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A METHOD FOR MODAL IDENTIFICATION OF EXISTING STRUCTURES USING MICROTREMORS

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Abstract

Evaluating the structural performance of the existing structures is important to confirm the structural soundness after natural disaster or validity of the seismic retrofit. Modal identification techniques based on vibration records are usually applied to evaluate dynamic characteristics of the target structure. In this technique, we will determine natural frequency, modal vector and modal damping ratio, called modal characteristics as parameters. For architectures and civil engineering structures, modal identification method using microtremor records is desirable, since these records can be observed readily and frequently, comparing with other vibration records such as earthquake records and excitation records.

Although the modal identification is generally implemented based on frequency response function which can be obtained from input-output relation, it is difficult to specify and record input vibrations into a structure from microtremor data. Some alternative approaches have been proposed, which is premised on using only output records without the use of the input records. The Frequency domain decomposition (FDD) method proposed by R.Brincker is one of the output records only modal identification method. In this method, the modal parameters are identified by taking the singular value decomposition (SVD) of a power spectral density matrix under assumption of white noise input. Despite the difficulty of input specification, many of methods, which use only output motions, assume input vibrations as white noise. Therefore, accuracy of parameters identified by these methods depends on whether the assumption of input motions are satisfied.

A new technique is proposed for the modal identification, which does not require any assumption to the input vibrations. Cross correlation analysis is applied to three records in order to calculate spectral ratio between records at two target sites. The proposed technique for modal identification is presented on the basis of the cross correlation analysis. It is demonstrated analytically under an assumption of individual modes that ratios of cross correlations can reduce the influences from incoherent noise and input motions, which are unknown. A closed form relationship is derived between the ratios of cross correlations and modal vectors. An inversion technique is applied to the relationship and the modal vectors are estimated.

The proposed method is applied to a numerical simulation data with five-degree-of-freedom system. It is confirmed that the method can identify modal vectors without any information on input motions. Some difficulties, however, are also recognized for the identification of modal damping ratios. This means that some improved algorithms are required for more accurate identification of structural damping.

Keywords: Modal characteristics; Microtremor; Cross correlation analysis; Structural health monitoring (SHM)

1. Introduction

Evaluating the structural performance of the existing structures is important to confirm the structural soundness after natural disaster or validity of the seismic retrofit [1]. Many kinds of techniques are proposed for modal identification, in which many of them are based on vibration records obtained from excitation experiments, earthquake, microtremors, and so on, and they are usually applied to evaluate dynamic characteristics of a target structure. Generally, the objective of the modal identification is to determine natural frequency, modal vector and modal damping ratio, called modal characteristics as parameters. For architectures and civil engineering structures, modal identification method using microtremor records is desirable, since microtremors can be observed readily anytime and anywhere. Techniques based on microtremor data, furthermore, are also expected to apply the structural health monitoring (SHM) to identify the properties for linear response.

The modal identification is generally implemented based on frequency response function which can be obtained from input-output relation, thus it is assumed that input motions is earthquake ground motions observed on ground surface near a target structure in a case of using earthquake ground motions. It, however, is difficult to identify the input motions during the earthquake because of effects of interactions between soil and structure, and structural responses may include non-linear effects for huge ground motions. On the other hand, in a case using microtremors, we have to take account of not only ground motions but also many kinds of sources such as local forces by wind. Of course, it is impossible to observe whole the input motions to the target structure. We, thus, face limitations on the applicability of input-output relations by microtremors.

To avoid the problems specific to microtremors, some alternative approaches have been proposed, which is premised on using only output records without the use of the input records [2, 3]. Despite the difficulty of input specification, many of methods, which use only output motions, assume input vibrations as white noise. Therefore, accuracy of parameters identified by these methods depends on whether the assumption is satisfied for input motions.

The three parameters for modal characteristics, that is, natural frequency, modal vector and modal damping ratio, are important to estimate dynamic behaviors of structure during earthquake, though modal damping ratio is less sensitivity to structural response than the other two parameters. This leads less accuracy of identification of damping ratio and estimation of too small structural response. To develop a technique is very important to identify modal damping ratio, though this is still difficult for us to solve. Thus, this work focuses development of a method which uses microtremor data and does not require any assumptions on input motions. Cross correlation analysis is applied to three records, at least, which are observed simultaneously at different levels of a target structure. As a first step, a technique is proposed to identify modal vectors using real part of a ratio for cross correlations. The proposed method is applied to numerically simulated data and discuss its applicability and limitations.

2. Theoretical Background of Proposed Method

2.1 Frequency response function

Frequency response function provides relationships between input and response in frequency domain. The modal identification is usually performed based on the frequency response function. A multi-degree-of-freedom (MDOF) system with N degree is considered. Let $X_q(\omega)$ be input motion at q-th mass and $Y_p(\omega)$ be response motion at p-th mass. The frequency response function $H_{pq}(\omega)$ between these two masses can be represented as

$$H_{pq}(\omega) = \frac{Y_p(\omega)}{X_q(\omega)} = \sum_{r=1}^{N} \left(\frac{u_{rp} u_{rq} / M_r}{\omega_r^2 - \omega^2 + 2 j h_r \omega_r \omega} \right), \tag{1}$$

where ω is circular frequency, u_{rp} and u_{rq} are r-th eigenmode of p- and q-th mass, respectively, M_r is r-th modal mass, ω_r is r-th eigen frequency, h_r is r-th modal damping ratio, and $j = \sqrt{-1}$. In a case where the system is excited by ground motions, relationships between input and response can be represented by using Eq.(1). To simplify the representation, we introduce modal participation factor β_r and simplified frequency response function $H_{p0}(\omega)$ for relationships between ground motion $X_0(\omega)$ and response $Y_p(\omega)$ at p-th mass:

$$H_{p0}(\omega) = \frac{Y_p(\omega)}{X_0(\omega)} = \sum_{r=1}^{N} \left(\frac{\beta_r u_{rp}}{\omega_r^2 - \omega^2 + 2jh_r\omega_r\omega} \right). \tag{2}$$

In the following sections, relationships will be analytically derived between eigenmodes and cross correlations using the frequency response function.

2.2 Analysis for cross correlations

Cross spectra is discussed in frequency domain between responses at two masses. A ratio between two cross spectra is introduced to reduce noise such as observation noise and to identify accurately spectral ratios of responses with respect to common input motions. Thus, simultaneous observations are necessary at least three masses, in which two records of them should be obtained at target masses and the other one at an arbitrary mass of the structure. Let $Y_s(\omega)$ be Fourier spectrum of $y_s(t)$ for $s = 1, 2, \ldots, N$, and we introduce cross spectral ratio $R_{pq}(\omega)$ for the targets of p- and q-th masses using another mass, which is ℓ-th, as follows:

$$R_{pq}(\omega) = \frac{S_{p\ell}(\omega)}{S_{q\ell}(\omega)}, \tag{3}$$

where $S_{mn}(\omega) = \langle Y_m(\omega)Y_n^*(\omega)\rangle$ $(m, n = p, q, \ell)$ is cross spectrum between responses at m- and n-th masses, and * and $\langle \cdot \rangle$ stand for complex conjugate and expectation operator in the meaning of ensemble, respectively. $R_{pq}(\omega)$ is a function of frequency response functions and free from observation noise. As a special case where input motion can be only ground motion, we can obtain $R_{pq}(\omega) = \dfrac{H_{p0}(\omega)}{H_{q0}(\omega)}$, which is ratio of frequency response functions with respect to ground motion.

2.3 A method for modal identification

The modal characteristics are identified using the cross spectral ratio $R_{pq}(\omega)$ defined as Eq.(3) for multi-degree-of-freedom (MDOF) systems, in which it is assumed that the damping term is proportional viscous damping. In a case where different modes can be divided into different parts around the vicinity of each natural frequency, the MDOF system can be approximated as sum of one degree-of-freedom (1DOF) system with a natural frequency for each fundamental and higher mode. This approximation will be acceptable for a case where the modal damping ratio is enough small such as around 1%. For example, Eq.(2) yields for s-th mode with natural frequency ω_s as,

$$H_{p0}(\omega) = \sum_{r=1}^{N} \left(\frac{\beta_r u_{rp}}{\omega_r^2 - \omega^2 + 2jh_r\omega_r\omega} \right) \approx \frac{\beta_s u_{sp}}{\omega_s^2 - \omega^2 + 2jh_s\omega_s\omega}. \tag{4}$$

Under this approximation, real part of the cross spectral ratio $R_{pq}(\omega)$ at frequency ω_s for s-th mode provide ratio of modal amplitudes between p- and q-th masses:

$$\Re[R_{pq}(\omega_s)] \approx \frac{u_{sp}}{u_{sq}}, \tag{5}$$

where u_{sp} and u_{sq} are the s-th modal vectors at p- and q-th masses, respectively, and $\Re[\,\cdot\,]$ stands for real part of a complex number.

Furthermore, it is derived analytically that imaginary part of the cross spectral ratio $\Im[R_{pq}(\omega)]$ has higher sensitivity for modal damping ratio than its real part. This suggests that we can identify modal damping ratio using $\Im[R_{pq}(\omega)]$, however, we will leave this topic for future works.

From these results, an algorithm is proposed to identify the modal parameters as follows:

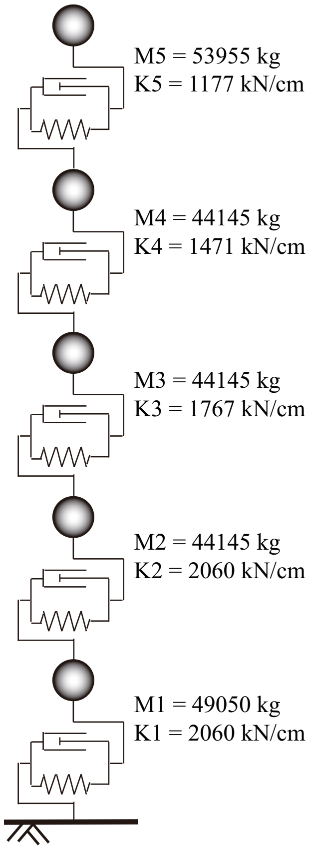


Fig. 1 A model of 5DOF system for numerical simulation.

Table 1 – Modal parameters for numerical model of 5DOF system

Mode No.	r	1	2	3	4	5
Natural frequency	ω_r [Hz]	2.82	7.55	11.98	15.74	19.21
Mode damping ratio	h_r	0.040	0.040	0.035	0.035	0.030
Modal vectors	u_{r1}	1.00	1.00	1.00	1.00	1.00
	u_{r2}	1.93	1.46	0.65	-0.32	-1.47
	u_{r3}	2.85	1.18	-0.68	-1.07	1.00
	u_{r4}	3.69	0.04	-1.12	1.19	-0.41
	u_{r5}	4.32	-1.42	0.70	-0.34	0.07

Table 2 – Input motions for numerical simulations

	ground motion	external force to mass	forced mass(es)	note
Case 1	white	–	–	
Case 2	non-white	non-white	#5	uncorrelated
Case 3	–	non-white	#3, #4, #5	correlated

Table 3 – Natural frequencies estimated from peaks of Fourier amplitude spectra

Mode No. r	1	2	3	4	5
Target	2.82	7.55	11.98	15.74	19.21
Case 1	2.78	7.63	11.77	15.81	19.95
Case 2	2.83	7.67	11.86	15.49	19.86
Case 3	3.00	7.48	12.08	14.97	19.15

(1) Identify the natural frequencies from Fourier amplitude spectrum, in which the natural frequencies are determined at independent peaks of the spectrum,

(2) Identify the modal vector from real part of the cross spectral ratio, $\Re[R_{pq}(\omega)]$.

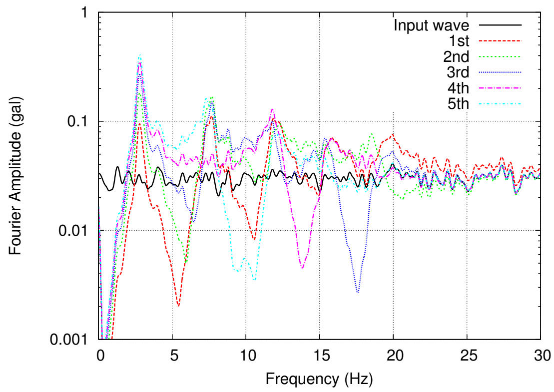
3. Numerical Simulations

3.1 Problem settings

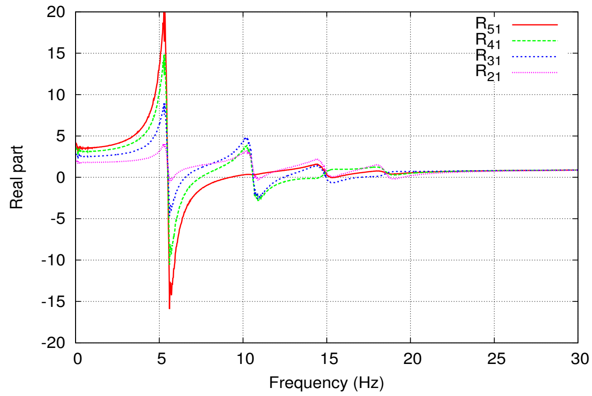
The proposed method for modal identification is applied to numerically simulated microtremor data for 5-degree-of-freedom (5DOF) system to confirm its applicability. Fig.1 and Table 1 shows the 5DOF model and its modal parameters for the numerical simulation. Structural responses are numerically calculated from the mode superposition method, which is conventional technique for the modal analysis. Input motions, which is simulated microtremor, are generated applying Rice's representation [4, 5] with a given power spectrum and random phase angles. The input motions are generated for long duration with sampling rate or 200 Hz. The accelerations of structural response are divided into 20 portions with 40-second length. We average 20 cross spectra calculated from the Fourier transforms of each 40-second portion and apply the averaged cross spectrum to the proposed technique to estimate modal vectors. Some different cases are considered for the numerical simulations as shown in Table 2.

3.2 Case 1 (input from only ground with white noise)

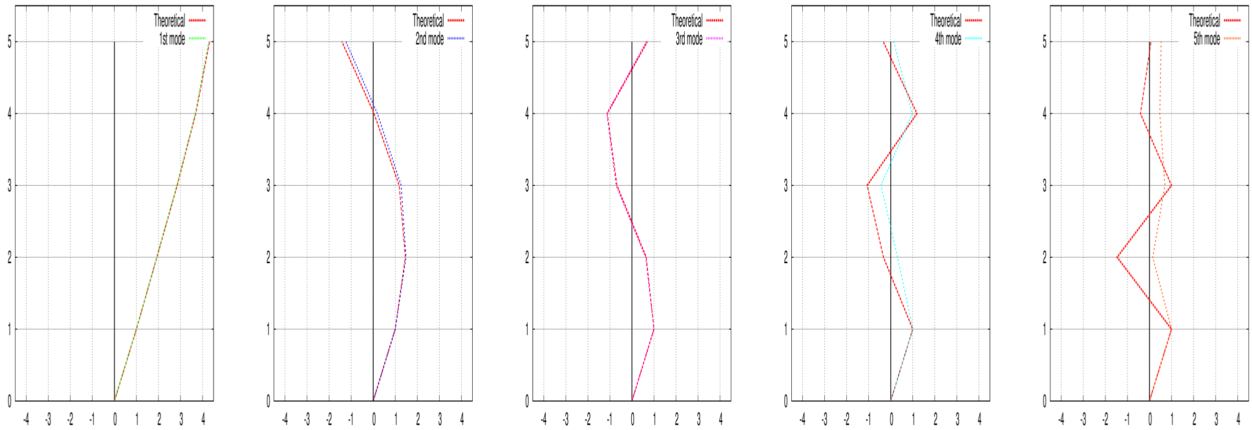
Let us, firstly, discuss Case 1 whose input motion is only ground motion of white noise. Fig.2(a) shows the Fourier amplitude spectra for input (black solid line) and response motions at each mass. From this figure, the natural frequencies can be identified readily for 1st to 3rd modes, though the peaks for 4th and 5th modes are not clear. However, the natural frequencies is determined as listed on Table 3. Fig.2(b) shows real part of cross spectral ratios



(a) Fourier amplitude spectra for input and response motions



(b) Real part of cross spectral ratios



1st 2nd 3rd 4th 5th

(c) Identified modal vectors

Fig. 2 – Modal parameters identified through the proposed method (Case 1).

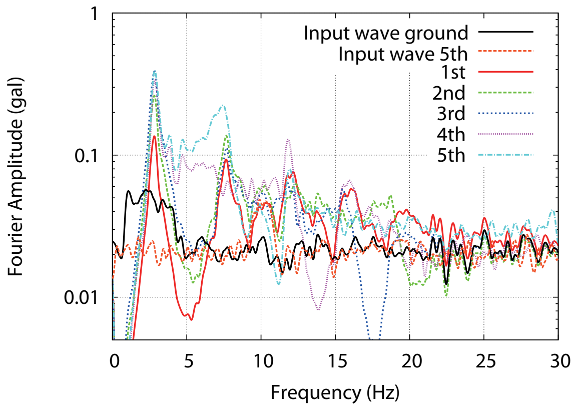
defined as Eq.(5), whose subscripts are $q = 1$ and $p = 2$, 3, 4, and 5. The identified mode vectors are shown in Fig.2(c). In this figure, the red lines are target vectors and the others are identified. Mode vectors for 1st to 3rd modes agree with target ones very well, though the shapes for 4th and 5th modes are similar to target and their amplitudes shows not so good agreements. This seems to be caused by the excitations are not enough for 4th and 5th modes and suggests that the accuracy of eigen frequency is important to identify the mode vectors.

3.3 Case 2 (input from ground and 5th mass with uncorrelated non-white noise)

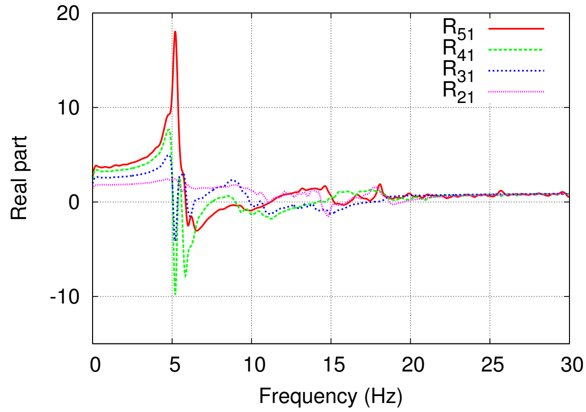
Non-white noise inputs which are uncorrelated each other and have a different frequency characteristic are imposed as a ground motion and external force to the 5th mass. Fig.3(a) shows the Fourier amplitude spectra for input motions and responses at each mass. Identified natural frequencies are listed on Table 3 and Figs.3(b) and (c) show real part of cross spectral ratios and identified mode vectors, respectively. The modal vectors were accurately estimated through the proposed method except for the 5th mode vector. This comes from the enough excitation for less than 4th mode because more power is provided from input at 5th mass. Furthermore, the results are independent of the spectral properties of the input motions, in which this point is an advantage of the proposed method.

3.4 Case 3 (input from 3rd, 4th, and 5th masses with correlated non-white noise)

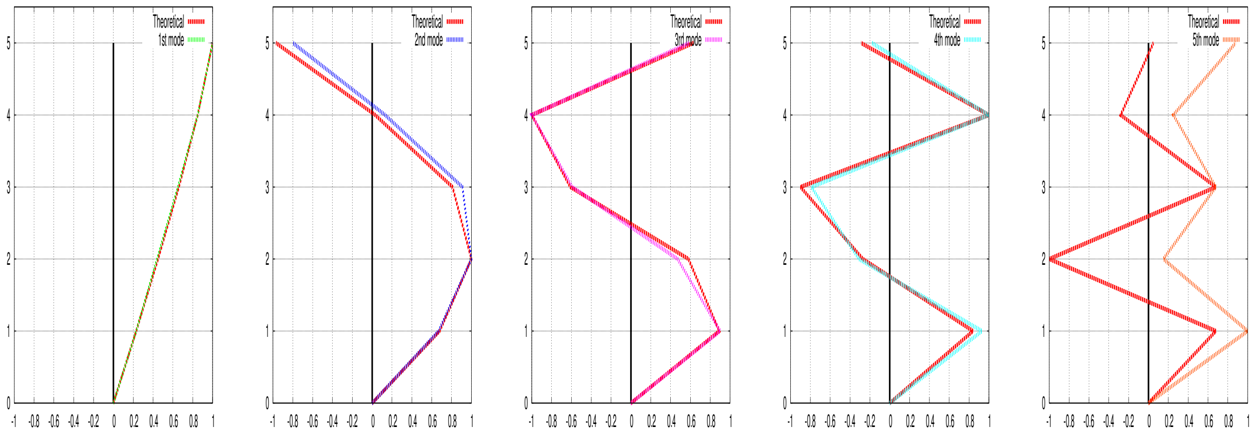
The Fourier amplitudes are different for three masses in the frequency range of 4 to 7 Hz, though the phase angles are same each other. Fig.4(a) shows Fourier amplitude spectra for input and output motions. The results are shown in Table 3 for the identified natural frequencies and Figs.4(b) and (c) for real part of cross spectral ratios and mode vectors, respectively. In this case, the cross spectral ratios are very stable because of identical phases. As the results,



(a) Fourier amplitude spectra for input and response motions



(b) Real part of cross spectral ratios

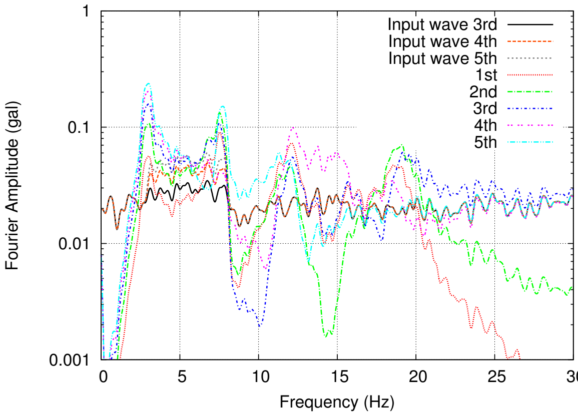


1st 2nd 3rd 4th 5th

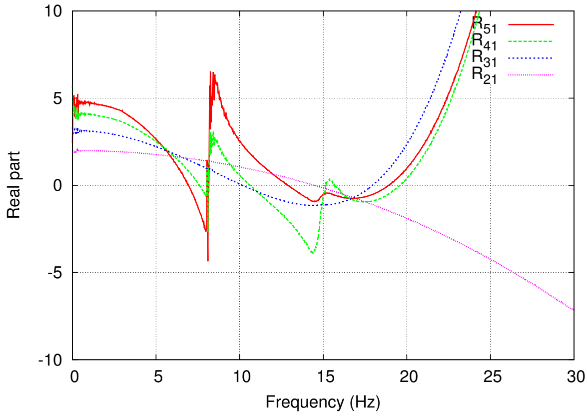
(c) Identified modal vectors

Fig. 3 – Modal parameters identified through the proposed method (Case 2).

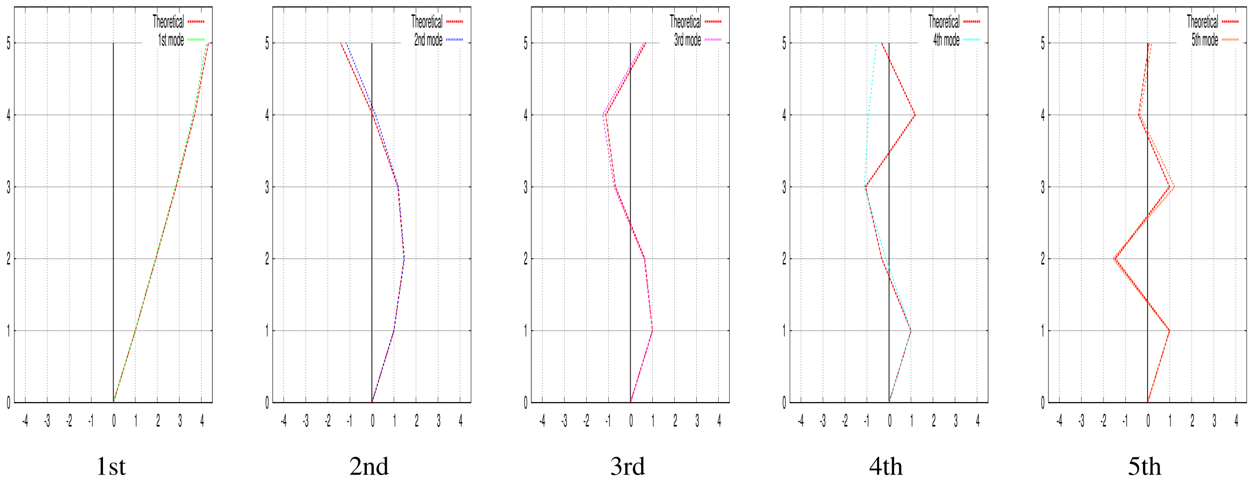
mode vectors are estimated perfectly for 1st to 5th modes. Of course, there are very few cases with identical phases in real phenomena, however, this results indicate that the proposed method is free from properties of input motion: mode vectors can be identified without any information on input motions.



(a) Fourier amplitude spectra for input and response motions



(b) Real part of cross spectral ratios



(c) Identified modal vectors

Fig. 4 – Modal parameters identified through the proposed method (Case 3).

4. Concluding Remarks and Future Developments

A modal identification method is proposed on the basis of cross spectral ratio in this study. This method uses only output motions at masses of multi-degree-of-freedom system and does not require any information on input motions: locations of input, time histories or spectra for input motions, correlations among input motions. The proposed method can be applied under assumptions of small modal damping ratios. It has been confirmed that modal vectors can be identified using the real part of cross spectral ratios through numerical simulations with a few cases of input motions.

Furthermore, the concept of cross spectral ratio suggests that the imaginary parts of cross spectral ratios have better sensitivity to identify modal damping ratio than its real parts. A technique will be developed to identify the modal damping ratio in near future.

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