

A Study on Optimum Distribution of Story Shear Force Coefficient for Seismic Design of Multi-story Structure

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Abstract

Typically, the story shear force acting horizontally on a multi-story structure is proportionally calculated to the height of and weight acting upon each story. The story shear force distributions of most seismic design codes generally reflect the influences of higher vibration modes based on the elastic deformations of structures. However, as the seismic design allows for the plastic behavior of a structure, the story shear force distribution shall be effective after it is yielded due to earthquake excitation. Hence, this study conducted numerical analyses on the story shear force distributions of most seismic design codes to find out the characteristics of how a structure is damaged between stories. By conducting the static and dynamic response analyses based on the calculation formula for the story shear force distribution, which is proposed by the seismic design code of several countries, in this paper we weigh the behaviors of a multi-story structure including plastic deformation, and analyze the characteristics of damage distribution caused by ground motion. Considering the nature of distribution of story shear force coefficient, which appears in the story-specific distribution, a beam with infinite stiffness and the 2D framework were applied to the model shape for response analysis in order to facilitate the story-specific design and the analysis of response characteristics. Analysis results show that the more forces are distributed onto high stories, the lower its concentration is and the more energy is absorbed. In comparison of the distribution of cumulative plastic deformation ratios of analytical models according to a variety of input earthquake motions, the models KBC2009 and IBC2006 with the linear type distribution of story shear force coefficients showed that the damage was tended to extremely concentrated in top portion. The models UBC97, NZS1170, AIJ and H.Akiyama with the distribution of story shear force coefficients in curvilinear form showed that the layer damage was relatively well dispersed compared to the models KBC2009 and IBC2006. In analyzing the layer damage intensity and the energy absorption at limit state, the seismic efficiency of H.Akiyama model with the largest increase in distribution of story shear force coefficients in top portion was excellent. We drew the optimum distribution of story shear force coefficients, implementing the optimum distribution of layer damage in which the damage distribution uniformly appears on all stories for analytical models having a variety of natural periods, and proposed the calculation formula with variables resulted from analyzing the optimum distribution of story shear force according to the changes of natural periods. With the analytical models in the same conditions, the damage distribution by the proposed calculation formula showed the most equitable form in comparison to the layer damage distribution obtained from the distribution of story shear force coefficients based on the seismic design code of several countries. From the results, this study proposes the optimum story shear force distribution and its calculation formula that make the damages uniformly distributed onto whole stories. Consequently, the story damage distribution from the optimum calculation formula was considerably more stable than existing seismic design codes.

Keywords: Optimum Distribution of Story Shear Force Coefficient, Accumulative Plastic Deformation Ratio



1. Introduction

Typically, the story shear force acting horizontally on a multi-story structure is proportionally calculated to the height of and weight acting upon each story. However, if dynamic load such as earthquake is acting on it, this rule does not apply to the distributed load acting upon every story under the influence of higher vibration modes. For this reason, seismic design code of several countries around the world analyze the characteristics of load distribution caused by higher vibration modes in various ways and provide various forms of the modification factor of the story shear force distribution accordingly.

Most of the seismic design code use the analysis method based on Elastic Strain Range such as Response Spectrum Analysis Method and Mode Superposition Method in order to analyze the effects of higher vibration modes. Since the plastic deformation is allowed in seismic design in some degree up to the collapse of building, the needs for securing the enough stability arise in consideration of elastic deformation as well as the behaviors after the occurrence of plastic deformation in analyzing the seismic load.

By conducting the static and dynamic response analyses based on the calculation formula for the story shear force distribution, which is proposed by the seismic design code of several countries, in this paper we weigh the behaviors of a multi-story structure including plastic deformation, and analyze the characteristics of damage distribution caused by ground motion. Based on these results, the point of this paper is to propose the optimized calculation formula for the story shear force distribution in order to improve the seismic performance by distributing the seismic load evenly on all stories.

2. Comparisons of story shear force distribution of seismic design code around the world

2.1 Comparing the story shear force distribution

Examining the story shear force calculation formula of the seismic design code around the world reveals that the base shear calculation formula and the seismic load distribution formula are not in constant manner, which in turn affect the size and distribution type of the story shear force in various forms. For this reason, we specify structure models, as shown in Table 1, calculate the story shear force based on each seismic design code, and compare the distributions in this study. The calculated results of story shear force distributions are shown in Fig.1.

Category	Structural design requirements		
Use	General Office Building		
Structure Classification	Steel ordinary moment frames		
Shape	Plane (width \times height) = 8m \times 8m / story height = 3m		
Weight	(Sum of general dead load and live load :)		
Area & Ground characteristics	Using the most dominant region and modulus subgrade reaction in each standa		
Stories(Aspect Ratio)	3-story (1.125), 5-story (1.875), 10-story (3.75), 20-story (7.5)		

Table 1 - Structural conditions for calculating the story shear force distributions

Since the modulus subgrade reaction is differently applied for each region, the size of base shear calculated for each seismic design code is different from each other. With this in mind, based on the result value of story shear force from Fig.1, the story shear force distribution of each story, normalized by the size of the base shear force, is shown in Fig.2, and the distribution types are compared accordingly.



Fig. 1 – Story shear force distribution based on the seismic design code



Fig. 2 - Normalized story shear force distribution based on the seismic design code

The "Seismic design of a structure according to Energy Balance Method¹)" proposes the concept of yield story shear force coefficient in order to control and evaluate the layer damage. The yield story shear force coefficient α_i is defined as shown in Eq. (1):

$$\alpha_i = \frac{Q_{yi}}{\sum_{j=i}^N W_j} \tag{1}$$

when, Q_{yi} : Yield story shear force on the *i* story, $\sum_{j=i}^{N} W_j$: Total weight on *i* story of upper structure

The distribution of yield story shear force coefficient $\overline{\alpha_i}$ refers to the ratio of the yield shear force coefficient on each story to the 1st story, as shown in Eq. (2):

$$\overline{\alpha_i} = \alpha_i / \alpha_1 \tag{2}$$

when, $\overline{\alpha_i}$: Yield shear force coefficient at the $\frac{1}{2}$ story, α_1 : Yield shear force coefficient of 1st story



The plastic deformation energy can be used to measure the damage. The definition of cumulative plastic deformation ratio on each story \P is as shown in Eq. (3).

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$$\eta_{i} = \frac{w_{pi}}{Q_{yi}\delta_{yi}}$$
(3)
when, W_{pi} : Plastic deformation energy on i story, Q_{yi} : Yield story shear force on i story, δ_{yi} : Yield strain on i story

2.2 Distribution of story shear force coefficients by seismic design code

In the Eq. (1)~(2), we substitute the yield shear force Q_{yi} with the story shear force Q_i , propose the distribution of story shear force coefficient s^{α_i} , and calculate the story shear force distribution of each seismic design code. The equation for the story shear force coefficient s^{α_i} and the distribution of story shear force coefficient s^{α_i} are as shown in Eq. (4) and (5) respectively.

$${}_{s}\alpha_{i} = \frac{Q_{i}}{\sum_{j=i}^{N} W_{j}} \tag{4}$$

When, Q_i : Story shear force of i story, $\sum_{j=i}^{N} W_j$: Total weight of the upper portion of i story

$$\overline{{}_{s}\alpha_{i}} = {}_{s}\alpha_{i}/{}_{s}\alpha_{1} \tag{5}$$

When, s^{α_i} : Story shear force coefficient of i story, s^{α_1} : Story shear force coefficient of i story

The Fig.3 shows the distribution of story shear force coefficient according to the story shear force (Fig.1) based on each seismic design code. As to the overall shape of distribution of story shear force coefficient, the difference in the low-rise structure is not significant between the seismic design code, but the difference can be certainly identified in the high-rise structure.

Having the distribution of linear form, both KBC2009 and IBC2006 represent the same load distribution type, and the distribution patterns appear almost identically. On the other hand, showing in curvilinear form, the distributions of UBC97, NZS1170, AIJ and H.Akiyama's have the different types of calculation formulas, and show the different patterns of distribution. As to the distributions of UBC97 and H.Akiyama's, an increase of shear force is higher in top portion, and the shear force distribution of NZS1170 is lower in upper-middle.



Fig. 3 – Distribution of story shear force coefficient of each seismic design code



3. Response analysis of structure according to story shear force distribution

3.1 Analytical model design

Considering the nature of distribution of story shear force coefficient, which appears in the story-specific distribution, a beam with infinite stiffness and the 2D framework were applied to the model shape for response analysis in order to facilitate the story-specific design and the analysis of response characteristics. Thus, the response of columns represents the corresponding story, and the story shear force distribution based on a seismic design criterion can be reflected to designing the analytical model by controlling the load bearing of columns. The basic shape of the analytical model is as shown in Fig.4.

Provided the damage is concentrated in the hinge area of both ends of column member, the framework was modelled with flexure spring on both ends of column member. The restoring force characteristic of flexure spring was set to bi-linear of perfect elasto-plasticity.

The story shear force distribution of each analytical model is as shown in Fig.5. As to the distribution of the story shear force of each model, the story shear force of the models KBC2009 and IBC2006 with low load bearding increase appears high in middle as well as lower portions and low in upper portion compared to other criteria. The shear force distributions of the models UBC97, NZS1170, AIJ and H.Akiyama drastically increase in upper portion, and tend to be concentrated in top portion.



Fig. 5 – Designed story shear force based on distribution of story shear force coefficient



3.2 Analysis Method

The analysis program CANNY is based on the Newmark- β method for the dynamic response analysis. The damping ratio of frame is considered as 2%.

The "Seismic design of a structure according to Energy Balance Method" defines the state of perfect plasticity as $\eta = 10$ level, where η is the cumulative plastic deformation ratio of the plastic deformation capacity of structural member. Conservatively applying $\eta = 6$ as the limit state in this study, we conducted the analysis by multiplying the acceleration ratio to the input earthquake motion until the value of the cumulative plastic deformation ratio on each story reached 6 at the maximum.

The El Centro NS, Hachinohe NS, Kobe NS and Taft EW earthquake waves are used as input ground motions.

3.3 Analysis Result

As for the damage assessment criterion, the cumulative plastic deformation ratio by plastic deformation energy at limit state was used, and the story distribution values for the cumulative plastic deformation ratio of the entire structure are as shown in Fig.6 and Fig.7.

As it was studied during the analysis process of distribution of story shear force coefficient, the layer damage distribution of each analytical model clearly showed the differences in reflecting the degree of load bearing increase in upper portion. The KBC2009 and IBC2006 models with less distribution of load bearing in upper portions showed high stability in low-rise models, and the damage was extremely concentrated in top portions as the number of stories increases. The more the load bearing distribution had in upper portion of a model, the more evenly the damage distribution appeared in middle and high-rise models.

4. Optimum distribution of story shear force coefficients

On the response characteristics analysis of a structure in accordance with the story shear force distribution of domestic and international seismic design code, we found that the seismic safety of a structure was largely affected by the shear force distribution in upper portion of higher-rise model, through this, we were able to predict the distribution tendency of story shear force affected by higher vibration modes to a certain extent.





Fig. 7 – Distribution of cumulative plastic deformation ratio (20-story)



The Eq. (6) using the distribution of story shear force coefficient and the weight of upper structure. By counterplotting the calculation process of story shear force, summarizing the Eq. (6) for the coefficient distribution $\mathfrak{s}^{\mathfrak{A}_{1}}$, and applying the story shear force $\mathfrak{p}^{\mathfrak{Q}_{1}}$ in the state of optimum distribution of layer damage will yield the optimum distribution of story shear force coefficient, $\mathfrak{p}^{\mathfrak{A}_{1}}$. The calculation formula of optimum distribution of story shear force coefficient is as shown in Eq. (7).



$$Q_i = {}_{z} \alpha_1 \overline{}_{s} \overline{\alpha_i} \sum_{j=i}^{N} W_j \tag{6}$$

When, $z\alpha_1$: Story shear force coefficient on 1st story according to the local conditions

 $\mathfrak{s}^{\mathfrak{A}_k}$: Distribution of story shear force coefficient on \mathfrak{k} story

 $\sum_{j=i}^{N} W_j$: Total weight of upper structure on *i* story

$$\overline{{}_{p}\alpha_{i}} = \frac{{}_{p}q_{i}}{{}_{z}\alpha_{1}\sum_{j=i}^{N}W_{j}}$$

$$\tag{7}$$

When, p^{α_1} : Optimum distribution of story shear force coefficient

 pQ_i : Story shear force of i story in the state of optimum distribution of layer damage

 z^{α_1} : Story shear force coefficient of 1st story depending on local conditions

 $\sum_{j=i}^{N} W_j$: Total weight of upper portion of i story

By using the above equation, the optimum distribution of story shear force coefficient p^{α_1} in accordance with each input earthquake motion is estimated, and the graph is as shown in Fig.8.

A look at the tendency according to the number of stories of analytical model suggests that the optimum distribution of story shear force coefficient in 3-story model is in almost linear fashion, and the higher-rise model is more likely to show a decrease in story shear force from lower portion to middle portion and a big increase in the shear force distribution. In the extreme case of 20-story model, the distribution of story shear force coefficient was estimated less than 1 in middle portion. In addition, the distribution pattern according to each input earthquake motion appeared in many forms, but the corresponding tendency was not clearly identified.

4.2 Drawing generalized curves

In order to comprehensively reflect the characteristics of optimum distribution of story shear force coefficient graph for each input earthquake motion, we drew the approximation formula of quadric function.



Fig. 8 - Optimum distribution of story shear force coefficient

Representing the correlation between the layer coordinates and by using the least square method. In the process, we applied 1 to all the sections where the distribution of story shear force coefficient was less than 1.



The generalized curve of optimum distribution of story shear force coefficient calculated is as shown in Fig.9. As to the generalized curve of optimum distribution of story shear force coefficient, the higher-rise analytical model is more likely to show an increase in distribution value of top portion and the curvature of the curve.





- 4.3 Proposal and verification of optimum distribution of story shear force coefficient
- 4.3.1 Proposal of calculation formula

The analysis models are added for investigating the tendency of optimum distribution of yield story shear force coefficients. The natural periods of added models are as shown in Table 2.

Table 2 – Natural	vibration pe	eriod of ad	ded models

Optimum	3-story	5-story	10-story	20-story
Natural vibration period	0.40s	0.61s	1.00s	2.08s

Accordingly, the generalized curve for optimum distribution of story shear force coefficient is found by using the least square method for the models as shown in Fig.10.

The generalized curve of optimum distribution of story shear force coefficient drawn in accordance with a variety of natural periods showed the longer the natural period is more likely to clearly show an increasing pattern of the distribution value and the curvature of graph in top portion. As shown in Fig.11, we showed the optimum distribution of story shear force coefficient and the natural period in overlapped manner and analyzed the pattern accordingly.

The calculation formula of optimum distribution of story shear force coefficient is described using an exponential function and the natural period as shown in Eq. (8).

$$\overline{{}_{p}\alpha_{i}} = 1 + \{A \times (x')^{B}\}$$

$$\tag{8}$$

When, \mathfrak{X}^{\prime} : Height up to the ⁴th story from the ground, \mathbb{T} : Natural period

 $A = -0.08 \times T^{2} + 1.05 \times T + 0.60$ $B = 0.03 \times T^{2} + 0.80 \times T + 0.87$



Fig. 10 - Generalized curve of optimum distribution of story shear force coefficient by natural period.



Fig. 11 - Pattern of optimum distribution of story shear force coefficient according to natural period



Fig. 12 - Optimum distribution of story shear force coefficient by the proposed calculation formula

The comparison between the graph of optimum distribution of story shear force coefficient estimated and that of story shear force coefficient based on the seismic design code is as shown in Fig.12.

4.3.2 Verification on the proposed calculation formula

The verification on the proposed calculation formula was conducted in the following manner: First, design a structure by applying the optimum distribution of story shear force coefficient in accordance with the distribution of story shear force coefficient, based on the seismic design code, as well as the proposed calculation formula. Compare and analyze the damage distribution accordingly. The dynamic response analysis was carried out in the same way as described in section 3.

The Fig.13 show the comparison results of the damage distribution based on the proposed calculation formula to that of based on the seismic design code respectively.

Although the distribution pattern of damage appeared in various forms, the average value for the damage distribution by the input earthquake motion is represented in order to analyze the overall distribution pattern and displayed together in diagrams. As for the analytical models in the same conditions though, comparing the damage distribution based on the proposed calculation formula and that of based on the seismic design code, the former showed the most evenly distributed layer damage among all analytical models.



Fig. 13 – Verification of the proposed predict formula



5. Conclusion

In this study, the characteristics of damage distribution based on the seismic design code were compared, and the response analysis of structures applied with these characteristics were carried out. In addition, we drew the optimum distribution of story shear force coefficient in diagrams, which uniformly distributed the damage to all stories, and proposed the calculation formula accordingly. The results thus obtained were as follows.

1) In comparison of the distribution of cumulative plastic deformation ratios of analytical models according to a variety of input earthquake motions, the models KBC2009 and IBC2006 with the linear type distribution of story shear force coefficients showed that the damage was tended to extremely concentrated in top portion.

2) The models UBC97, NZS1170, AIJ and H.Akiyama with the distribution of story shear force coefficients in curvilinear form showed that the layer damage was relatively well dispersed compared to the models KBC2009 and IBC2006. In analyzing the layer damage intensity and the energy absorption at limit state, the seismic efficiency of H.Akiyama model with the largest increase in distribution of story shear force coefficients in top portion was excellent.

3) We drew the optimum distribution of story shear force coefficients, implementing the optimum distribution of layer damage in which the damage distribution uniformly appears on all stories for analytical models having a variety of natural periods, and proposed the calculation formula with variables resulted from analyzing the optimum distribution of story shear force according to the changes of natural periods.

4) With the analytical models in the same conditions, the damage distribution by the proposed calculation formula showed the most equitable form in comparison to the layer damage distribution obtained from the distribution of story shear force coefficients based on the seismic design code of several countries.

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