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Theoretical Solution to Liquefaction-Induced Change of Site Predominant Period

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Abstract

Not only the ground failure but also the change of natural frequency of the site is induced by the liquefied layer, while from the vibration point of view the structure seismic response from the vibration point of view would be effected by the change of the site natural frequency due to the liquefaction. In this paper, the analytical solution to the site predominant period with a liquefied interlayer is presented by simplifying the actual horizontal site into the three-mass system with a liquefiable mass. The natural frequency decreasing ratio of the site (NFDRS) due to the liquefaction is derived and the influence of the main factors on NFDRS is demonstrated. The effects of the liquefied layer on NFDRS is ranked to four degree. The results in the paper indicate that the natural frequency of the site would be decreased by the liquefied interlayer and the NFDRS depends on three factors: the two thickness ratio of the liquefied layer to the non-liquefied upper layer , the non-liquefied sub layer to the liquefied layer, and shear modular ratio of the liquefiable soil before and after liquefaction. The effect of liquefied layer on NFDRS is heavy in most cases, medium in some cases, very heavy in a few cases and slight in few cases. NFDRS is mainly depends on the thickness ratio of the liquefied layer to the non-liquefied upper layer , and the relation between NFDRS and λ_1 can be describe in three modes. The thickness ratio of the non-liquefied sub layer to the liquefied layer play the second prominent part in NFDRS and the curve of NFDRS with λ_2 could be divided into two stages, i.e., rapid growth and smooth growth. Although NFDRS increases with the increasing of the shear modular ratio of liquefiable soil before and after liquefaction, the maximum increasing amplitude of NFDRS is less than 0.15. For the thin hard site, the liquefied layer shows the significant effect on NFDRS. While the liquefied layer exists the lower part of the site the effect is very significant. For the medium site, NFDRS due to the liquefaction varies gradually from the slight level to the medium degree and to the significant degree with increasing of the thickness and depth of the liquefiable layer. For the deep soft site, NFDRS due to the liquefaction varies gradually from the slight degree to the medium degree with increasing of the thickness of the liquefiable layer.

Keywords: liquefaction; NFDRS; theoretical solution; three-mass system

1. Introduction

A large number of survey data show that many earthquake damage is due to the same or similar natural period of engineering structure and the predominant period of site, which can cause resonance effect. As one of the key factors of the dynamic characteristics of the ground, the predominant period of the site is an essential parameter to be considered in the seismic design. For the regular site, the effect of predominant period of site on the structure has been systematically and in-depth studied. For the liquefaction site, a lot of research aimed at the destruction caused by the instability or settlement of soil liquefaction, the study on the liquefaction of the soil layer and its influence on the site from the perspective of vibration is still rare[1-3]. In fact, some of the domestic and international seismic damage investigation, including the 1976 Tangshan earthquake in China, there are many examples of the liquefaction site to reduce the earthquake damage to the rigid structure. At the same time, the recent earthquake records on the liquefied site also showed that the long period component of the ground motion was significantly enlarged, and it was obvious would increase the seismic damage to the long period structure from aspect of vibration. In New Zealand Ms6.3 Earthquake on Feb. 22, 2011, CTV building was severely damaged, according to the site investigation, ruled out due to liquefaction induced ground failure may be caused by the destruction of the building should be due to a seismic vibration effect. The CTV building natural period is about 0. 7S, the analysis shows that, the predominant period of acceleration response spectrum of ground motion was significantly longer, about from the beginning of 0. 1 ~ 0. 3S increased to 0. 5 ~ 1. 0s, and building natural period of agreement[4,5].

For liquefiable site, general taking such as piling, deep foundation reinforcement measures to resist liquefaction soil instability and settlement, unless replacing all of the liquefiable soil otherwise site characteristics would not been changed and will still be liquefied. From the point of view of fluctuation, the natural vibration characteristics of the soil layer will be changed under earthquake loading and have an important effect on the ground motion. And then, it will effect on the amplitude and mode of vibration of the structure. Recently, some international standard have noticed this problem. In Building Code International standard, it is suggested that the ground motion for the liquefiable site need to consider the impact of liquefied soil layer and carry out special calculations. In NEHRP, the similar suggest also been proposed. It should be noted that although researchers began to pay attention to the high frequency component of ground motion decreasing in the liquefied site, while the low frequency component increasing. But only the suggest of considering these two effect is presented, its mechanism and the law have not been formed completely. So the special regulation cannot been given. Obviously, one of the most fundamental issues in this area is how to describe the changing of the natural vibration characteristics of the site. Due to the complexity of the problem, the influence of the liquefied soil layer on the characteristics of the site remains in a simple qualitative understanding, the relevant theoretical research is still few.

In this paper, the analytical solution to the site predominant period with a liquefied interlayer is presented by simplifying the actual horizontal site into the three-mass system with a liquefiable mass, in order to explore the influence of liquefaction layer on the site predominant period.

2. Theoretical model

The actual of the horizontal site with a liquefied interlayer is simplified to a three-mass system as shown in Figure 1. G_1, ρ_1, V_{s1} are shear modular, density and shear velocity respectively of upper layer. G_2, ρ_2, V_{s2} are shear modular, density and shear velocity respectively of liquefiable layer. G_3, ρ_3, V_{s3} are shear modular, density and shear velocity respective of sub layer. h_1, h_2, h_3 are thickness of upper layer, liquefiable layer and sub layer. m_1, m_2, m_3 are quality of mass for upper layer, liquefiable layer and sub layer. K_1 and K_3 are stiffness of upper layer and sub layer respectively. K_L and K_LL are stiffness of before liquefied and sfter liqufied for liquefiable layer.

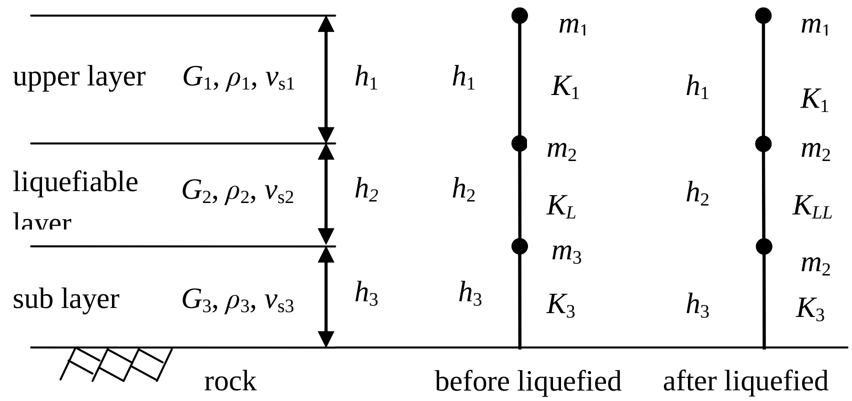


Fig. 1 – Three-mass model

3. Solution

According to the elastic dynamics, the vibration differential equation of the simplified model shown in fig.1 can been written as

$$\begin{cases} m_1\ddot{y}_1 + k_{11}y_1 + k_{12}y_2 + k_{13}y_3 = 0 \\ m_2\ddot{y}_2 + k_{21}y_1 + k_{22}y_2 + k_{23}y_3 = 0 \\ m_3\ddot{y}_3 + k_{31}y_1 + k_{32}y_2 + k_{33}y_3 = 0 \end{cases} \tag{1}$$

where

$$k_{11} = K_1 \quad k_{12} = -K_1 \quad k_{13} = 0$$

$$k_{21} = -K_1 \quad k_{22} = K_1 + K_L \quad k_{23} = -K_L \tag{2}$$

$$k_{31} = 0 \quad k_{32} = -K_L \quad k_{33} = K_L + K_3$$

Eq. (2) into Eq. (1) yields coefficient matrix

$$\begin{vmatrix} K_1 - m_1\omega^2 & -K_1 & 0 \\ -K_1 & K_1 + K_L - m_2\omega^2 & -K_L \\ 0 & -K_L & K_L + K_3 - m_3\omega^2 \end{vmatrix} = 0 \tag{3}$$

expansion

$$\omega^6 - \left[\frac{K_1}{m_1} + \frac{K_L + K_3}{m_3} + \frac{K_1 + K_L}{m_2}\right]\omega^4 + \left[\frac{K_1(K_L + K_3)}{m_1 m_3} + \frac{K_1 K_L}{m_1 m_2} + \frac{K_1 K_L + K_1 K_3 + K_L K_3}{m_2 m_3}\right]\omega^2 - \frac{K_1 K_L K_3}{m_1 m_2 m_3} = 0 \tag{4}$$

where

$$K_1 = \frac{G_1}{h_1} = \frac{\rho_1 v_{s1}^2}{h_1} \tag{5}$$

$$K_L = \frac{G_2}{h_2} = \frac{\rho_2 v_{s2}^2}{h_2} \tag{6}$$

$$K_3 = \frac{G_3}{h_3} = \frac{\rho_3 v_{s3}^2}{h_3} \tag{7}$$

assume as：

$$\rho_1 = \rho_2 = \rho_3$$

suppose as:

$$\frac{b}{a} = -\left(\frac{K_1}{m_1} + \frac{K_L + K_3}{m_3} + \frac{K_1 + K_L}{m_2} \right) \tag{8}$$

$$\frac{c}{a} = \left(\frac{K_1 \left(K_L + K_3 \right)}{m_1 m_3} + \frac{K_1 K_L}{m_1 m_2} + \frac{K_1 K_L + K_1 K_3 + K_L K_3}{m_2 m_3} \right) \tag{9}$$

$$\frac{d}{a} = -\frac{K_1 K_L K_3}{m_1 m_2 m_3} \tag{10}$$

Eq. (6)-(7) into Eq. (8)-(10) yields

$$\frac{b}{a} = -\left(\frac{v_{s1}^2}{h_1^2} + \frac{v_{s2}^2}{h_2 h_3} + \frac{v_{s3}^2}{h_3^2} + \frac{v_{s1}^2}{h_1 h_2} + \frac{v_{s2}^2}{h_2^2} \right) \tag{11}$$

$$\frac{c}{a} = \left(\frac{v_{s1}^2 v_{s2}^2}{h_1^2 h_2 h_3} + \frac{v_{s1}^2 v_{s3}^2}{h_1^2 h_3^2} + \frac{v_{s1}^2 v_{s2}^2}{h_1^2 h_2^2} + \frac{v_{s1}^2 v_{s2}^2}{h_1 h_2^2 h_3} + \frac{v_{s1}^2 v_{s3}^2}{h_1 h_2 h_3^2} + \frac{v_{s2}^2 v_{s3}^2}{h_2^2 h_3^2} \right) \tag{12}$$

$$\frac{d}{a} = -\frac{v_{s1}^2 v_{s2}^2 v_{s3}^2}{h_1^2 h_2^2 h_3^2} \tag{13}$$

Eq.(4) can been simplified as

$$\omega^6 + \frac{b}{a}\omega^4 + \frac{c}{a}\omega^2 + \frac{d}{a} = 0 \tag{14}$$

solve the Eq. (14)， obtain the minimum value as natural frequency

$$\omega^2 = \frac{-1 + i\sqrt{3}}{2} \sqrt[3]{-\frac{q}{2} + \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} + \frac{-1 - i\sqrt{3}}{2} \sqrt[3]{-\frac{q}{2} - \sqrt{\left(\frac{q}{2}\right)^2 + \left(\frac{p}{3}\right)^3}} - \frac{b}{3a} \tag{15}$$

where

$$p = \frac{c}{a} - \frac{b^2}{3a^2} \tag{16}$$

$$q=\frac{2b^3}{27a^3}-\frac{bc}{3a^2}+\frac{d}{a} \tag{17}$$

the natural period of site, T, can been written as

$$T=\frac{2\pi}{\omega}=\frac{2\pi}{\left(\frac{-1+i\sqrt{3}}{2}\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\frac{-1-i\sqrt{3}}{2}\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}-\frac{b}{3a}\right)^{\frac{1}{2}}} \tag{18}$$

Similarly， the natural period of site after liquefied, T', can been written as

$$T'=\frac{2\pi}{\left(\frac{-1+i\sqrt{3}}{2}\sqrt[3]{-\frac{q'}{2}+\sqrt{\left(\frac{q'}{2}\right)^2+\left(\frac{p'}{3}\right)^3}}+\frac{-1-i\sqrt{3}}{2}\sqrt[3]{-\frac{q'}{2}-\sqrt{\left(\frac{q'}{2}\right)^2+\left(\frac{p'}{3}\right)^3}}-\frac{1}{3}\left(\frac{b}{a}\right)'\right)^{\frac{1}{2}}} \tag{19}$$

where

$$p'=\left(\frac{c}{a}\right)'-\frac{1}{3}\left(\frac{b}{a}\right)'^2 \tag{20}$$

$$q'=\frac{2}{27}\left(\frac{b}{a}\right)^3-\frac{1}{3}\left(\frac{b}{a}\right)'\left(\frac{c}{a}\right)'+\left(\frac{d}{a}\right)' \tag{21}$$

$$\left(\frac{b}{a}\right)'=-\left(\frac{v_{s1}^2}{h_1^2}+\frac{v_{s2}'^2}{h_2h_3}+\frac{v_{s3}^2}{h_3^2}+\frac{v_{s1}^2}{h_1h_2}+\frac{v_{s2}'^2}{h_2^2}\right) \tag{22}$$

$$\left(\frac{c}{a}\right)'=\left(\frac{v_{s1}^2v_{s2}'^2}{h_1^2h_2h_3}+\frac{v_{s1}^2v_{s3}^2}{h_1^2h_3^2}+\frac{v_{s1}^2v_{s2}'^2}{h_1^2h_2^2}+\frac{v_{s1}^2v_{s2}'^2}{h_1h_2^2h_3}+\frac{v_{s1}^2v_{s3}^2}{h_1h_2h_3^2}+\frac{v_{s2}'^2v_{s3}^2}{h_2^2h_3^2}\right) \tag{23}$$

$$\left(\frac{d}{a}\right)'=-\frac{v_{s1}^2v_{s2}'^2v_{s3}^2}{h_1^2h_2^2h_3^2} \tag{24}$$

where v'_{s2} is the shear velocity of the soil layer after liquefied.

natural frequency increasing ratio of the site δ：

$$\delta=\frac{T'-T}{T}=$$

$$\left(\frac{\frac{-1+i\sqrt{3}}{2}\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\frac{-1-i\sqrt{3}}{2}\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}-\frac{b}{3a}}{\frac{-1+i\sqrt{3}}{2}\sqrt[3]{-\frac{q'}{2}+\sqrt{\left(\frac{q'}{2}\right)^2+\left(\frac{p'}{3}\right)^3}}+\frac{-1-i\sqrt{3}}{2}\sqrt[3]{-\frac{q'}{2}-\sqrt{\left(\frac{q'}{2}\right)^2+\left(\frac{p'}{3}\right)^3}}-\frac{1}{3}\left(\frac{b}{a}\right)'}\right)^{\frac{1}{2}}-1 \tag{25}$$

assume

$$\lambda_1=\frac{h_1}{h_2}=\frac{m_1}{m_2} \tag{26}$$

$$\lambda_2=\frac{h_2}{h_3}=\frac{m_2}{m_3} \tag{27}$$

$$P_1=\frac{G_1}{G_2}=\frac{v_{s1}^2}{v_{s2}^2} \tag{28}$$

$$P_2=\frac{G_2}{G_3}=\frac{v_{s2}^2}{v_{s3}^2} \tag{29}$$

$$P_L=\frac{G_2'}{G_2}=\frac{v_{s2}'^2}{v_{s2}^2} \tag{30}$$

λ_1 is the thickness ratio of the liquefied layer to the non-liquefied upper layer, λ_2 is the thickness ratio of the non-liquefied sub layer to the liquefied layer, P_1 is the shear modular ratio of the liquefied layer to the non-liquefied upper layer, P_2 is the shear modular ratio of the non-liquefied sub layer to the liquefied layer, P_L is the shear modular ratio of before and after liquefaction for the liquefiable soil layer.

the general relationship of shear velocity V_s with depth h can be expressed as[6]

$$v_s=ah^b \tag{31}$$

where b=0.2, the shear module at 2/3h is taken as the averaged shear module：

$$P_1=\frac{G_1}{G_2}=\frac{v_{s1}^2}{v_{s2}^2}=\left(\frac{a\left(\frac{2}{3}h_1\right)^{0.2}}{a\left(h_1+\frac{2}{3}h_2\right)^{0.2}}\right)^2=\left(\frac{\frac{2}{3}\lambda_1}{\lambda_1+\frac{2}{3}}\right)^{0.4} \tag{32}$$

$$P_2=\frac{G_2}{G_3}=\frac{v_{s2}^2}{v_{s3}^2}=\left(\frac{a\left(h_1+\frac{2}{3}h_2\right)^{0.2}}{a\left(h_1+h_2+\frac{2}{3}h_3\right)^{0.2}}\right)^2=\left(\frac{\lambda_1+\frac{2}{3}}{\lambda_1+1+\frac{2}{3\lambda_2}}\right)^{0.4} \tag{33}$$

Eq.(25) can been simplified as：

$$\delta=\left(\frac{\frac{-1+i\sqrt{3}}{2}L_1+\frac{-1-i\sqrt{3}}{2}L_2-\frac{M_1}{3}}{\frac{-1+i\sqrt{3}}{2}L_3+\frac{-1-i\sqrt{3}}{2}L_4-\frac{N_1}{3}}\right)^{\frac{1}{2}}-1 \tag{34}$$

where

$$M_1=-P_1P_2-\lambda_1\lambda_2\left(P_2\lambda_1+\lambda_1\lambda_2\right)-\lambda_1\left(P_1P_2+P_2\lambda_1\right) \tag{35}$$

$$M_2=P_1P_2\lambda_1\lambda_2\left(P_2\lambda_1+\lambda_1\lambda_2\right)+P_1P_2{}^2\lambda_1^2+\lambda_1^2\lambda_2\left(P_1P_2^2\lambda_1+P_1P_2\lambda_1\lambda_2+P_2\lambda_1^2\lambda_2\right) \tag{36}$$

$$M_3=-P_1P_2^2\lambda_1^4\lambda_2^2 \tag{37}$$

$$N_1=-P_1P_2-\lambda_1\lambda_2\left(P_2P_L\lambda_1+\lambda_1\lambda_2\right)-\lambda_1\left(P_1P_2+P_2P_L\lambda_1\right) \tag{38}$$

$$N_2=P_1P_2\lambda_1\lambda_2\left(P_2P_L\lambda_1+\lambda_1\lambda_2\right)+P_1P_2{}^2P_L\lambda_1^2+\lambda_1^2\lambda_2\left(P_1P_2^2P_L\lambda_1+P_1P_2\lambda_1\lambda_2+P_2P_L\lambda_1^2\lambda_2\right) \tag{39}$$

$$N_3=-P_1P_2^2P_L\lambda_1^4\lambda_2^2 \tag{40}$$

$$L_1=\sqrt[3]{-\frac{1}{2}\left(\frac{2M_1{}^3}{27}-\frac{M_1M_2}{3}+M_3\right)+\sqrt{\frac{1}{4}\left(\frac{2M_1{}^3}{27}-\frac{M_1M_2}{3}+M_3\right)^2+\frac{1}{27}\left(M_2-\frac{M_1{}^2}{3}\right)^3}} \tag{41}$$

$$L_2=\sqrt[3]{-\frac{1}{2}\left(\frac{2M_1{}^3}{27}-\frac{M_1M_2}{3}+M_3\right)-\sqrt{\frac{1}{4}\left(\frac{2M_1{}^3}{27}-\frac{M_1M_2}{3}+M_3\right)^2+\frac{1}{27}\left(M_2-\frac{M_1{}^2}{3}\right)^3}} \tag{42}$$

$$L_3=\sqrt[3]{-\frac{1}{2}\left(\frac{2N_1{}^3}{27}-\frac{N_1N_2}{3}+N_3\right)+\sqrt{\frac{1}{4}\left(\frac{2N_1{}^3}{27}-\frac{N_1N_2}{3}+N_3\right)^2+\frac{1}{27}\left(N_2-\frac{N_1{}^2}{3}\right)^3}} \tag{43}$$

$$L_4=\sqrt[3]{-\frac{1}{2}\left(\frac{2N_1{}^3}{27}-\frac{N_1N_2}{3}+N_3\right)-\sqrt{\frac{1}{4}\left(\frac{2N_1{}^3}{27}-\frac{N_1N_2}{3}+N_3\right)^2+\frac{1}{27}\left(N_2-\frac{N_1{}^2}{3}\right)^3}} \tag{44}$$

Therefore, the natural frequency increasing ratio of the site δ depends on three factors: the two thickness ratio of the liquefied layer to the non-liquefied upper layer , λ_1, the non-liquefied sub layer to the liquefied layer, λ_2, and shear modular ratio of the liquefiable soil before and after liquefaction, P_L.

4. Effect of liquefied layer on site predominant frequency

Eq.(34) gives a theoretical solution to the natural frequency increasing ratio of the site due to liquefaction. In order to correspond to the predominant frequency of ground motion, the expression of the natural frequency increasing ratio of the site is changed to the natural frequency decreasing ratio of the site(NFDRS),

$$\Delta=(f-f^{'})/f=\delta/(1+\delta) \tag{45}$$

where *f* and *f’* are predominant frequency before and after liquefied of site.

It should be explained that the dynamic response of the soil layer under strong earthquake is a nonlinear problem, therfore the three particle system is obviously not representative of the general situation. However, taking into account the liquefaction leads to a significant decrease in the stiffness of the soil layer, which is far beyond the general nonlinear soil stiffness changes, that is, the general nonlinear lead to the reduction of soil stiffness is less than the liquefaction of soil stiffness decreased. Therefore, the model of this paper should have a greater reliability of the relative reduction of the site stiffness caused by the liquefied layer.

The effects of the liquefied layer on NFDRS is ranked to four degree, slight (NFDRS<0.2), medium (0.2<NFDRS<0.5), significant (0.5<NFDRS<0.8) and very significant (0.8<NFDRS) degrees.

4.1 Comparison of several factors

Fig. 2 shows the relationship of NFDRS with λ_1 and λ_2 according to Eq. (34) for P_L=1/80. In Fig. 2, the coordinates (X, Y, Z) represent(λ_1, λ_2, NFDRS). As shown in Figure 2, natural frequency decreasing ratio of the site increases with the increasing of the thickness ratio of the non-liquefied sub layer to the liquefied layer , and decreases with the increasing of the thickness ratio of the liquefied layer to the non-liquefied upper layer. NFDRS is sensitive to λ_1 than λ_2.

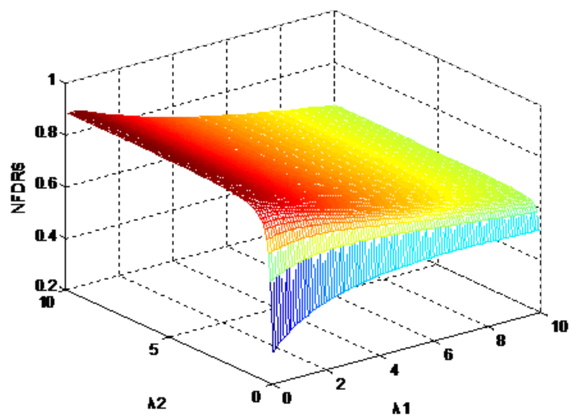
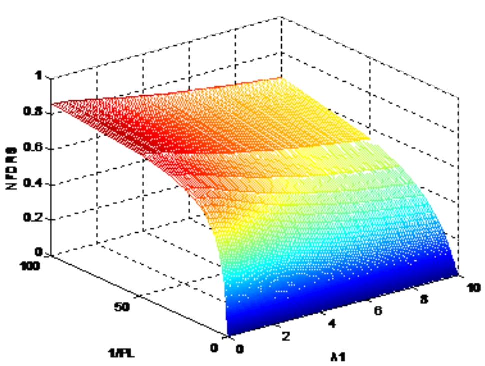


Fig. 2 – The relationship of NFDRS with λ_1 andλ_2.

Furthermore, Fig. 3 shows the relationship of NFDRS with λ_1 and $1/P_L$ for λ_2=1 and Fig. 4 shows the relationship of NFDRS with λ_2 and $1/P_L$ for λ_1=1. As shown in Figure 3, natural frequency decreasing ratio of the site decreases with the increasing of the thickness ratio of the liquefied layer to the non-liquefied upper layer. It can be seen from Fig.4 that NFDRS increases with the increasing of the thickness ratio of the non-liquefied sub layer to the liquefied layer and increases with the P_L.



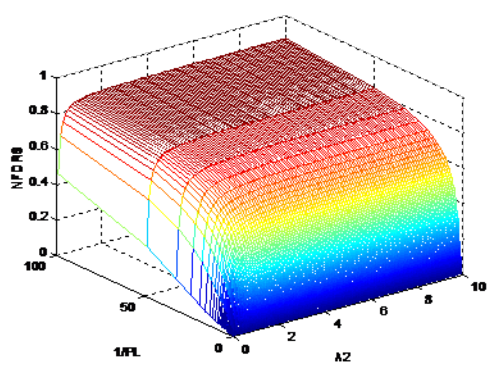


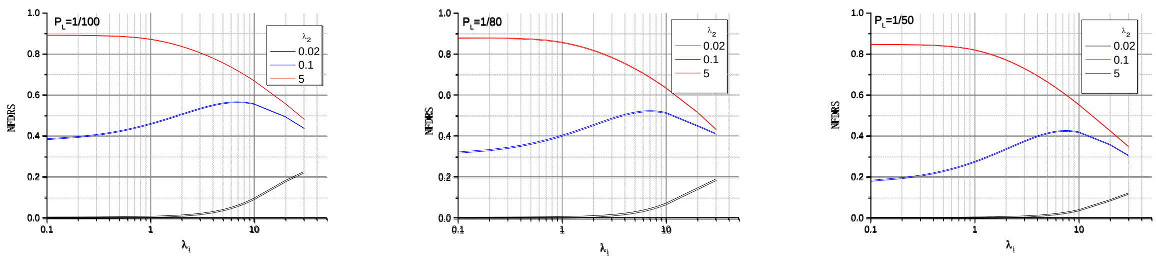
Fig. 3 – The relationship of NFDRS with λ_1 and $1/P_L$ Fig. 4 – The relationship of NFDRS with $1/P_L$ and λ_2.

A comparison between Fig.3 and Fig.4, can be found for the same $1/P_L$, the relationship between NFDRS and λ_1 , and the relationship between NFDRS and λ_2 are different. NFDRS with the increasing of λ_1 shows a steady decrease and with the increasing of λ_2, NFDRS rapidly grow in the first and then steady grow.

4.2 Effect of upper layer

For three kinds of soften degree of liquefied layer, the effect of the thickness ratio of upper layer and liquefied layer on the natural frequency decreasing ratio of the site is shown in Fig. 5.

It can be seen from Fig. 5 that the influence of the thickness ratio of upper layer and liquefied layer on the natural frequency decreasing ratio of the site is approximately the same, that is, the influence of softening degree of the liquefaction layer on the relationship of between NFDRS and λ_1 is not very significant. At the same time, it can be seen that the influence of the thickness ratio of the overburden layer and liquefied layer on the natural frequency decreasing ratio of the site is closely related to the thickness ratio of the liquid layer and the sub layer. When λ_2 is larger(λ_2=5), NFDRS is very large(above 0.4), and with the increasing of λ_1 NFDRS decreases, that is, while the liquefaction layer and the bottom layer thickness ratio is large, natural frequency decreasing ratio of the site is very large, and with the increasing of the thickness ratio of the upper layer and the liquefaction layer is reduced. The relationship between NFDRS and λ_1 is the exponential. When λ_2 is smaller(λ_2=0.02), NFDRS is small(less than 0.2), and NFDRS increases with the increasing of λ_1. The two also showed an exponential relationship. When λ_2 is medium(λ_2=0.1), the NFDRS is fluctuant in a range(about 0.3-0.4), and increase first and then decrease with the λ_1 increasing.

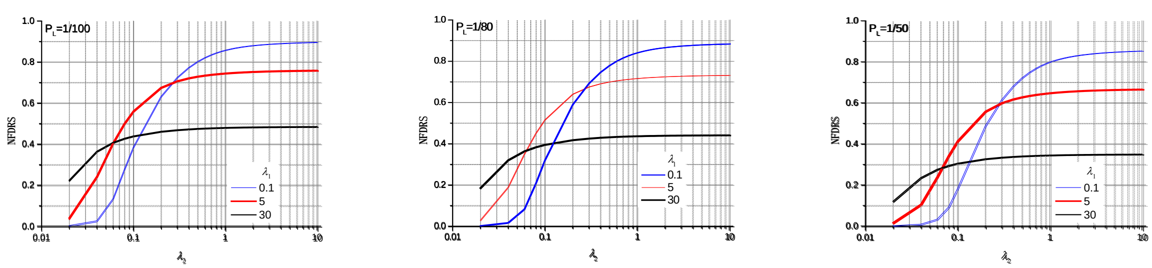


(a) P_L=1/100 (b) P_L=1/80 (c) P_L=1/50

Fig. 5 –Relationship between NFDRS and λ_1

4.3 Effect of sub layer

For three kinds of soften degree of liquefied layer, the effect of the thickness ratio of liquefied layer and the sub layer on the natural frequency decreasing ratio of the site is shown in Fig. 6.



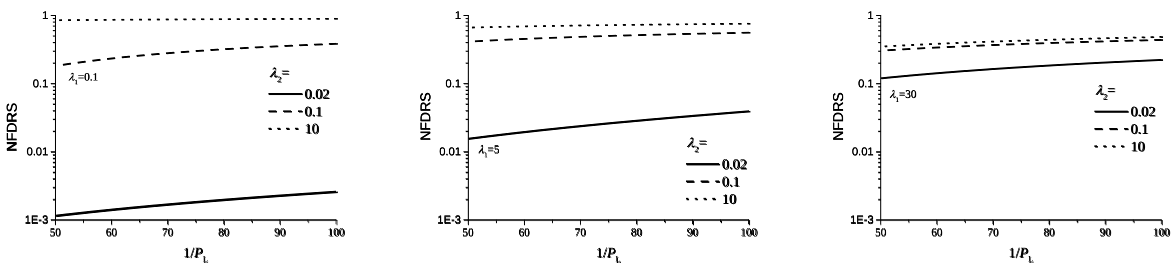
(a) P_L=1/100 (b) P_L=1/80 (c) P_L=1/50

Fig. 6 –Relationship between NFDRS and λ_2

It can be seen from Fig. 6 that the influence of the thickness ratio of liquefied layer and sub layer on the natural frequency decreasing ratio of the site is approximately the same, that is, the influence of softening degree of the liquefaction layer on the relationship of between NFDRS and λ_2 is not very significant. At the same time, it can be seen that the influence of the thickness ratio of the sub layer and liquefied layer on the natural frequency decreasing ratio of the site is closely related to the thickness ratio of the liquid layer and the upper layer. On the whole, with the increasing of λ_2, the curve can be divided into two stages, one is the rapid rise stage, one is the steady growth phase, and the conversion point is closely related to λ_1. When λ_1 is smaller (λ_1=0.1), conversion point is at λ_2=1. When λ_1 is larger (λ_1=5), conversion point appears at λ_2=0.2. With the further increasing of λ_2, that is, the depth of the liquefaction layer increasing, the rapid rise stage change into the steady growth phase at about λ_1=0.1.

4.4 Effect of liquefied layer softening degree

For three kinds of the thickness ratio of liquefied layer and upper layer, the effect of the softening degree of liquefied layer on the natural frequency decreasing ratio of the site is shown in Fig. 7. It can been seen from Fig. 7 that NFDRS and $1/P_L$ are in an exponential form. Regardless of the thickness of the covering layer and the underlying layer, the natural frequency decreasing ratio of the site and the softening degree of the site are exponentially increasing. Although NFDRS increases with $1/P_L$ increasing, but the absolute increment is not large,about 0.001-0.15. As a result, the softening degree of the liquefied layer has little influence on the natural frequency decreasing ratio of the site in the conventional range.



(a) λ_1 =0.1 (b) λ_1 =5.0 (c) λ_1 =30

Fig. 7 –Relationship between NFDRS and $1/P_L$

5. Conclusion

In this paper, the actual soil layer is regarded as a horizontal layered, and the three mass system represents a site with the liquefied layer. The theoretical solution to the natural frequency increasing ratio of the site due to liquefaction is proposed and the influence law of the characteristic quantity of the soil layer and the liquefied layer on the site predominant frequency was obtained. The conclusions of the paper can be expressed as follows:

1. The presence of the liquefaction layer will cause a drop in the site's predominant frequency.The natural frequency increasing ratio of the site depends on three factors: the two thickness ratio of the liquefied layer to the non-liquefied upper layer , λ_1, the non-liquefied sub layer to the liquefied layer, λ_2, and shear modular ratio of the liquefiable soil before and after liquefaction.

2. The effects of the liquefied layer on NFDRS is ranked to four degree, slight (NFDRS<0.2), medium (0.2<NFDRS<0.5), significant (0.5<NFDRS<0.8) and very significant (0.8<NFDRS) degrees. The effect of liquefied layer on NFDRS is significant in most cases, medium in some cases, very significant in a few cases and slight in few cases.

3. NFDRS is mainly depends on the thickness ratio of the liquefied layer to the non-liquefied upper layer , λ_1, and the relation between NFDRS and λ_1 can be describe in three modes, i.e. the increase exponentially with λ_1, the decrease exponentially with λ_1, and the fluctuation of the increase first and then decrease in a small range.

4. The thickness ratio of the non-liquefied sub layer to the liquefied layer, λ_2 play the second prominent part in NFDRS and the curve of NFDRS with λ_2 could be divided into two stages, i.e., rapid growth and smooth growth, and the conversion point between the two stages is related to $\boldsymbol{\lambda}_1$.

5. Although NFDRS increases with the increasing of the shear modular ratio of liquefiable soil before and after liquefaction, the maximum increasing amplitude of NFDRS is less than 0.15.

6. To be clear, there is non-limitations of layers's height to use in this model and the layers thickness of the example mentioned in this paper is about 1m-50m.

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