\frac{\phi_{21}{}^2k_{22}}{m_2}+\cdots+\frac{\phi_{n1}{}^2k_{nn}}{m_n}\right) & 0 & \cdots & 0 \\ 0 & \left(\frac{\phi_{12}{}^2k_{11}}{m_1}+\frac{\phi_{22}{}^2k_{22}}{m_2}+\cdots+\frac{\phi_{n2}{}^2k_{nn}}{m_n}\right) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \cdots & 0 & \left(\frac{\phi_{1n}{}^2k}{m_1}+\frac{\phi_{2n}{}^2k_{22}}{m_2}+\cdots+\frac{\phi_{nn}{}^2k_{nn}}{m_n}\right) \end{bmatrix}$$

Where $k_{11}=k_1+k_2$, $k_{22}=k_2+k_3$; ...; $k_{nn}=k_n$ ; $\omega_i{}^2=\dfrac{k_{ii}}{m_i}$; $\phi_j{}^2\omega_i{}^2=\sum_{i=1}^{n}\phi_{ij}{}^2\dfrac{\kappa_{ii}}{m_i}$; $\Omega^2=\left\lfloor W_{ii}{}^2\right\rfloor$

With $$\left[\Phi_j{}^2W_i{}^2\right]=[\Phi]^T[M]^{-1}[K][\Phi]=\left[\Phi_j\right]^T\left[W^2\Phi_j\right]=[\Phi]^T\Omega^2[\Phi]$$

Then $$[\Phi]^T[\Phi]\{\ddot{z}\}+[\Phi]^T\Omega^2[\Phi]\{z\}=[\Phi]^T\{\ddot{y}_s\} \tag{8}$$

From Eq. (2) $\{u\}=[\Phi]\{z\}$ replacing in Eq. (8).

$$[\Phi]^T\{\ddot{u}\}+[\Phi]^T\Omega^2\{u\}=[\Phi]^T\{\ddot{y}_s\} \tag{9}$$

Then $$\{\ddot{u}\}+\Omega^2\{u\}=\{\ddot{y}_s\} \tag{10}$$

$$\ddot{u}_i+\omega_i{}^2u_i=y_s$$

Then it has been determined the expression that relates the ground acceleration with the movement of the structure. To solve this equation is necessary to know the function of basal acceleration, which arises from a seismic known record.

## 2. Basal Movement Simulation Function

In order to define a function that represents a basal solicitation, a known basal solicitation record is used, characterized by its mean frequency, pulse numbers and amplitudes.

Considering a hypothetical basal acceleration record as shown in Fig. 1, between $t_0$ and $t_5$, there are 5 cycles with similar $\Delta t_1$, and 4 cycles between $t_5$ and $t_9$ with similar $\Delta t_2$, then each cycle correspond to $2\pi$, i.e.

In $t=0$, $\pi/2$, $\pi$, $3\pi/2$, $2\pi$, then 2 series of sawtooth functions with $\overline{\omega}_{S1}$ and $\overline{\omega}_{S2}$ mean frequencies representing the seismic event have been defined.

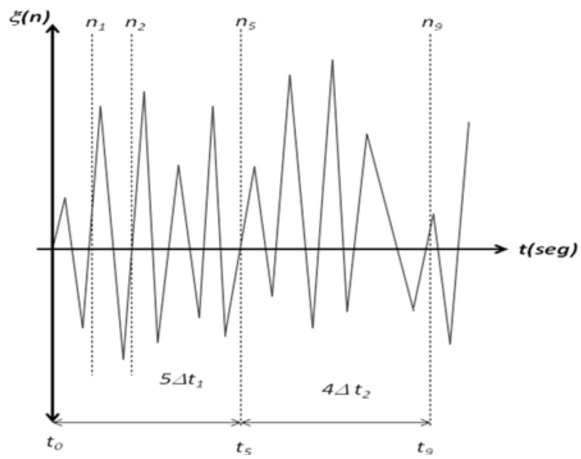



Fig. 1 - Displacement record

Where $\overline{T}_i = \left(\dfrac{t-t_0}{n_i}\right)$; $\overline{\omega}_s = \dfrac{2\Pi}{T_i}$; $\Delta t = \dfrac{T_i}{4}$; $t_n = n\Delta t$ with $n = 1,2,...,n$

Each period consists of 4 fourth of a period, then it is possible to evaluate the displacements in each fourth of a period or cycle in which the movement conditions are defined. Under this conditions it is possible to define a sawtooth, periodic secant-sen function with unitary amplitude for every cycle given by Eq. (11).

$$\eta(t_n) = sen\left(\overline{\omega}_s n \frac{\overline{T}_i}{4}\right) = sen(\overline{\omega}_s t_n) \tag{11}$$

With:

$\overline{\omega}_s$ : Mean cycle frequency

$\overline{T}_i$ : Mean cycle time

$\xi$ : Seismic displacement record

The maximum values are obtained for integer numbers of the fourth period as shown in the following example in Table 1 and Fig 2.

Table 1 - Unitary values

| n | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| $t_n$ | $t_0$ | $t_1$ | $t_2$ | $t_3$ | $t_4$ | $t_5$ |
| $\eta(t_n)$ | 0 | 1 | 0 | -1 | 0 | 1 |

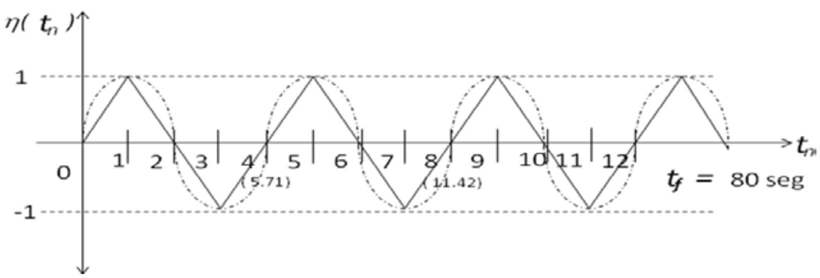



Fig. 2 - Secant-Sen function (Sawtooth)

$$\overline{T}_s = \left(\frac{80}{3{,}5}\right) = 22.85\sec \Rightarrow \Delta t = \frac{22.85}{4} = 5{,}71\sec$$

$$\text{For } t_3 = 3\Delta t = 3\times 5{,}71 = 17{,}14\sec$$

Multiplying sawtooth function $\eta(t_n)$ by an amplitude vector $\{\xi\}$ of seismic record, the basal movement is given by:

$$\{y_s(t)\} = \{\xi\}\eta(t_n) = \{\xi\}sen(\overline{\omega}_s t_n)$$

Note that developed expressions are a mathematical representation of the image, seismic record, i.e. the same function may represent displacement, velocity or acceleration. The matter is that the parameters coherently represent the recorded phenomena.

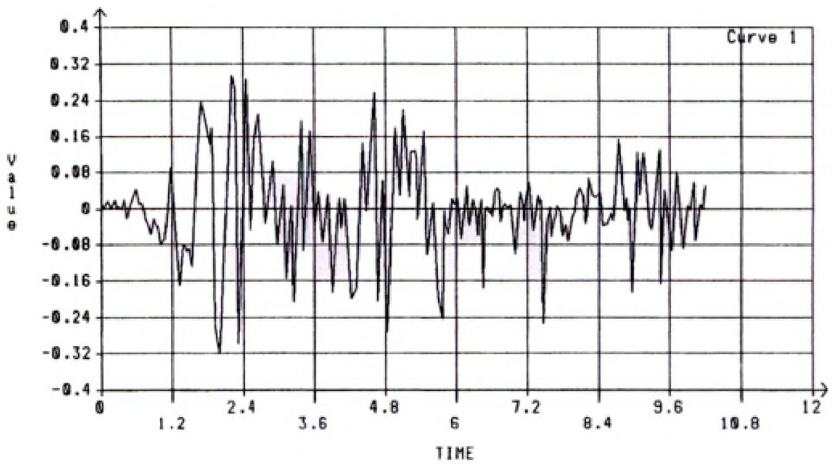



**Fig. 8.21** Ground acceleration for the first 10 sec recorded for the north–south component of the 1940 El Centro earthquake.

Then, the mean frequency its equal to the basal pulse record with same amplitudes than the seismic record, then the function that simulates the seismic event is obtained.

Displacement: $\{y_s(t)\} = \{\xi\} sen(\overline{\omega}_s t_i)$, $\{\xi\}$ Seismic amplitude vector (12)

Example, Applying the displacement function considering 7 seconds period:

$$\overline{T}_s = \left(\frac{t_f - t_0}{N}\right) = 1.75\text{sec} \Rightarrow \Delta t = \frac{T_s}{4} = 0{,}4375\text{sec};\ \ t_n = n\Delta t$$

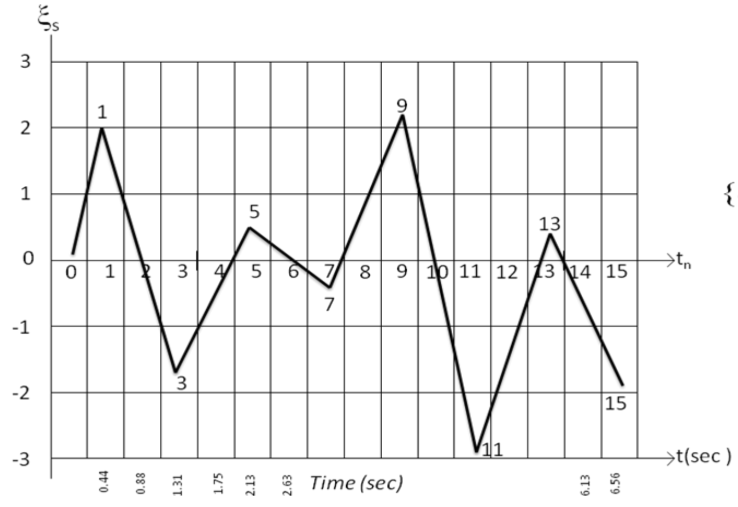
The mean angular frequency it's given by:

$$\overline{\omega}_s = \frac{2\Pi}{T_s} = \frac{6.38}{1.75} = 3{,}5903\, rad/\text{sec}$$

Then considering an amplitude vector of known basal movement record $\{\xi_s\}$, for each interval $n\Delta t$ the relation between values is checked in Table 2 and Fig 3.

Table 2 - Basal amplitude movement for each interval

<table>
<tr><td>n</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>6</td><td>7</td><td>8</td><td>9</td><td>10</td><td>11</td><td>12</td><td>13</td><td>14</td><td>15</td></tr>
<tr><td>$t_n = n\Delta t$</td><td>0</td><td>0,44</td><td>0,88</td><td>1,31</td><td>1,75</td><td>2,19</td><td>2,63</td><td>3,06</td><td>3,5</td><td>3,94</td><td>4,38</td><td>4,81</td><td>5,25</td><td>5,69</td><td>6,13</td><td>6,56</td></tr>
<tr><td>$\{\xi_s(t)\}$</td><td colspan="2">(0→1)<br>2,0</td><td colspan="2">(1→3)<br>-3,7</td><td colspan="2">(3→5)<br>1,7</td><td colspan="2">(5→7)<br>-0,8</td><td colspan="2">(7→9)<br>2,5</td><td colspan="2">(9→11)<br>-5,0</td><td colspan="2">(11→13)<br>3,3</td><td colspan="2">(13→15)<br>-2,4</td></tr>
</table>




$$\{\xi_{s}(t_n)\} = \begin{Bmatrix} 2{,}0 \\ -3{,}7 \\ 1{,}7 \\ -0{,}8 \\ 2{,}5 \\ -5{,}0 \\ \vdots \end{Bmatrix}$$

Fig. 3 - Basal movement

Then, the amplitude in $n_0, n_1$ interval is $\Delta\xi_1 = \xi_1 - \xi_0$; in $n_1, n_2$ is $\Delta\xi_2 = \xi_2 - \xi_1$, etc. Each line represents the basal amplitude movement, displacement, velocity or acceleration, in either positive or negative way, upward positive, downward negative, then each line is a movement itself in which the initial value is the end value of the former movement.

Considering basal pulses transmitted to the structure by with the basal frequency, the solicitation in presented with the frequency imposed, then the equation of motion is given by,

$$\ddot{u}_i + \omega^2{}_i u_i = \xi sen \overline{\omega}_s t \tag{13}$$

The solution of this equation is given by numerical or analytical method, from which the second is presented.

An analytical solution of the equation is given by:

$$u(t) = u_c(t) + u_p(t) \tag{14}$$

$u_c(t)$ : Complementary solution that satisfies the homogeneous equation $\Rightarrow u_c(t) = A\cos\omega t + Bsen\omega t$;

$u_p(t)$ : Particular solution from the second member of the equation $\Rightarrow u_p(t) = Ysen\overline{\omega}_s t$;

Replacing $u_p(t)$ in the differential Eq. (13)

$$-Y\overline{\omega}_s{}^2 sen\overline{\omega}_s t + \omega^2 Ysen\overline{\omega}_s t = \xi sen\overline{\omega}_s t$$

$$Y\left(\omega^2 - \overline{\omega}_s{}^2\right) = \xi \Rightarrow Y = \frac{\xi}{\omega^2 - \overline{\omega}_s{}^2} \tag{15}$$

Replacing in Eq. (14) it is get

$$u(t) = A\cos\omega t + Bsen\omega t + \frac{\xi}{\omega^2 - \overline{\omega}_s{}^2} sen\overline{\omega}_s t \tag{16}$$

$$\text{In } t = 0 \Rightarrow u(t) = 0 \Rightarrow A = 0$$

$$\text{In } t = 0 \Rightarrow \dot{u}(t) = 0 \Rightarrow B = -\frac{\xi}{\omega^2 - \overline{\omega}_s{}^2}$$

Replacing in Eq. (16)

$$u(t) = \frac{\xi}{\omega^2 - \overline{\omega}_s{}^2}\left(sen\overline{\omega}_s t - sen\omega t\right) \tag{17}$$

In this development, the phase difference $\psi$ between basal pulse and level solicitation is not considered.

## 3. Transversal Vibrations in Prismatic Sections

A beams that vibrates in one of its inertial axe and flex with elastic curve $\eta(x)$, in every section there is a bending moment $M(x)$ and shear $Q(x)$ that corresponds to a ficticious charge $q(x)$ that represents the inertial forces, then,

$$\frac{d^2\eta}{dx^2} = -\frac{M}{EI} \Rightarrow EI\eta'' = -M \Rightarrow \frac{d}{dx}(EI\eta'') = -Q \Rightarrow ;\ \frac{d^2}{dx^2}(EI\eta'') = q(x)$$

The inertial fictitious charge that acts by unit length, it's given by: $-\frac{m\partial^2\eta}{\partial t^2} = -.DA\frac{\partial^2\eta}{\partial t^2}$

D is density, A is the transverse section

Due to $\eta(x,t)$ is a function of x and t, it satisfies: $\frac{\partial^2}{\partial x^2}\left(EI\frac{d^2\eta}{dx^2}\right) = -.DA\frac{\partial^2\eta}{\partial t^2}$

For constant section it is have:

$$\frac{\partial^2\eta}{\partial t^2} + \frac{\alpha^2\partial^4\eta}{\partial x^4} = 0 \qquad (18)$$

with $\alpha^2 = \frac{EI}{DA}$

The solution of the differential equation is given by a wave propagation in the increasing x direction represented by the function $\eta(x,t) = f_1(x-ct)$. Considering the solution given by:

$$\eta(x,t) = \eta sen\left(2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right) = \eta sen\left(\frac{2\pi}{\lambda}(x-ct)\right) \qquad (19)$$

It is verified $\frac{\partial^2\eta}{\partial t^2} = -\frac{4\pi^2}{T}$ and $\frac{\partial^4\eta}{\partial x^2} = \frac{16\pi^4}{\lambda^4}$, replacing in (18) it is get

$$c = \frac{n\pi}{\lambda}\sqrt{\frac{Er^2}{D}};\ r = \frac{I}{A}$$

c: Propagation speed

n: Number of waves

r: Turning radius

In the case of transverse vibrations on the structure, the velocity c is dependent of the wavelength $\lambda$ of the perturbation, which is different from the longitudinal vibration, which doesn't depend on it.

Then, analyzing the structure response of a basal pulse, is important to consider the reflected wave given by the solution of the differential equation that could increase the solicitation.

## 4. Conclusions

So far, the solution of the dynamic equilibrium's equations have rested upon the idea about normalize those equations under the orthogonal condition. However, this work has not considered this paradigm, due to it is not possible to apply this principle over the mass matrix to obtain a unitary matrix as result and, subsequently, to be applied over the rigidity matrix - which are forces - to obtain as result the natural vibrating frequencies.

Therefore, this work shows the procedure independently of this practice and succeed to decouple the movement equilibrium's equations respecting it's dimensionality.

In the evaluation of the phenomena of spreading seismic waves over the highest levels of the structure; applying the common practice, the results do not show significant differences. However, when inertial masses and wavelength are considered; this effect should be taken into account and be studied in further works.

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