16th World Conference on Earthquake, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 624

Registration Code: S-B1462860243

A new reliability method combining Kriging and Probability Density Evolution Method

Zhongming. Jiang[1], Jie. Li [2]

[1] *Ph.D. Candinate, School of Civil Engineering, Tongji University ,20101602156@cqu.edu.cn.com*
Position, affiliation, e-mail address
[2] *Ph.D, Distinguished Professor, School of Civil Engineering, Tongji University, lijie@tongji.edu.cn*
…

Abstract

Dynamic response analysis of non-linear structures involving random parameters under earthquakes has been an important and challenging problem for a long time. As a newly developed method, probability density evolution method (PDEM) is capable of capturing the instantaneous probability density function (PDF) of stochastic dynamic responses of structures. However, as the demand for accuracy of numerical model increase, engineering problems involve more and more complex computer codes and the calculating of the reliability of structure may require very time-consuming computations. Therefore, minimizing the number of calls to the numerical models become one of the most important challenges in this area. Many response surface methods (RSM) such as Least Square Regression, Polynomial Chaos, Support Vector Machine are introduced to solve this problem. Recently, the Gaussian process regression (GPR) or so called Kriging method has received increasing attention in the field. Unlike most response surface methods, Kriging method is an exact interpolation method and capable of giving the confidence of its result. The aim of this paper is to propose a new approach named K-PDEM based on probability density evolution method and Kriging metamodel to assess the reliability of structure under earthquake. The result shows that the new method is efficient and accurate for calculating the reliability of structure especially when the number of calls to numerical model is small.

Keywords: response surface methods, Kriging, probability density evolution method

1. Introduction

Stochastic dynamic analysis of nonlinear structures is an open research question especially when the degree of randomness is large. Because of the great difficulty in analytic calculation, a variety of numerical methods such as the Monte Carlo simulation (MCS) method[1,2], the random perturbation technique[3] and the orthogonal polynomials expansion method[4,5] have been introduced to satisfy the requirements of engineers and scientists. When the problem comes to the highly nonlinearity, Monte Carlo simulation is the most applicable method. However, the computational effort of MCS is sometimes unbearable for the stochastic analysis of sophisticated structure numerical model. For random perturbation technique, the computational effort is relatively lower compare to the MCS, but the application of the technique is restricted by the small variation assumption about the random parameters. The orthogonal polynomials expansion method in some ways achieved the balance between the computational effort and applicability.

In general, the methods which can provide the high-order statistical quantities and has acceptable computation effort are not available yet. Recently, On base of the principle of preservation of probability, a series of numerical probability density evolution methods has been developed[6]. The researches shows that the probability density evolution method (PDEM) can meet the requirement of fair accuracy and efficient when the dimension of random parameters and the number of the representative point set is acceptable [7].
However, for complex and time-consuming structure model, the number of representative points is still relative large for engineering purpose[8]. To further decrease the number of the representative points on the premise of reasonable accuracy, the surrogate model method are introduced. The surrogate model can be roughly divide into two categories: parameter surrogate model and nonparametric surrogate model. The earlier application of surrogate model is parameter regression[9]. However, the result of the parameter surrogate model is sensitive to the experiment design and the application of this method is limited by its weak generalization ability[10,11]. To overcome the above limitations, nonparametric surrogate models have been proposed such as Polynomial Chaos[12], Neural Network[13] and Support Vector Machine[10].

Aside from these surrogate models, Kriging method has been intensively investigated because of its stochastic property and extensive applicability. In the 1970s, Krige and Matheron[14] first developed Kriging method in the field of geostatistics. Unlike most response surface methods, Kriging method is an exact interpolation method and capable of giving the confidence of its result. The applications of Kriging method to structures analysis are rather recent. Romero introduced the Kriging method in civil engineering for solving stochastic problem in 2004[15]. According to his result, Kriging method is more efficient than polynomial regression and finite-element interpolation. Following their pioneer work, Kaymaz[16] investigates the effectiveness of Kriging method and compare it to classic response surface method. The result shows that the reliability results can be improved by carefully choosing the Kriging parameters.

In this article, a new method are proposed for stochastic dynamic analysis result by introducing Kriging method into the numerical calculation framework of PDEM. The idea of this method can be divided into two stage. The first stage of the method consists of selecting the representative points set with small size and training the Kriging estimators at each time step. The training of Kriging estimator is based on the MATLAB toolbox called DACE[17]. Another set of enriched representative points with large size is selected and the response of each time step can be achieved by using Kriging estimator. Then, the probability density function (PDF) of the responses can be calculated by the application of traditional PDEM.

This article is framed in five sections. The theory and numerical implementation of probability density evolution method is briefly introduced in Section 2. Following this, the basic idea of Kriging method is introduced in Section 3. Section 4 propose the framework of new method (K-PDEM) and its details. In Section 5, three numerical examples are investigated to validate K-PDEM. The result is compared to theoretical solution and Monte Carlo Simulation. According to the test results, K-PDEM shows fair accuracy when the number of calculation to the response is relatively small compared to the traditional PDEM.

2. PROBABILITY DENSITY EVOLUTION METHOD

For simplicity, a dynamical system with stochastic parameters is written as

$$\dot{\mathbf{X}}(t) = \mathbf{\Phi}(\mathbf{X}, \mathbf{\Theta}, t) \tag{1}$$

with the initial condition

$$\mathbf{X}(t)\big|_{t=0} = \mathbf{x}_0 \tag{2}$$

where $\mathbf{X} = (X_1, X_2, \cdots, X_N)^{\mathrm{T}}$ is the state vector consisting of N components $X_i(t)$, $\mathbf{\Phi} = (\Phi_1, \Phi_2, \cdots, \Phi_N)^{\mathrm{T}}$ the dynamic operator, $\mathbf{\Theta} = (\Theta_1, \Theta_2, \cdots, \Theta_s)^{\mathrm{T}}$ the s-dimensional random vector with the joint probability density function (PDF) $p_{\mathbf{\Theta}}(\mathbf{\theta})$, $\mathbf{x}_0 = (x_{0,1}, x_{0,2}, \cdots, x_{0,s})^{\mathrm{T}}$ the initial value vector with the PDF $p_{\mathbf{X}_0}(\mathbf{x})$.

It is well known that, under certain regularity conditions, the solution of Eq. (1) and Eq. (2) exists and is unique. This solution is expressible in the form

$$\begin{aligned} &\mathbf{X}(t) = \mathbf{H}(\mathbf{\Theta}, t) \quad \text{or} \\ &X_j(t) = H_j(\mathbf{\Theta}, t), \quad j = 1, 2, \cdots, N \end{aligned} \tag{3}$$

Then, the generalized density evolution equation can be deduced from the principle of preservation of probability [6]

$$\frac{\partial p_{\mathbf{X\Theta}}(\mathbf{x}, \mathbf{\theta}, t)}{\partial t} + \sum_{j=1}^{N} \dot{X}_j(\mathbf{\theta}, t) \cdot \frac{\partial p_{\mathbf{X\Theta}}(\mathbf{x}, \mathbf{\theta}, t)}{\partial x_j} = 0 \tag{4}$$

with the initial condition

$$p_{\mathbf{X\Theta}}(\mathbf{x}, \mathbf{\theta}, t) = \delta(\mathbf{x} - \mathbf{x}_0) p_{\mathbf{\Theta}}(\mathbf{\theta}) \tag{5}$$

Because the difficulty in obtaining the close-from expression of $\dot{X}_j(\mathbf{\theta}, t)$, analytically solving the Eq. (4) extremely hard. However, as a partial differential equation, its numerical solution is usually available [6].

3. KRIGING METHOD

3.1 Kriging method

The Kriging methods is a class of basic best linear interpolator with statistical property. It is developed by Krige in geostatistics in the 1950s[27]. Then, Romero introduced the Kriging method into the structural reliability problems[15]. The basic form of the Kriging estimator can be expressed as

$$Z^*(\mathbf{u}) - m(\mathbf{u}) = \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha [Z(\mathbf{u}_\alpha) - m(\mathbf{u}_\alpha)] \tag{6}$$

where $\mathbf{u}$ is the vectors of estimation point, $\mathbf{u}_\alpha$ is the points near the estimation point, is the number of points which in the nearby area of estimation point, $m(\mathbf{u})$, $m(\mathbf{u}_\alpha)$ expected values of $Z(\mathbf{u})$ and $Z(\mathbf{u}_\alpha)$, $\lambda_\alpha(\mathbf{u})$ is the weight value of each $\mathbf{u}_\alpha$ for estimation location $\mathbf{u}$, different estimation location will receive different weight.

The basic idea of Kriging method is that the $Z(\mathbf{u})$ is a random field with a trend component $m(\mathbf{u})$ and a residual component $R(\mathbf{u}) = Z(\mathbf{u}) - m(\mathbf{u})$. The residual of predicted value of Kriging method at $\mathbf{u}$ is the sum of the weighted residuals of surrounding data points at $\mathbf{u}_\alpha$. So, the most crucial part of Kriging method is to determine the weights λ_α which minimize the variance of the estimator

$$\sigma_E^2(\mathbf{u})=\mathrm{Var}\{Z^*(\mathbf{u}) - Z(\mathbf{u})\} \tag{7}$$

under the constraint

$$\mathrm{E}\{Z^*(\mathbf{u}) - Z(\mathbf{u})\} = 0 \tag{8}$$

As the Kriging estimator has been divided into two parts

$$Z(\mathbf{u}) = R(\mathbf{u})+m(\mathbf{u}) \tag{9}$$

Suppose the residual component $R(\mathbf{u})$ is a stationary Gaussian process with zero mean

$$\mathrm{E}\{R(\mathbf{u})\} = 0 \tag{10}$$

and the covariance between point $\mathbf{u}$ and $\mathbf{u}+\mathbf{h}$

$$\mathrm{Cov}\{R(\mathbf{u}),R(\mathbf{u}+\mathbf{h})\}=\mathrm{E}\{R(\mathbf{u}) - R(\mathbf{u}+\mathbf{h})\}=C_R(\mathbf{h}) \tag{11}$$

where $C_R(\mathbf{h})$ is the correlation function. The definition of correlation function of different Kriging model is decided by the different input semivariogram model

$$C_R(\mathbf{h}) = C_R(\mathbf{0}) - \gamma(\mathbf{h}) \tag{12}$$

where $\gamma(\mathbf{h})$ is the semivariogram model.

There are three main kinds of Kriging methods, simple, ordinary and Kriging with trend. The difference between them is the assumption about the trend component $m(\mathbf{u})$. For convenience, all the following equations are based on ordinary Kriging theory. For ordinary kriging, the trend component $m(\mathbf{u})$ is constant in the local neighborhood of each estimation point. In this case, Eq.(14) become

$$Z^*(\mathbf{u})=m(\mathbf{u}) + \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha [Z(\mathbf{u}_\alpha) - m(\mathbf{u})]=\sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u}) Z(\mathbf{u}_\alpha) + \left[1 - \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u})\right] m(\mathbf{u}) \tag{13}$$

For ordinary Kriging, the sum of weights is requiring to be 1

$$Z^*(\mathbf{u})=\sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u}) Z(\mathbf{u}_\alpha) \text{ with } \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u}) = 1 \tag{14}$$

According to the rule of variance, the Eq.(15) can be written as

$$\begin{aligned}\sigma_E^2(\mathbf{u}) &= \mathrm{Var}\{R^*(\mathbf{u})\} + \mathrm{Var}\{R(\mathbf{u})\} - 2\mathrm{COV}\{R^*(\mathbf{u}), R(\mathbf{u})\} \\ &= \sum_{\alpha=1}^{n(\mathbf{u})}\sum_{\beta=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u})\lambda_\beta(\mathbf{u})C_R(\mathbf{u}_\alpha - \mathbf{u}_\beta) + C_R(0) - 2\sum_{\beta=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u})C_R(\mathbf{u}_\alpha - \mathbf{u})\end{aligned} \tag{15}$$

In order to minimize the variance in Eq.(15) with the unit-sum constraint on the weight, the objective function of this problem is error variance plus an additional term involving a Lagrange parameter, $\mu(\mathbf{u})$

$$L = \sigma_E^2(\mathbf{u}) + 2\mu(\mathbf{u})\left[1 - \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u})\right] \tag{16}$$

Take the derivative respect to μ , the constraint obey

$$\frac{1}{2}\frac{\partial L}{\partial \mu} = 1 - \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u}) = 0 \tag{17}$$

Then, take the derivative of the Eq.(24) with respect to each of the Kriging weights and set each derivative to be zero, we have the equations for Kriging weights

$$\begin{cases} \sum_{\beta=1}^{n(\mathbf{u})} \lambda_\beta(\mathbf{u})C_R(\mathbf{u}_\alpha - \mathbf{u}_\beta) + \mu(\mathbf{u}) = C_R(\mathbf{u}_\alpha - \mathbf{u}) \\ \qquad \alpha = 1,\ldots,n(\mathbf{u}) \\ \sum_{\beta=1}^{n(\mathbf{u})} \lambda_\beta(\mathbf{u}) = 1 \end{cases} \tag{18}$$

By solving Eq.(26), the Kriging weights can be obtained and the Kriging estimator in Eq.(14) is available. As mentioned before, the estimator of the variance of the predictions

$$\sigma_E^2(\mathbf{u}) = C_R(0) - \sum_{\alpha=1}^{n(\mathbf{u})} \lambda_\alpha(\mathbf{u})C_R(\mathbf{u}_\alpha - \mathbf{u}) \tag{19}$$

can be also received by substituting the Kriging weights into the Eq.(27).

4. PROPOSED METHOD: K-PDEM

The new method which combine probability density evolution method and Kriging is named K-PDEM. Unlike the traditional PDEM[6] , the K-PDEM use Kriging as surrogate model to replace the time-consuming deterministic analysis of numerical model. For these cases which the number of calculations to numerical model is small, the K-PDEM can greatly improve the results compare to the traditional PDEM. Besides, because of the exact interpolation characteristic of Kriging method, the new method is especially well-suited for the reliability analysis of structural numerical model which is a relatively noise-free model[19].

By combing the Kriging method and probability density evolution method, the detail of the K-PDEM is given in Figure.1. It consists 7 stage:

(i) Select the representative points $\boldsymbol{\theta}_{kr}$ in the domain $\Omega_{\boldsymbol{\Theta}}$, where $kr = 1, 2, \cdots, N_{kr}$, N_{kr} is the total number of selected points for Kriging estimator.

(ii) For a given $\boldsymbol{\theta}_{kr}$, carry out the deterministic analysis to obtain the response $\mathbf{X}(\boldsymbol{\theta}_{kr}, t_k)$, where $t_k = k \cdot \Delta t \ \ (k = 0,1,2,...)$, Δt is the time step.

(iii) For every $t_k \ \ (k = 0,1,2,...)$, calculating the Kriging estimator $\mathbf{X}^*(\boldsymbol{\theta}_{kr}, t_k)$ according to the representative points $\boldsymbol{\theta}_{kr}$ and $\mathbf{X}(\boldsymbol{\theta}_{kr}, t_k)$. This stage can be performed by the DACE toolbox. The corresponding correlation functions used in this paper is Gaussian.

(iv) Select another enriched representative points $\boldsymbol{\theta}_q$ in the domain $\Omega_{\boldsymbol{\Theta}}$, where $q = 1, 2, \cdots, N_{sel}$, N_{sel} is the total number of selected points. Generally, N_{sel} is much large than N_{kr} . For each point, assign a probability P_q as its assigned probability.

(v) Using Kriging estimator $\mathbf{X}^*(\boldsymbol{\theta}_{kr}, t_k)$ to predict the response $\mathbf{X}^*(\boldsymbol{\theta}_q, t_k)$.

(vi) Differentiate $\mathbf{X}^*(\boldsymbol{\theta}_q, t)$ with respect to t . Introducing $\dot{\mathbf{X}}(\boldsymbol{\theta}_q, t)$ to the discrete version of Eq.(4) and (5) and solving the equation with the finite difference method to obtain the numerical solution of $p_{\mathbf{X}\boldsymbol{\Theta}}(\mathbf{x}, \boldsymbol{\theta}_q, t)$.

(vii) Numerical integral by $p_{\mathbf{X}}(\mathbf{X}, t) = \sum_{q=1}^{N_{sel}} p_{\mathbf{X}\boldsymbol{\Theta}}(\mathbf{x}, \boldsymbol{\theta}_q, t)$

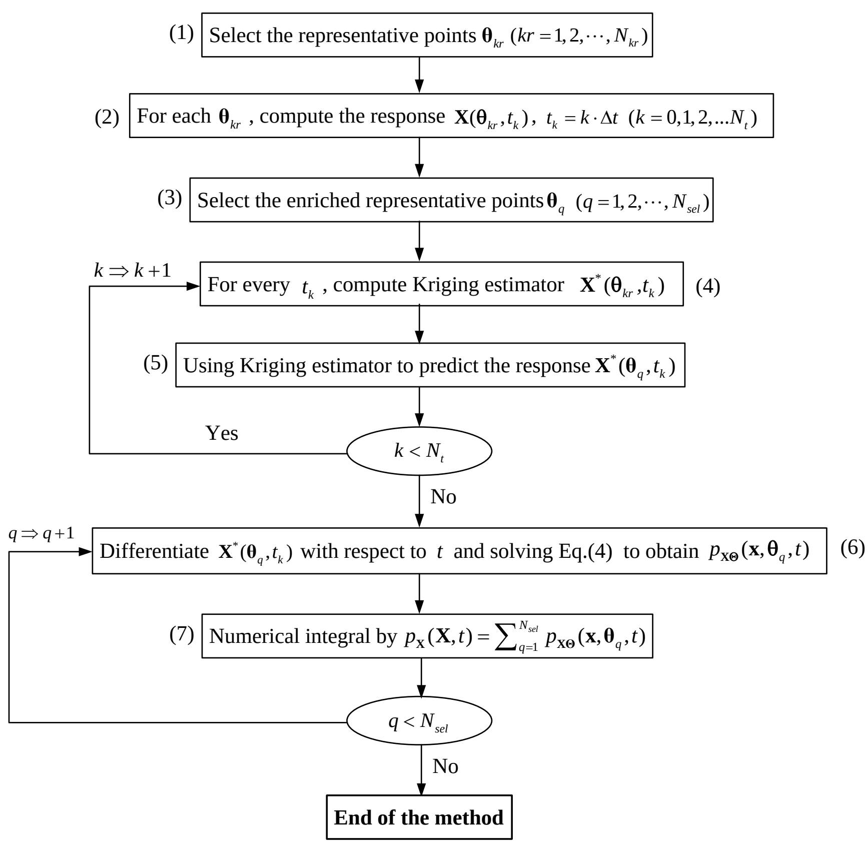


Fig. 1 –K-PDEM flowchart

5. NUMERICAL EXAMPLES

In order to check the performance of the K-PDEM method, several examples are studied: first, a SDOF system with random frequency are tested to observe the method's behavior. Following this, a 10-story space RC frame with 7 random variables, designed in accordance with the Chinese design code, is simulated to test the effectiveness of K-PDEM.

5.1 Example 1: A linear SDOF system

The free oscillation equation of an linear SDOF system reads

$$\dot{X} + \omega^2 X = 0 \tag{20}$$

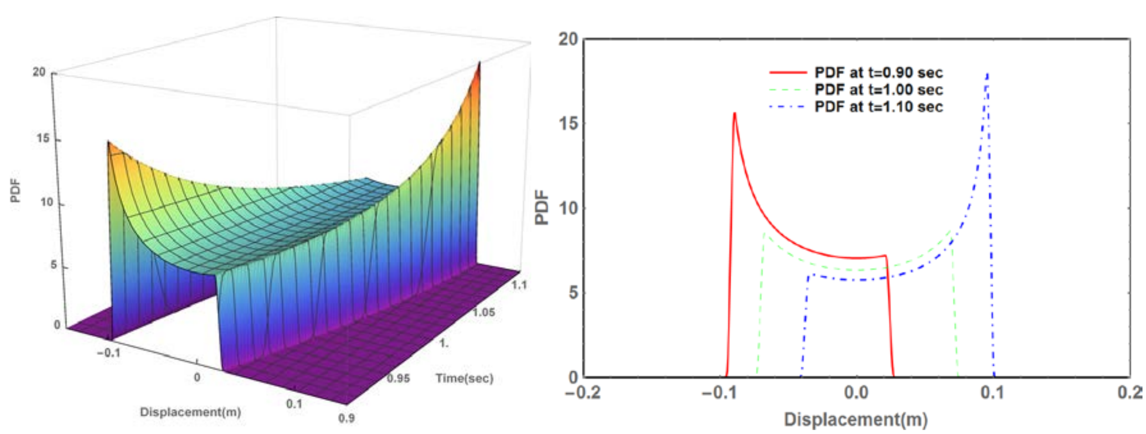
with the initial condition

$$X(0) = 0.1,\ \ \dot{X}(0) = 0 \tag{21}$$

the natural frequency of the system ω is a uniformly distributed random variable in $[5\pi/4, 7\pi/4]$. The analytical solution of the system and the probability density function of the response X were presented in the previous work[6]. In stochastic analysis, 50 representative points and 500 enriched representative points are selected and the corresponding assigned probabilities are computed according to the number theoretical method based algorithm[18].

With the proposed method, the instantaneous PDF and some probabilistic information is in Figure.2. Figure.2(a) shows the evolution of PDFs of the first-floor displacement with respect to time in the range of $[0.90, 1.10]$. Figure.2(b) shows the PDFs of the first-floor displacement at three certain time moment.

Besides, K-PDEM is compared with the traditional PDEM[6] and analytical solution. The results shows in Figure.3 and Table.1 prove that the proposed K-PDEM is of high accuracy. The K-L (Kullback–Leibler) information distance are introduced to quantify the difference between two probability distribution functions. According to the comparison in Table 1, K-PDEM is significantly better than the traditional PDEM when the number of calls to the displacement is relatively small.



(a) PDF evolution surface (b) Typical PDF at certain time instants

Fig. 2 –The PDFs of the response of the system

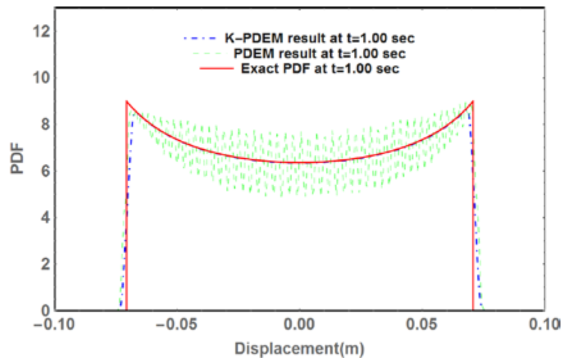


Fig. 3 –Comparison between K-PDEM,PDEM and theoretical solution

Table 1 –K-L distance between theoretical solution and numerical solution

K-L distance	*t=0.90*	*t=1.00*	*t=1.10*
K-PDEM	*0.0062824*	*0.0054720*	*0.0027323*
PDEM	*0.0181406*	*0.0263626*	*0.0254710*

5.1 Example 2: 10-story space RC frame

The second example considers a 10-story space RC frame, designed in accordance with the Chinese design code (Ministry of Construction of the People's Republic of China, 2010), is simulated to assess its earthquake-resistant capacity under earthquake excitations. The OpenSees model of this building is shown in Figure.4. Story heights of the frame are 4.5m for the first floor and 3.5m for the other floors. Columns are spaced at 5m in the long direction of the floor plan while in the short direction they are spaced at 6m, 3m and 6m, respectively.

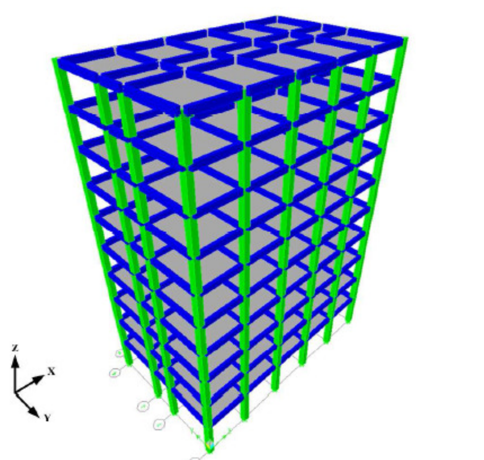


Fig. 4 –Configuration of 10-story RC frame

The heights of the first floor is 4.5m and 3.5m for the other floors. Columns are spaced at 5m in the long direction of the floor plan while in the short direction they are spaced at 6m, 3m and 6m, respectively. The dimensions and the reinforcement details are given in Table 2. Stirrups for both beams and columns are diameter 8mm with a spacing of 100mm.

Table 2 –Dimensions and the reinforcement details for 10-story RC

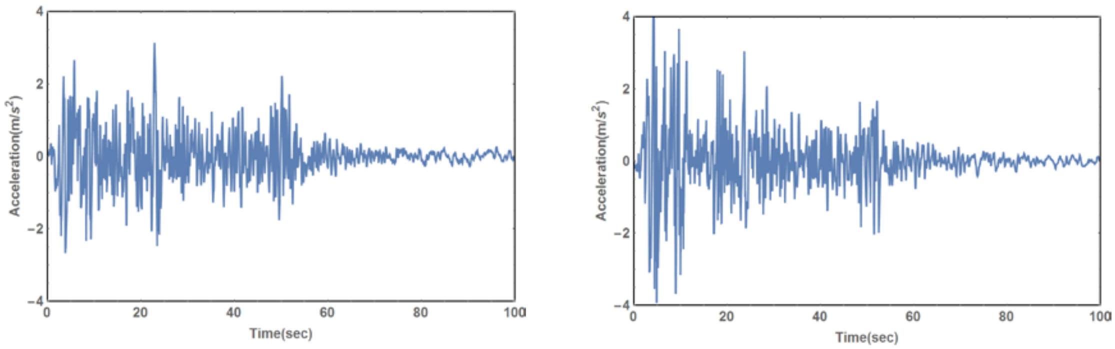
Story No.	*Exterior Beam*		*Interior Beam*		*Column*	
	Cross Section Dimension	*Reinforcement Area*	*Cross Section Dimension*	*Reinforcement Area*	*Cross Section Dimension*	*Reinforcement Area*
1-4	*300×600mm*	$2274mm^2$	*250×500mm*	$2274mm^2$	*600×600mm*	$3768mm^2$
5-10	*300×600mm*	$2035mm^2$	*250×500mm*	$2035mm^2$	*600×600mm*	$3768mm^2$

Seven material properties parameters are considered as random variable in this model. The mean value and the coefficient of variation of the random variables are listed in Table.3. All random variable are normal distributed.

Table 3 –The probabilistic information of the random variables

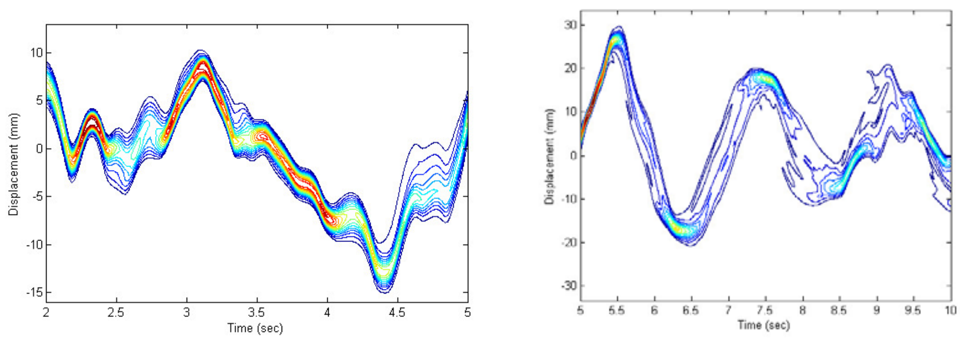
Type	*Mean*	*Coefficient of variation*
Compressive strength f_c	*36MPa*	*0.15*
Compressive peak strain $\varepsilon_{c,p}$	*0.002*	*0.15*
Compressive residual strain $\varepsilon_{c,r}$	*0.0015*	*0.15*
Tensile strength f_t	*3.6MPa*	*0.15*
Tensile residual strain $\varepsilon_{t,r}$	*0.01*	*0.15*
Elastic modulus E_s	$2\times10^5 MPa$	*0.15*
Yield strength f_y	*400MPa*	*0.15*

The structure is subjected to bidirectional earthquake ground motions. The north-south component of the ground motion in the 1940 Imperial Valley California earthquake recorded at the El Centro Station is applied along the long direction of the frame’s floor plan while the east-west component of the same ground motion is simultaneously applied along the short direction. Figure.5 shows the time history of the accelerations recorder in the El Centro earthquake.

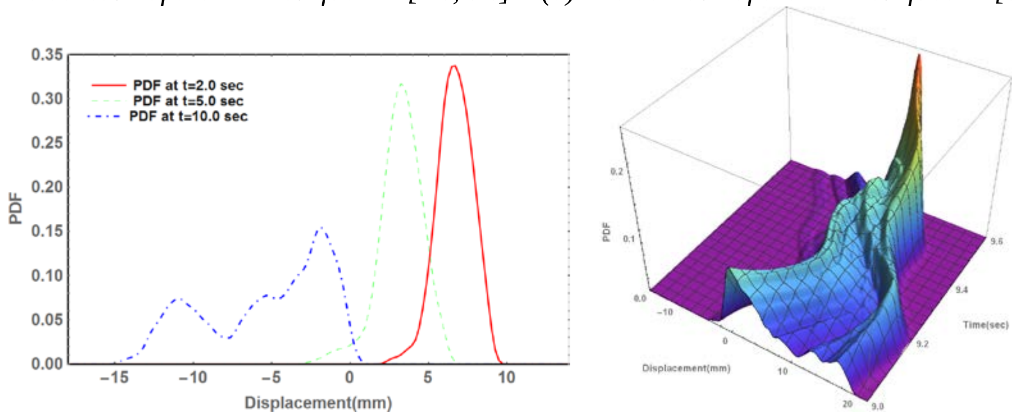


(a) north-south component *(b) north-south component*

Fig. 5 –Acceleration time history



(a) The contour of the PDF surface in [2 s, 5 s] *(b) The contour of the PDF surface in [5 s, 10 s]*



(c) Typical PDF at certain time instants *(d) PDF evolution surface*

Figure 7:PDF of the first inter-story drift

In Figure.6 shown is the probabilistic information of the first inter-story drift. Figure.6(a)-(b) are the contours of the probability density in the time intervals [2 s, 5 s] and [5 s, 10 s], respectively. The irregularity of the contour in these figures means that the probability density changes greatly against time. This can also be seen clearly from the typical PDFs at different instants of times (Figure.6(c)). Figure.6(d) shows the evolution of PDFs of the first inter-story drift with respect to time in the range of [9.0 s, 10.0 s].

6. CONCLUSIONS

A new stochastic dynamic response analysis for structure named K-PDEM is developed by combining the Probability Density Evolution Method (PDEM) and Kriging surrogate model method. The proposed method improve the accuracy of the PDEM by using representative points as the training point for Kriging estimator. To implement the K-PDEM, the representative points are first utilized to train the Kriging estimator, and the Kriging estimator is then used to prediction the response of the enriched representative points. After the above process, the number of the representative points with assigned probability can be greatly increased. Three examples have been analyzed and the results verify the efficiency and accuracy of the proposed method. In all the numerical examples, the K-PDEM shows significant improvement compare with the traditional PDEM. When the number of samples is relatively small, the advantage of K-PDEM is more obvious.

Accordingly, some issue need to be further studied. For instance, when the dimension of the random variable is too high, most of response surface method will encounter "curse of dimensionality" problem which will tremendously decrease the accuracy of prediction. Using dimension reduction technique to overcome this problem is ongoing investigation.

7. Acknowledgements

Financial supports from the National Natural Science Foundation of China (Grant Nos. 51261120374 and 51538010) are gratefully appreciated.

8. Copyrights

9. References

[1] Spanos P, Zeldin B. Monte Carlo treatment of random fields: a broad perspective. *Applied Mechanics Reviews* 1998; **51**(3): 219-37.
[2] Shinozuka M, Jan C-M. Digital simulation of random processes and its applications. *Journal of sound and vibration* 1972; **25**(1): 111-28.
[3] Contreras H. The stochastic finite-element method. *Computers & Structures* 1980; **12**(3): 341-8.
[4] Jensen H, Iwan W. Response of systems with uncertain parameters to stochastic excitation. *Journal of Engineering Mechanics* 1992.
[5] Ghanem R, Spanos P. Polynomial chaos in stochastic finite elements. *Journal of Applied Mechanics* 1990; **57**(1): 197-202.
[6] Li J, Chen J. Stochastic dynamics of structures: John Wiley & Sons; 2009.

[7] Li J, Chen JB. The probability density evolution method for dynamic response analysis of non‐linear stochastic structures. *International Journal for Numerical Methods in Engineering* 2006; **65**(6): 882-903.
[8] Li J, Chen J, Sun W, Peng Y. Advances of the probability density evolution method for nonlinear stochastic systems. *Probabilistic Engineering Mechanics* 2012; **28**: 132-42.
[9] Rajashekhar MR, Ellingwood BR. A new look at the response surface approach for reliability analysis. *Structural safety* 1993; **12**(3): 205-20.
[10] Hurtado JE. An examination of methods for approximating implicit limit state functions from the viewpoint of statistical learning theory. *Structural Safety* 2004; **26**(3): 271-93.
[11] Guan X, Melchers R. Effect of response surface parameter variation on structural reliability estimates. *Structural Safety* 2001; **23**(4): 429-44.
[12] Ghanem RG, Spanos PD. Stochastic finite elements: a spectral approach: Courier Corporation; 2003.
[13] Hurtado JE, Alvarez DA. Neural-network-based reliability analysis: a comparative study. *Computer methods in applied mechanics and engineering* 2001; **191**(1): 113-32.
[14] Matheron G. The intrinsic random functions and their applications. *Advances in applied probability* 1973: 439-68.
[15] Romero V, Swiler L, Giunta A. Construction of response surfaces based on progressive-lattice-sampling experimental designs with application to uncertainty propagation. *Structural Safety* 2004; **26**(2): 201-19.
[16] Kaymaz I. Application of kriging method to structural reliability problems. *Structural Safety* 2005; **27**(2): 133-51.
[17] Lophaven SN, Nielsen HB, Søndergaard J. DACE-A Matlab Kriging toolbox, version 2.0, 2002.
[18] Van Beers W, Kleijnen JP. Kriging interpolation in simulation: a survey. Simulation Conference, 2004 Proceedings of the 2004 Winter; 2004: IEEE; 2004.