



## EVALUATION OF RC DOME DYNAMIC PARAMETERS BASED ON FULL-SCALE TESTING BY VIBRATION MACHINE

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### **Abstract**

Analysis of natural vibration periods, modes and corresponding damping ratios of long-span spatial structures can provide fast and reliable indication about their real state. Experimental investigation of such structures under vibration loading can provide the required information about their current dynamic parameters that can be used to calibrate theoretical models, develop appropriate modelling techniques, etc. Sometimes the structural assessment can include non-destructive testing and additionally finite element modeling. Based on the obtained results, proper methods for retrofitting of existing long span spatial structures, damaged by earthquakes, impact and other dynamic excitations can be developed. In this study a spherical RC dome with a 30 m diameter has been investigated experimentally and theoretically. The study was aimed at developing and verifying an accurate theoretical model for assessing dynamic parameters of roofing RC domes. The investigated structure was subjected to dynamic loads, generated by a vibration machine. It allows testing structural dynamic response at various loading intensity levels up to plastic deformations or even local damages. The available experimental results are compared with analytical ones and with those of preliminary finite elements analysis. It is shown that the results, obtained by all methods (experimental, analytical and FE analysis), are very close. The theoretical model that was developed can be recommended for design of roofing RC domes.

*Keywords: RC dome; vibration machine; dynamic parameters; dynamic loading intensity level*



## 1. Introduction

Monitoring of long span spatial structures is an important topic that has found a relief reflection in results of modern studies in the last decade. Testing a structure under vibration or impulse loading can provide valuable information about its dynamic parameters. This information can be further used to calibrate theoretical models, develop modelling techniques, and verify theoretically predicted damage. Such testing yields information about the dynamic parameters like natural vibration periods, vibration modes and damping ratios [1]. One of the advantages of this approach is that it enables to estimate nonlinear structural behaviour of a full scale structure.

Dynamic characteristics of long span spatial structures depend on as-built or current conditions. Sometimes the structural assessment can include non-destructive testing and additionally finite element modeling. Impact vibration experiments of concrete spherical shell of the planetarium in Funabashi, Japan were performed [2]. The shell has a diameter of 23.2 m, a rise of 8.11 m and a 12.6 m radius of curvature; the shell thickness is 8 cm. Structural velocities in the vertical and two horizontal directions were measured. Damping characteristics of this shell and other spatial structures have been analyzed. It was reported that the damping ratio of this spherical shell is 2.8% and an empirical expression for calculating this ratio depending on the span of spatial structures was proposed.

Long span structures that were retrofitted, using modern seismic protection systems, like base isolation, dampers, etc., also need monitoring in order to check the efficiency of such protection. For example, monitoring of “Our Lady of Tears Shrine” church in Syracuse, Italy, was recently performed [3]. The height of the structure is 74.3 m from the covering plane. The church consists of a conical dome, which rises from a base ring, supported by 22 columns. The upper part of the church was seismically isolated from the lower structure, by substituting the pre-existent bearings with new sliding seismic isolators [4]. It was reported that experimental results have fundamental importance to calibrate and validate the numerical model of the structure.

As known, if a long span structure is constructed on rather soft soils, the seismic wave length can be of the same order like the length of the structure. In such case out of phase supports’ vibrations can appear not only in horizontal, but in vertical direction too. This phenomenon has taken place, for example, in 1952 in one of the buildings analyzed and described by Housner [5]. The out of phase supports’ vibrations can be superposed by different forms of natural vibrations of the structure, which can increase or decrease the overall vibrations’ amplitudes. Therefore the phenomenon should be taken into account in design of long span structures.

In the frame of the present study dynamic behaviour of a spherical RC dome in Dushanbe (Tajikistan) is measured, analysed and discussed. The dome was excited by a vibration machine that has applied vertical dynamic loads. The experimental results are compared with analytical ones.

## 2. Research aims and scope

Experimental results on dynamic behaviour of a long span RC dome are presented and discussed. The main aim of dynamic testing is obtaining dynamic characteristics of the investigated structure and to select the dominant direction of vibrations, in which the structure should be designed. These characteristics allow calculation of corresponding dynamic loads, acting on the structure.

Analytical expression for assessment of natural vibration frequencies of the dome is proposed. This is one of the first attempts for taking into account simultaneously the natural vibrations of long span structures analytically. The analytical results were verified using the experimental data, obtained in the frame of this study, as well as the available results for other similar structures.

## 3. Vibration testing of a long span RC dome

A dome of a public building in Dushanbe (Tajikistan) was tested under dynamic loading, applied by a vibration machine. The general view of the tested structure is presented in Fig. 1.



Fig. 1 – A general view of the tested dome

The dome is a shallow thin walled spherical RC shell. Its outside diameter is 30 m and the inside one is 26.5 m, the supporting ring's width is 1.75 m. A structural scheme of the RC dome is shown in Fig. 2. The height of the dome is 3.6 m. The dome has a thickness of 6 cm and a system of meridian and parallel ribs. The meridian ribs limit the sector with a central angle of  $12^\circ$  and come to the upper ring with a diameter of 3.3 m. This ring has a 19 cm height (excluding the dome thickness) and 20 cm width rib along its perimeter. This ring is reinforced by two longitudinal periodic shape steel rods with a diameter of 14 mm and links with a diameter of 8 mm and a step of 18 cm. The part of the dome, located inside this ring, is reinforced by a net made of 8 mm diameter steel rods placed with a 20 cm step in both directions.

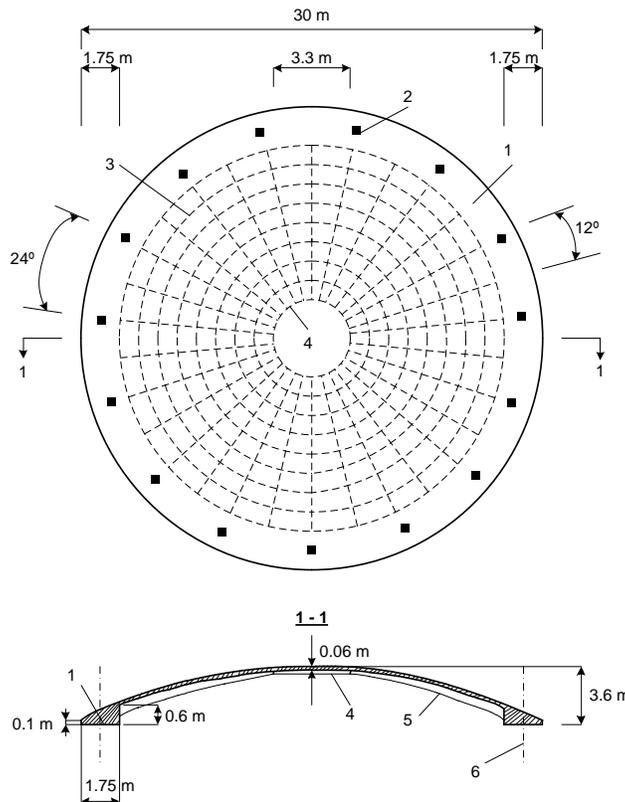


Fig. 2 – A structural scheme of the investigated dome: 1 – supporting ring; 2 – columns; 3 – parallel ribs; 4 – upper ring; 5 – meridian ribs; 6 – column axes

The meridian ribs of the dome have a variable height (from 14 to 44 cm), increasing in the direction of the supporting ring. The ribs' width is 8 cm. These ribs are reinforced by two 25 mm periodic shape rods and 8 mm



links, placed with a 10 – 15 mm step. The parallel rings are located with a step of 1.5 m and have a section of 8 x 8 cm. These rings are reinforced by 18 mm periodic shape rods and 8 mm links, placed with a step of 20 cm.

The dome itself is reinforced between the ribs by a net, made of 6 mm diameter steel rods placed with a 15 cm step. The supporting ring is reinforced by a net, made of 8 mm steel rods and a spatial steel frame made of steel rods with a diameter of 28 mm and a 10 cm step along the height of the ring. The height of the supporting ring varies from 10 to 60 cm.

The dome is supported by 15 supports located along the perimeter as shown in Fig. 2. The structure is designed for a zone with seismic activity corresponding to degree 8 according the Modified Mercally scale. Therefore experimental investigation of the dome's dynamic parameters (natural vibration modes and corresponding periods, damping ratios) in vertical and horizontal directions was important for assessment of the structure's seismic resistance.

The dynamic load was applied to the shell using a vibration machine (VM), located at the centre of the dome. The VM caused vertical and horizontal vibrations of the structure. The frequency of the applied dynamic load was smoothly changed to cause resonant vibrations, allowing identification of the dome's natural dynamic parameters.

For recording the vibrations of the dome velocities (OSP) and displacements (K001) were measured. A scheme of the measuring equipment location is shown in Fig. 3a. Two types of devices were used in the experiments: K001 for measuring accelerations and OSP for velocities. The dynamic parameters of the dome were measures in X and Z directions.

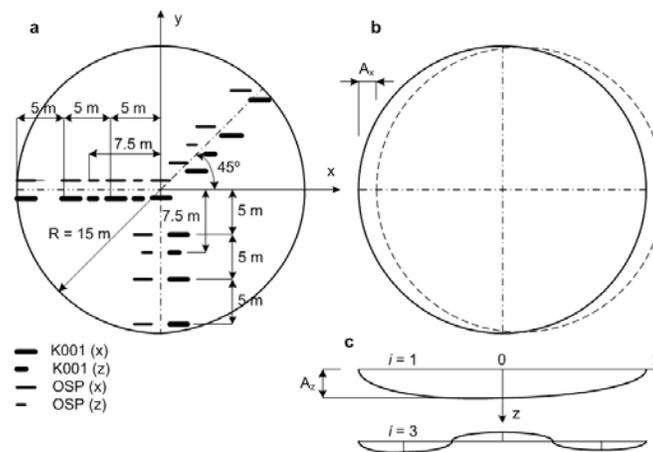


Fig. 3 – Location of measuring equipment on the dome (a); natural mode shapes in horizontal (b) and vertical (c) directions

The eccentric masses, used in the VM, were selected so that nonlinear behaviour of the dome would not occur. Only the first vibration mode was identified in the horizontal direction (Fig. 3b) and two symmetrical modes – in the vertical direction (Fig. 3c). Table 1 presents the results of dynamic testing.

Following the obtained results, the vibrations of the structure in the horizontal direction can be simulated as for a single degree of freedom system. In other words, the RC spherical shell itself is stiff enough in horizontal direction, in spite that it is shallow and thin-walled. At the same time, in the vertical direction the first two symmetrical natural vibration modes were identified. It shows that this direction is the design one. Proving this fact was the main goal of this study that was focused on vertical vibrations of long span structures.



#### 4. Analytical investigation of long span domes' vertical vibrations

One of the first analytical investigations of shallow spherical shells' natural frequencies and mode shapes was performed by Reisner [6]. Practical aspects in dynamic theory of RC shells were developed [7] and further applied for seismic design [8]. Following these approaches, the expression for calculating the natural vibration frequencies was proposed:

$$\omega_{mn}^2 = [g / (\gamma h)] [D (\lambda_n^2 + \mu_m^2)^2 + E h / R^2] \quad (1)$$

where  $m$  and  $n$  are number of half-waves in the vibration modes in X and Y directions, respectively; R is the shell's radius of curvature; D is the cylindrical stiffness of a shell that is obtained as follows:

$$D = E h^3 / [12 (1 - \nu^2)] \quad (2)$$

Here E is the elasticity modulus of concrete;  $h$  is the shell thickness;  $\nu$  is the Poisson's coefficient. Additionally,

$$\lambda_n = n \pi / d \quad (3)$$

$$\mu_m = m \pi / d \quad (4)$$

where  $d$  is the dome's diameter.

It can be shown that the first term,  $D (\lambda_n^2 + \mu_m^2)^2$ , in Eq. (1) presents the contribution of bending, whereas the second term,  $E h / R^2$ , presents the contribution of membrane forces. It will be further shown that for thin walled shells the first term is negligible, compared to the second one.

Let analyse the above discussed spherical RC dome of a public building in Dushanbe using Eq. (1). Taking into account that the shell is spherical, it will vibrate according to symmetric modes only and therefore  $n = m = 1, 3$ , etc. Considering that for concrete  $\nu_c^2 \ll 1$ , Eq. (1) takes the following form:

$$\omega_{mn}^2 = [g/(\gamma h)] [(E_c h^3/12) (2 \lambda_n^2)^2 + E_c h / R^2] = [(g E_c / \gamma)] [(h^2/3) \lambda_n^4 + 1/R^2] \quad (5)$$

For  $n = m = 1$  and  $1/R^2 = 64 f^2 / d^4$ , where  $f$  is the shell's height.

$$\omega_{11}^2 = (g E_c / (\gamma d^4)) [(\pi^4 h^2 / 3) + 64 f^2] = [g E_c / (\gamma d^4)] (32.47 h^2 + 64 f^2) \quad (6)$$

For thin-walled shells  $h^2 \ll f^2$ . Therefore  $\omega_{11}^2 = 64 f^2 E_c g / (\gamma d^4)$  and in this case

$$\omega_{11} = (8 f / d^2) (E_c g / \gamma)^{0.5} = k / (E_c g / \gamma)^{0.5} \quad (7)$$

where  $k = 1 / R$  is the shell curvature. Correspondingly, the natural vibration period

$$T_{11} = 2 \pi / \omega_{11} = [\pi d^2 / (4 f)] [\gamma / (E_c g)]^{0.5} \quad (8)$$

Let define

$$c = 0.25 \pi [\gamma / (E_c g)]^{0.5} \quad (9)$$

Then  $1/c$ , m/s is a parameter of dynamic waves' velocity in concrete. Therefore

$$T_{11} = c d^2 / f \quad (10)$$



As known, the modulus of elasticity of concrete depends on concrete creep and the surrounding environment's temperature:

$$T_{11} = f(E_{c \text{ red}}) \quad (11)$$

where  $E_{c \text{ red}}$  is the reduced value of concrete elasticity modulus, taking into account the above mentioned factors [9]. Therefore

$$T_{11} = 2 \pi / \omega_{11} = [\pi d^2 / (4 f)] [\gamma / (E_{c \text{ red}} g)]^{0.5} \quad (12)$$

For  $n = m = 3$

$$\omega_{33}^2 = [g E_{c \text{ red}} / (\gamma d^4)] (2630 h^2 + 64 f^2) \quad (13)$$

Let compare the analytical and experimental results for the spherical RC dome of a public building in Dushanbe ( $d = 30 \text{ m}$ ,  $f = 3 \text{ m}$ ,  $\gamma = 24 \text{ kN/m}^3$ ,  $g = 9.81 \text{ m/s}^2$ ,  $E_c = 23800 \text{ MPa}$ ;  $h_m \approx 10 \text{ cm}$ ). Following [9],  $E_{c \text{ red}} = 0.375 E_c$ . For  $n = m = 1$  the corresponding natural vibration period following Eq. (31) is:  $T_{11} = [3.14 30^2 / (4 3)] [24 / (0.375 23800 10^3 9.81)]^{0.5} = 235.5 5.185 10^{-4} = 0.122 \text{ s}$ .

For  $n = m = 3$  and following Eq. (13),  $2630 h^2 + 64 f^2 = 10^5 (2.63 + 57.6)$ . Similarly to the case when  $n = m = 1$ , the first term is negligible, compared to the second ( $2.63 \ll 57.6$ ), therefore the first term can be neglected. Hence,  $\omega_{33} \approx \omega_{11}$ , or  $T_{33} = 0.122 \text{ s}$ . The measured natural vibration period value  $T_{11 \text{ exp}} = 0.106 \dots 0.220 \text{ s}$  (see Table 1).

Table 1 – Experimentally obtained natural dynamic parameters of the dome for the first natural vibration modes in horizontal and vertical directions

Dynamic parameter	Horizontal direction	Vertical direction
Natural period, sec.	0.22	0.106...0.22
Damping ratio	0.07	0.04...0.10
Peak displacement, mm	0.25	0.10...0.37
Peak velocity, cm/sec.	0.94...0.98	0.63...2.08

The theoretical value is within the experimentally obtained range. The eccentric masses in the dynamic experiments, carried out using the vibration machine, varied from 26.8 to 44 kg that caused an increase in nonlinear deformations of the tested dome, and correspondingly affected the measured values.

Similar analysis was carried out for an RC dome that was tested in Georgia [10]. The dome has the following parameters: diameter  $d = 19.65 \text{ m}$ , rise  $f = 1.6 \text{ m}$ , shell thickness  $h = 8 \text{ cm}$ , concrete class C25, concrete modulus of elasticity  $E_c = 23800 \text{ MPa}$ . Considering the creep of concrete,  $E_{c \text{ red}} = 0.5 0.87 E_c = 0.435 E_c$ . Then the natural vibration period of the dome can be found as:  $T_{11} = [3.14 19.65^2 / (4 1.6)] [24 / (0.435 23800 10^3 9.81)]^{0.5} = 0.092 \text{ s}$ . The experimentally obtained dynamic parameters of this dome [11] are:  $T_{11} = 0.097 \dots 0.114 \text{ s}$ . These results are rather close to the analytically obtained values.

Dynamic characteristics of a pre- cast pre-stressed spherical RC dome were investigated [2]. The dome has a radius of 11.60 m, a rise of 8.11 m and a 12.60 m radius of curvature. The shell thickness is 8 cm, the measured concrete Young's modulus values were  $(2.9, 3.3 \text{ and } 3.7) \times 10^4 \text{ MPa}$ . The concreted mass density was  $38 \text{ kN/m}^3$  and the Poisson's ratio equalled 0.2. The experimental frequency, corresponding to  $E_{c \text{ max}}$ , was 65.93 Hz.

For analysis purposes the following data was used:  $d = 23.2$  m,  $f = 8.11$  m,  $h = 8$  cm, concrete class C25, dome's curvature  $k = 0.079$  1/m. Taking into account concrete creep and humidity of the surrounding environment [9],  $E_{c \text{ red}} = 0.5 \cdot 0.87 E_c = 0.435 E_c = 0.435 \cdot 37000 = 16095$  MPa, the natural vibration period of the dome (see Eqs. 9 and 10)  $c = 0.25 \pi [\gamma / (E_{c \text{ red}} g)]^{0.5} = 0.78 [38 / (16095 \cdot 10^3 \cdot 9.81)]^{0.5} = 3.827 \cdot 10^{-4}$  s/m. Then  $T_{11} = c d^2 / f = 3.827 \cdot 10^{-4} \cdot 23.2^2 / 8.11 = 0.025$  s.

The experimental value of  $T_{11} = 0.015$  s. The difference between the analytical and experimental results are because the tested dome is pre-cast (it means that there are ribs around the pre-cast elements) and pre-stressed (in this case the elements' stiffness significantly increases).

### 5. Finite elements analysis of the dome natural vibration period

The finite elements (FE) analysis was aimed at numerical modelling of the investigated spherical RC dome in Dushanbe, excited by a VM. The numerical results are compared with the experimentally and analytically dynamic parameters of the dome that were obtained before.

ANSYS software was used for the dome analysis. SOLID65 element with three degrees of freedom was used for creating the FE model. The model of the dome includes 286785 nodes and 209836 elements. The supporting columns have a section of 40 x 40 cm and are located along the perimeter of the dome each 24°. The view of the geometrical model is shown in Fig. 4.

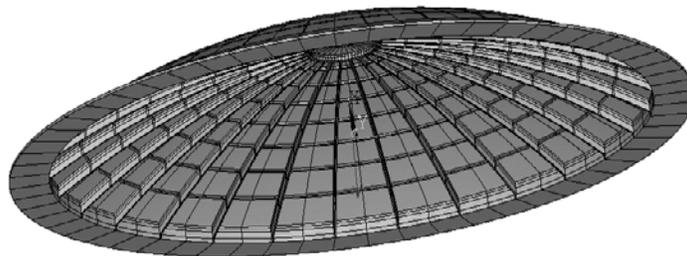


Fig. 4 – The view of the geometrical model for FE analysis

Table 2 demonstrates results obtained by all methods that were used in the present study. The experimental value is 0.106...0.220 sec (see Table 1) and the analytical result is 0.122 sec. As it follows from Table 2, results obtained by all methods are very close.

Fig. 5 presents the first and third natural vibration modes of the structure. Corresponding frequencies are 7.55 Hz and 9.12 Hz and natural vibration periods are 0.132 sec and 0.110 sec., respectively. As the analytical approach is rather simple, it can be used for design of such RC domes.

Table 2 – Natural vibration frequencies in vertical direction, sec.

Vibration mode	Experimental	Analytical	FE analysis
1	0.22	0.122	0.132
3	0.106	0.122	0.110

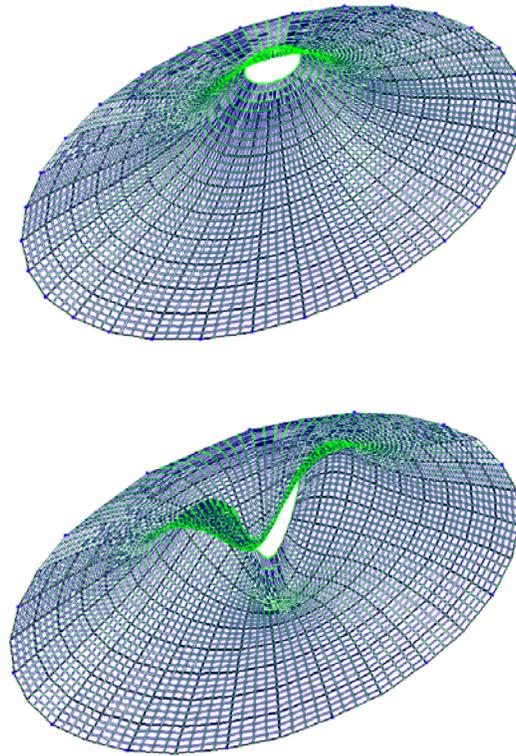


Fig. 5 – Mode shapes 1 and 3

## 6. Conclusions

Modern seismic design codes and provisions are mainly developed for tall buildings, for which horizontal loads are critical. Recommendations on seismic design of long span spatial structures in vertical direction are very limited. The current study presents results of full-scale RC dome testing under dynamic loadings and further analytical and FE investigation.

The dynamic loading for investigating the spherical RC shallow thin-walled dome with a 30 m diameter was applied by a vibration machine. Available experimental data on dynamic parameters of other RC domes were also used.

Two natural vibration modes in vertical direction were identified experimentally - the first and third symmetrical modes were evident, as the structure and the applied dynamic load were symmetrical.

The obtained experimental data were generalised analytically: the natural vibration periods were calculated. Further FE analysis was performed. Comparison of experimental, analytical and FE results shows that the analytical expressions allow predicting of the corresponding dynamic parameters with satisfactory accuracy.

Dynamic parameters that were obtained in the frame of the present study may be used for developing corresponding methods for proper design long span RC structures to vertical dynamic loads.

## 6. References

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