PHYSICALLY BASED PHASE SPECTRUM AND SIMULATION OF STRONG EARTHQUAKE GROUND MOTIONS

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Abstract

A physical method of modeling phase spectrum (i.e. Fourier phase spectrum) is derived through defining the relationships between the relative arrival time of wave groups and the Fourier amplitude. The concept of the relative arrival time highlights the phase properties of strong earthquake ground motions, i.e. the envelope delay. In this method, two parameters with clear physical meanings are introduced, say, the median arrival time and strong shock duration, which provide a logical basis for the modeling of phase spectrum in physical sense. For illustrative purposes, the time series of horizontal acceleration records of earthquake ground motions are investigated. Two techniques based on the discrete Fourier transform (DFT) and the continuous Fourier transform (CFT) are used to calculate the scatter diagram between the Fourier amplitude and relative arrival time. It is shown that using the DFT, the range of relative arrival time corresponding to the peak amplitudes hinges upon the time range of strong shock of ground motions, which is not held by the CFT. Numerical results reveal that the understanding of relative arrival time and its relevance to the Fourier amplitude of wave group provides a feasible means for the simulation of strong earthquake ground motions in engineering practice.

Keywords: phase spectrum; median arrival time; relative arrival time; discrete Fourier transform; ground motions

1. Introduction

Phase spectrum is well recognized as the critical component of spectral representation schemes for securing the non-stationary properties of earthquake ground motions. The phase held by frequencies had been considered as random variable uniformly distributed in the domain $[0,2\pi)$ before Ohsaki pointed out that phase difference did not submit to uniform distribution and its probability density function is closely related to the wave shape of ground motions[1]. Subsequently, much work was focused on revealing this interesting relationship and on developing methods of modeling phase spectrum utilizing the distribution of phase difference [2][3][4][5][6]. It is worth mentioning that Jin and Liao explained the physical meaning of a phase difference function which is so-called relative arrival time of wave group [4]. Furthermore, Thráinsson and Kiremidjian discovered the relationship of phase difference and Fourier amplitude, which provides a new perspective towards modeling phase spectrum [7][8]. In this regard, Boore introduced a concept of envelope delay with unit of time which is quite similar to phase difference, and investigated the relationship between envelop delay and amplitude as well as frequency [9].

In this paper, two techniques based on the discrete Fourier transform (DFT) and the continuous Fourier transform (CFT) are used to calculate the relative arrival time. The comparative studies of numerical results derived from the two techniques are proceeded. The relationship between relative arrival time and Fourier amplitude is verified using the data of ground motion records. Numerical results reveal that the relationship between relative arrival time of wave group and Fourier amplitude underlies the non-stationary properties of strong ground motions. A physical method of modeling phase spectrum is then derived based on the relationship of relative arrival time of wave group and Fourier amplitude. Following that, the simulation of ground acceleration is carried out by inverse discrete Fourier transform with phase spectrum and amplitude spectrum. Herein the amplitude spectrum of seismic ground motion is represented by the random function model developed by Wang and Li [10].

2. Relative arrival time of ground acceleration

The frequency-dependent relative arrival time of wave group (t_e) is also called envelope delay, which has the mapping formulation as follows [11]:

$$t_e(f) = -\frac{1}{2\pi}\frac{d\varphi(f)}{df} \tag{1}$$

where $\varphi(f)$ denotes the phase spectrum of strong ground motion. The value of t_e is not unique since $\varphi(f)$ is not unique. It has no influence on the time series of strong ground motion when $\varphi(f)$ changes in integer multiples of 2π.

Boore proposed a method to calculate t_e by using a Fourier transform without first unwrapping the phase, which guarantees the uniqueness of t_e [9]. This transform technique is so-called CFT method since it comes from continues Fourier transform. If the time series of earthquake ground acceleration $a(t)$ is given, its Fourier transformation $F(f)$ is then expressed by

$$F(f) = \int_{-\infty}^{+\infty} a(t)e^{-\mathrm{i}2\pi ft}\mathrm{d}t = R(F) + \mathrm{i}I(F) = A(f)e^{\mathrm{i}\varphi(f)} \tag{2}$$

where f denotes the natural frequency; $R(F)$ and $I(F)$ are real and imaginary parts of $F(f)$, respectively; $A(f)$ and $\varphi(f)$ denote the amplitude spectrum and phase spectrum of $a(t)$, respectively. Based on the CFT method, the relative arrival time can be calculated by

$$t_e(f) = -\frac{1}{2\pi}\frac{d\varphi(f)}{df} = -\frac{1}{2\pi}\frac{\frac{dI(F)}{df}R(F) - \frac{dR(F)}{df}I(F)}{R^2(F) + I^2(F)} \tag{3}$$

It is seen that $R(F)$, $I(F)$ and f are fixed when $a(t)$ is given, even though $\varphi(f)$ changes on the premise that $a(t)$ will not change. Then, there is a contradiction here. The middle part of Equation (3) will change when $\varphi(f)$ changes, while the right part won't change. Anyhow the CFT method offers a way to calculate the relative arrival time of ground motions.

Obviously, another method to calculate t_e needs to have phase first. Jin and Liao expressed t_e by discrete data:

$$t_e(f) = -\frac{1}{2\pi}\frac{\Delta\varphi(f)}{\Delta f} \tag{4}$$

where $\Delta\varphi$ and Δf are derived from discrete Fourier transform, $\Delta\varphi \in (-2\pi, 0)$, $\Delta f = 1/T$, $t_e \in (0, T)$,in which T denotes the duration of strong ground motion. Jin and Liao pointed out that when $\Delta\varphi \in (-2\pi, 0)$, $t_e \in (0, T)$. The method of calculating relative arrival time of wave group using Equation (4) in which $\Delta\varphi \in (-2\pi, 0)$ is so-called DFT method.

The two techniques are both used to calculate the relative arrival time of earthquake ground acceleration. To illustrate the range of relative arrival time, four ground acceleration records with different durations 6.85s, 39.99s, 76.705s, 129.995s are considered. The detailed information of ground motions are listed in Appendix. Corresponding to the ground acceleration durations, the relative arrival time ranges derived from DFT method are $[0, 6.174\text{s}]$, $[0, 39.706\text{s}]$, $[0, 76.573\text{s}]$, $[0, 129.388\text{s}]$, respectively. As shown in Fig. 1, the relative arrival time calculated from DFT method almost have the same time range as the ground motion time series; while the results derived from CFT method differ a lot. Therefore, the results of DFT method meet well with the meaning of 'relative arrival time'. It is also seen from Fig. 1 that the relative arrival time shows no strong dependence on frequency.

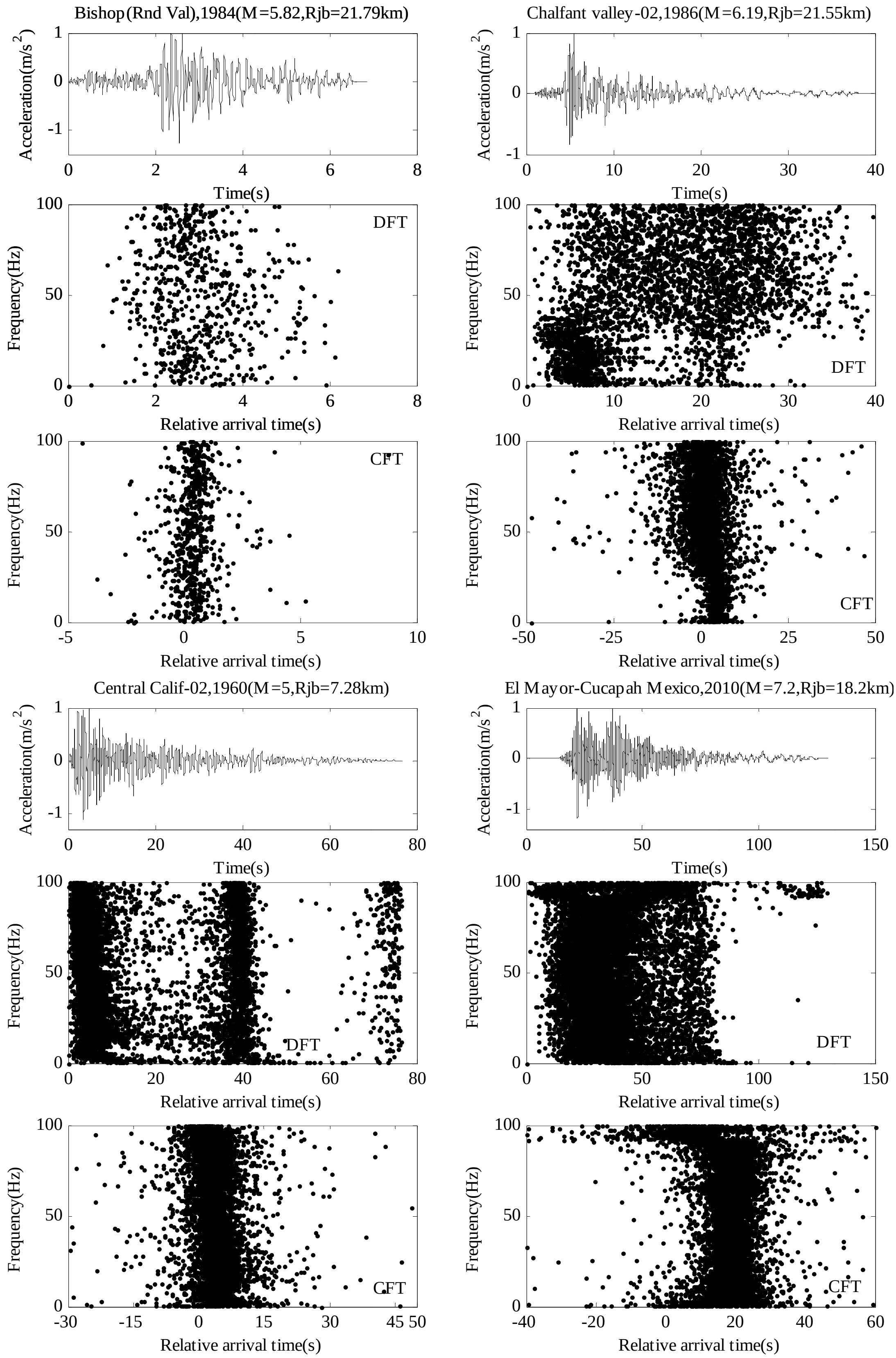

Fig. 1 Relative arrival times of ground acceleration records with different durations

3. Relationship between relative arrival time of wave group and Fourier amplitude

Although the relative arrival time shows a weak frequency-dependence; as seen in Fig. 1, the phase difference and relative arrival time give rise to be a strong amplitude-dependence according to references [8] and [9]. While the relative arrival time of ground accelerations shall have a linear relationship with phase difference owing to Equation (4). Therefore, the relative arrival time has the same relationship with Fourier amplitude as well. However, the unit of relative arrival time reveals the physical meaning of this relationship. To illustrate this special relationship, a couple of ground motion records and their scatter diagrams of relative arrival time and Fourier amplitude are shown in Fig. 2.

The relative arrival times shown in Fig. 2 are all computed using DFT method. It is shown that the wave groups with larger amplitudes arrive around the same time, which explains the strong non-stationary properties of strong ground motions. The strong shocks of Chi-Chi and Big Bear earthquakes occur, respectively, around 50s and 25s. The wave groups with large amplitudes of these two earthquakes arrive around the same time. Wenchuan earthquake, different from Chi-Chi earthquake, has two strong shocks, as shown in Fig. 2. The stronger one occurs around 50s, while the weaker one occurs between 100s and 150s. According to the scatter diagram of Wenchuan earthquake, wave groups with large amplitudes arrive around 50s and 125s, which strongly demonstrate the rationality of DFT method. The relative arrival time for lots of records are analyzed, which reveals the same properties. So it is not a coincidence though it can't be explained why $\Delta\varphi$ has a range of $(-2\pi, 0)$.

The relationship of relative arrival time against Fourier amplitude not only gives the reason why strong ground motions behaves non-stationary processes, but also reveals the temporal-dependence of phase. The Fourier amplitude spectrum governs the intensity of wave group, and the phase spectrum governs the arrival time of the wave group. The intensity and the arrival time are thus the two critical components representing the time series of ground accelerations.

As mentioned above, phase difference has a linear relation with the relative arrival time, so the distribution of phase difference has a same shape with that of relative arrival time. Considering the probability distribution of relative arrival time, the time with large probability represents the concentrated arrival time of wave groups. So the relationship between the distribution of phase difference and wave shape shows that strong shock occurs when wave groups arrive intensively. Meanwhile, the phase spectrum determines the arrival time of every wave group and the duration of whole strong ground motion due to temporal-dependence of phase.

4. Simulation of strong ground motions

The relationship between relative arrival time and Fourier amplitude provides a ready manner to simulate strong ground motions. In this paper, the strong ground motion is treated as a time series that is completely described by its Fourier transform. Thus, if the Fourier amplitude spectrum and phase spectrum are specified, the ground motion is obtained through the inverse Fourier transform given by

$$a(t) = \sum_{k=1}^{N} A_k \cos(2\pi f_k t + \varphi_k) \tag{5}$$

where A_k and φ_k denote Fourier amplitude and phase, respectively, and f_k is the corresponding frequency. The Fourier amplitude spectrum of ground motion has been studied extensively, which is newly represented by a physical random function model [12]. While the modeling of Fourier phase spectrum of ground motion still remains a challenge due to its complexity. A new method based on the relative arrival time is proposed in this paper to represent phase spectrum.

4.1 Modeling of Fourier amplitude spectrum

The physical random function model of Fourier amplitudes consists of 3 basic parts; say Equation (6) which is derived by Wang and Li [12]: the source amplitude A_s, transfer function of path H_{Ap} and transfer function of site H_{As}. These parts are derived from Brune's circular dislocation source [13], damping dissipation energy of homogeneous elastic medium and equivalent single degree-of-freedom system of site, respectively.

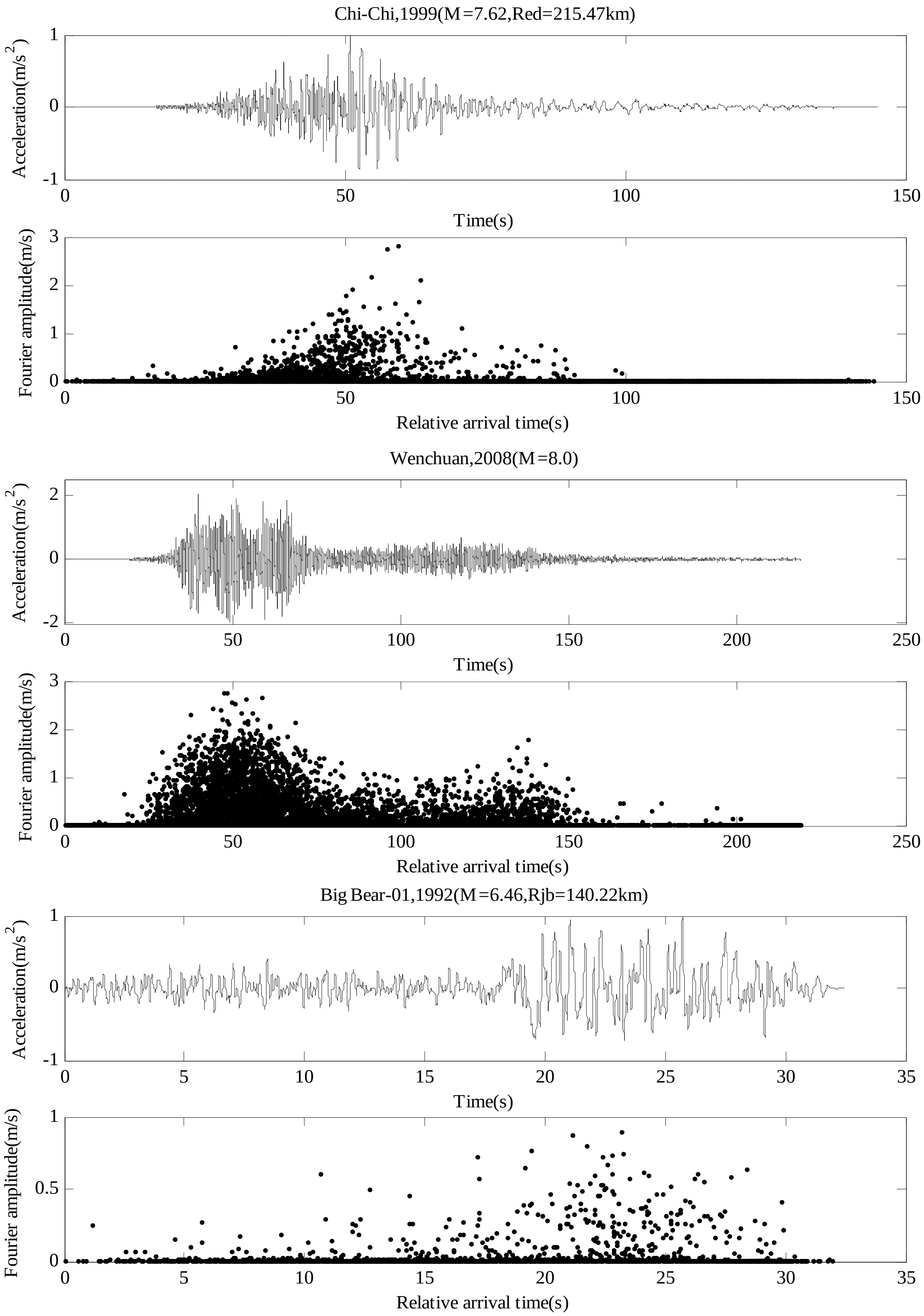

Fig. 2 Scatter diagrams of relative arrival time and Fourier amplitude(Frequency range is stated in appendix)

$$A = A_s \cdot H_{Ap} \cdot H_{As} \tag{6}$$

where

$$A_s = \frac{A_0}{\omega\sqrt{\omega^2 + \left(\frac{1}{\tau}\right)^2}} \tag{7}$$

$$H_{Ap} = \exp(-K\omega x) \tag{8}$$

$$H_{As} = \sqrt{\frac{1+4\xi_g^2(\omega/\omega_g)^2}{\left[1-(\omega/\omega_g)^2\right]^2 + 4\xi_g^2(\omega/\omega_g)^2}} \tag{9}$$

in which ω denotes circular frequency; x denotes the distance from source to local site; $A_0, \tau, \xi_g, \omega_g$ denote parameter of source intensity, parameter of Brune's source, predominant damping ratio of site and predominant circular frequency of site, respectively, which are the basic random variables representing the randomness inherent in the Fourier amplitude spectrum. The 4 basic random variables could be identified using the data of ground acceleration records.

4.2 Modeling of Fourier phase

Thráinsson and Kiremidjian [8] proposed a method to simulate earthquake ground motion considering the dependence of phase angle differences on Fourier amplitude. In their study, amplitudes are classified into three categories: small, intermediate and large. They found that in each category, the distribution of phase angle differences was different. Though the relationship between phases and amplitudes is utilized, the categories results in excessive parameters in the modelling scheme.

In this paper, all the relative arrival times uniformly distribute in different time range, which relies upon the shape of scatter diagram between relative arrival time and Fourier amplitude. Phase angles are derived from the integral of relative arrival time then. Only two parameters, i.e. strong shock duration and median arrival time, are needed in this method to simulate the phase spectrum.

While most of the earthquake ground motions have only one strong shock, revealing that the scatter diagram between relative arrival time and Fourier amplitude is unimodal, as shown in Fig. 3(Chi-Chi,1999). So a normal distribution curve could thus be used to represent the envelope of the scatter diagram. In Fig. 3, T denotes strong shock duration, and t_m denotes median arrival time.

The values of T and t_m are first calculated from ground acceleration records. It is indicated that one record corresponds to one set of T and t_m. According to the Plancherel theorem, T is defined as the range of time period corresponding to the most concentrated energy given by Equation (10) and (11); t_m is defined as the relative arrival time corresponding to the biggest amplitudes. To identify the value of t_m from recorded time history, the range of relative arrival time is uniformly separated into 100 parts. The amplitudes' quadratic sum corresponding to every part of relative arrival time is readily to be calculated. The mean value of relative arrival times is denoted by t_m of which the part corresponds to the max quadratic sum.

$$\sum_{t_e=t_i}^{t_j} A^2(t_e) = 95\% \sum_{t_e=\min(t_e)}^{\max(t_e)} A^2(t_e) \tag{10}$$

$$T = \min(t_j - t_i) \tag{11}$$

When simulating ground motion time histories, the mean value of the normal distribution curve equals to t_m, the standard deviation equals to $2T$/3, peak value of the normal distribution curve is defined as 2/3 of the largest amplitude which is derived from the physical random function model. All the parameters are determined completely empirical. Since the envelope is determined, every amplitude A corresponds to two time points: t_1 and t_2. Thus, the relative arrival time corresponding to this amplitude is uniformly distributed in $[t_1, t_2]$. So far, the amplitudes and phase angles are both determined, then the time series of earthquake ground motion can be obtained by inverse Fourier transform.

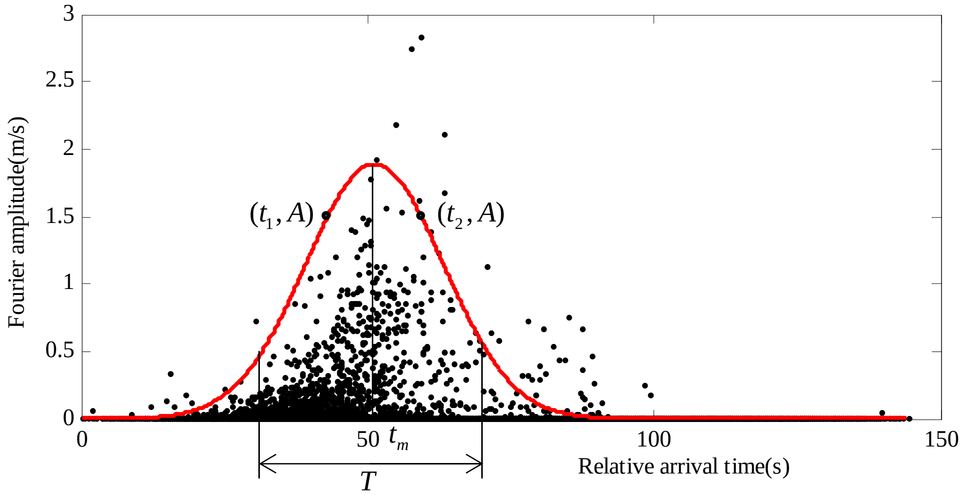

Fig. 3 Determination of relative arrival time based on Fourier amplitudes

(Frequency range is stated in appendix)

4.3 Simulation of ground motion accelerograms

For illustrative purposes, several tests are performed to evaluate the fit goodness of simulated ground motions to recorded ones. Two ground acceleration records are investigated herein, which expose to be large differences in duration and in spectral components from each other. The two records, meanwhile, both have one strong shock which is different from Wenchuan records. Although the presented method is not suitable for simulating ground acceleration with two strong shocks, fortunately most of the records just have single strong shock.

The strong shock durations and median arrival time of Chi-Chi ground acceleration are 23.35s and 50.9s, respectively; while they are 2.2s and 2.6s for Bishop ground acceleration, as shown in Fig. 4, where simulated processes with the same strong shock durations and median arrival time to the records are presented as well. The acceleration response spectra of the records and of 10 simulations are shown in Fig. 5. It reveals that the simulated samples match well with the records both in time histories and spectral properties.

5. Conclusions

The relationship between amplitudes and frequency-dependent phase derivatives is further illuminated in this article, by which the time property of phase is proved. The methods of simulating phases and ground motions are derived and demonstrated to be effective.

More work can be done based on this method, the strong shock duration T and median arrival time t_m can be regarded as random variables, which can be identified from recorded ground motions. The random variables of amplitude function can also be determined in the same way. The distribution of every random variable can be determined from lots of recordings if they are regarded as independent variables. Thus, ground motion time history can be simulated based on six random variables which can be determined by point generation.

The advantage of this method is that the phase can be derived based on the amplitude and only two parameters with clear physical meaning. However, the determination of parameters' value is empirical. Even so, it provides a rapid simulation of acceleration time histories for engineering purposes over a wide range of magnitudes and distances for various site types.

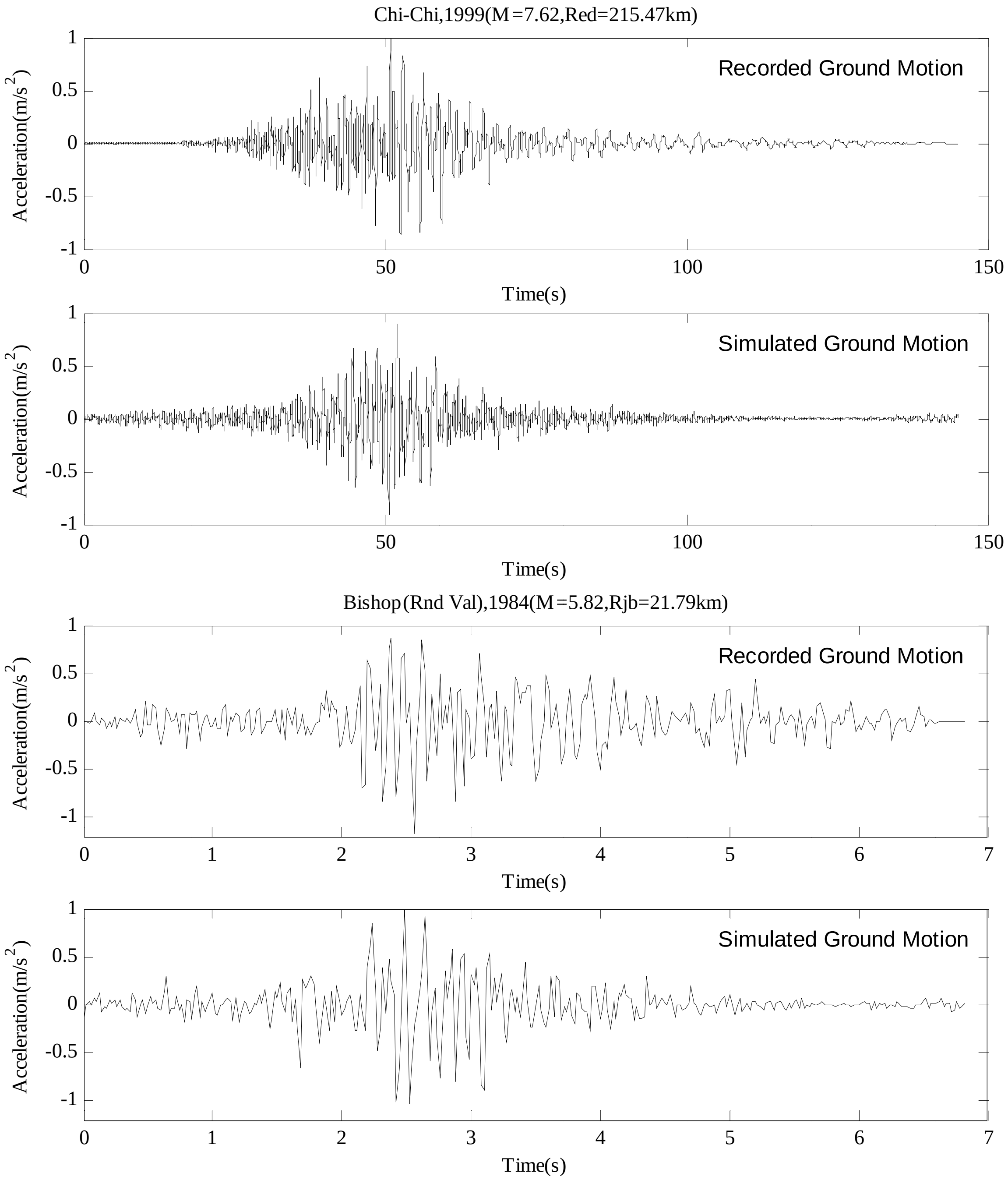

Fig. 4 Ground motion accelerograms and the corresponding simulated processes.

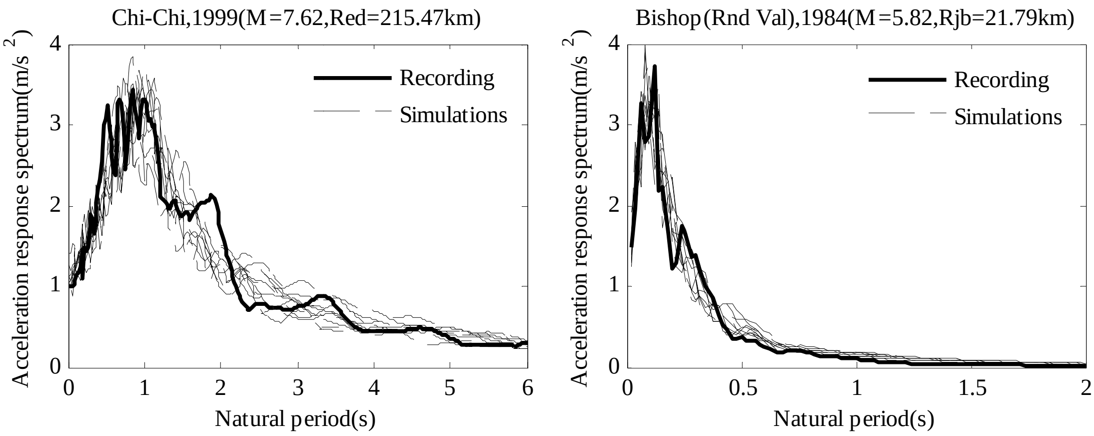

Fig. 5 Acceleration response spectrum of records and simulations (damping ratio 5%).

6. Appendix

Table 1 – Information of earthquake records used in numerical cases

Number	Earthquake Name	Year & Date	Magnitude	Station Name	Epicentral Distance (Km)	Time interval(s)
1	Bishop(Rnd Val)	1984-11-23	5.82	USGS 1661 McGee Creek	21.93	0.005
2	Chalfant Valley-02	1986-07-21	6.19	CDMG 54100 Benton	31.25	0.005
3	Central Calif-02	1960-01-20	7.28	USGS 1028 Hollister City Hall	8.01	0.005
4	El Mayor-Cucapah	2010-04-04	7.2	Chihuahua	20.63	0.005
5	Chi-Chi	1999-09-20	7.62	KAU043	215.47	0.005
6	Wenchuan	2008-05-12	8.0	Code of station: 051AXT	--	0.002
7	Big Bear-01	1992-06-28	6.46	LA-1955 1/2Purdue Ave. Bsmt	149.98	0.005

Note: i) All the recorded time series except Wenchuan wave are download from http://ngawest2.berkeley.edu/
ii) The Fourier amplitude spectrum, Fourier phase spectrum and relative arrival time of one record have the same frequency range and frequency interval in this article. The frequency range is $[0,1/2\Delta t]$, and the frequency interval is $1/T$, where T and Δt denote the ground motion duration and acceleration time interval.

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