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NONLINEAR SITE RESPONSE MODEL WITH IMPROVED DAMPING AND STRAIN SOFTENING

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Abstract

This paper presents the mathematical formulation of the one-dimensional nonlinear seismic site response program QUIVER, which contains two different analysis options. The first option performs a nonlinear site response analysis with a hyperbolic-type backbone curve and Davidenkov's model (1938) to describe the unloading-reloading behavior. The Davidenkov (1938) model gives narrower hysteresis loops than the traditional Masing rules, which result in less damping at large shear strains that more accurately reflects laboratory tests. In addition, the hyperbolic model can accurately estimate both small strain nonlinearity and a given shear strength at failure. The second option performs a nonlinear site response analysis using a visco-elastic linear loading-unloading model with a tri-linear model for strain softening post peak strength. The inclusion of a model for strain softening in Quiver is unique and not available in most seismic site response programs. Finally, this paper compares results from the first option with results from another commonly used one-dimensional site response program using ground motions with forward directivity, and demonstrates the effects and importance of strain softening on site response analyses.

Keywords: seismic site response; strain softening; Davidenkov model; forward directivity

1. Introduction

This paper presents the one-dimensional nonlinear seismic site response program QUIVER [1, 2], which contains two different analysis options. The first option addresses two concerns with site response analysis discussed by Stewart et al (2008) [3]. They showed that site response models using the extended Masing rules tend to over-predict damping at large shear strains and cannot match both shear modulus reduction curves and damping curves as measured in the laboratory. Second, they found that when laboratory shear modulus reduction curves are extended to large strains (1-10%) they often do not match the soil shear strength.

The second option addresses the issue of strain softening. Strain softening is the reduction of shear strength as the shear strain increases past the shear strain value where the peak shear strength occurs. Soils that have strong strain softening characteristics are most susceptible to complete failure during earthquake shaking. Strain softening occurs in many soils, however, the most susceptible are usually clays at high water content (> 0.9 × liquid limit) such as those in Eastern Canada and Scandinavia. In addition, marine clays may exhibit strain softening due to leaching of salt from the pore water, which reduces interparticle bonding. These clays are commonly known as quick clays.

This paper first describes the mathematical formulation of both analysis options in QUIVER. Then, it compares the results of the hysteretic model with a commonly used one-dimensional site response program. Finally, it demonstrates the effects and importance of strain softening on site response analyses.

2. Formulation of QUIVER

QUIVER is a one-dimensional total stress nonlinear site response analysis program. QUIVER models the soil profile as a unit soil column consisting of a series of shear beam finite elements resting on an elastic half space. Each element is represented by a mass, a spring, and a viscous dashpot with infinite lateral extent. The dynamic equation of motion for each node of the system is combined into the global equation of motion, which QUIVER solves in the time domain using the constant acceleration Newmark β method [4]. This is an implicit and unconditionally stable integration algorithm method. Small strain damping is simulated by conventional Rayleigh damping for two target frequencies.

QUIVER has two analysis options. The first option performs a nonlinear site response analysis using a hyperbolic-type backbone curve and Davidenkov's model (1938) [5] for unloading and reloading cycles. The second option performs a nonlinear site response analysis using a visco-elastic linear loading and unloading model with strain softening post peak strength. The following sections explain the two different models in detail.

2.1. Hysteretic model

QUIVER uses a normalized hyperbolic model based on the work of Hardin and Drnevich (1972) [6] for the backbone curve, described as:

$$\tau_N = \frac{\gamma_N}{1 + \gamma_N * \left(1 + a * exp(-b * \gamma_N)\right)} \tag{1}$$

$$\tau_N = \frac{\tau}{\tau_{max}} \tag{2}$$

$$\gamma_N = \frac{\gamma}{\gamma_r} \tag{3}$$

$$\gamma_r = \frac{\tau_{max}}{G_{max}} \tag{4}$$

where τ is shear stress, τ_N is normalized shear stress, τ_{max} is shear strength, γ is shear strain, γ_N is normalized shear strain, γ_r is the reference strain, G_{max} is the small strain shear modulus, and *a* and *b* are curve fitting parameters.

To describe the unload and reload behavior of the soil, QUIVER uses a modified version of the extended Masing rules [7, 8]. The extended Masing rules are:

1. The stress strain curve follows the backbone curve for initial loading.
2. The unloading and reloading curves have the same shape as the backbone curve, but enlarged by a factor of two with the origin shifted to the reversal point (γ_{rev}, τ_{rev}). This is expressed mathematically as:

$$\tau = 2 * F_{bb}\left(\frac{\gamma - \gamma_{rev}}{2}\right) + \tau_{rev} \tag{5}$$

where $F_{bb}\left(\frac{\gamma-\gamma_{rev}}{2}\right)$ is the model for the backbone curve evaluated at $\frac{\gamma-\gamma_{rev}}{2}$.

3. When the unloading or reloading curve exceeds the maximum past strain and intersects the backbone curve, the stress strain path follows that of the backbone curve until the next reversal point.
4. When the unloading or reloading curve intersects the curve from the previous cycle, then the stress-strain curve follows the path of the previous cycle.

Stewart et al (2008) [3] showed that site response models using the extended Masing rules tend to over-predict damping at large shear strains and cannot match both shear modulus reduction curves and damping curves as measured in the laboratory. To overcome these shortcomings, QUIVER uses an approach similar to the work of Muravskii (2005) [9], which is based on the model proposed by Davidenkov (1938). Instead of a reduction factor, QUIVER replaces Masing rule 2 and Eq. (5) with Eq. (6):

$$\tau = \tau_{rev} + H(\gamma - \gamma_{rev}) \tag{6}$$

where H is not the backbone curve, but a separate hysteretic function. The hysteretic function proposed by Davidenkov (1938) and used in QUIVER is:

$$H(u) = u * (1 - \beta * |u|^R) \tag{7}$$

$$u = \gamma - \gamma_{rev} \tag{8}$$

$$R = \frac{\tau_{N,bb} - \gamma_{N,bb} * F'_{bb}(\gamma_{N,bb})}{\gamma_{N,bb} - \tau_{N,bb}} \tag{9}$$

$$F'_{bb}(\gamma_{N,bb}) = \frac{\left(1 + a * b * \gamma_{N,bb}^2 * exp(-b * \gamma_{N,bb})\right)}{\left(1 + \gamma_{N,bb} * \left(1 + a * exp(-b * \gamma_{N,bb})\right)\right)^2} \tag{10}$$

$$\beta = \frac{\gamma_{N,bb} - \tau_{N,bb}}{\gamma_{N,bb} * (2 * \gamma_{N,bb})^R} \tag{11}$$

where τ_{Nbb} and γ_{Nbb} are the absolute values of τ_N and γ_N at the point where the unloading or reloading curve left the backbone curve, and $F'_{bb}(\gamma_{Nbb})$ is the derivative of the backbone curve evaluated at the point γ_{Nbb}.

Fig. 1 compares one unloading and reloading cycle (hysteresis loop) calculated using the original Masing rule number 2 to a cycle calculated using the Davidenkov (1938) model. Fig. 1 shows that the Davidenkov (1938) model gives a narrower hysteresis loop than the Masing model, which results in less damping because damping is a function of the area covered by the loop.

Fig. 2 shows the shear modulus reduction, damping, and normalized shear stress curves versus shear strain calculated using the Davidenkov (1938) model as implemented in QUIVER for several different combinations of

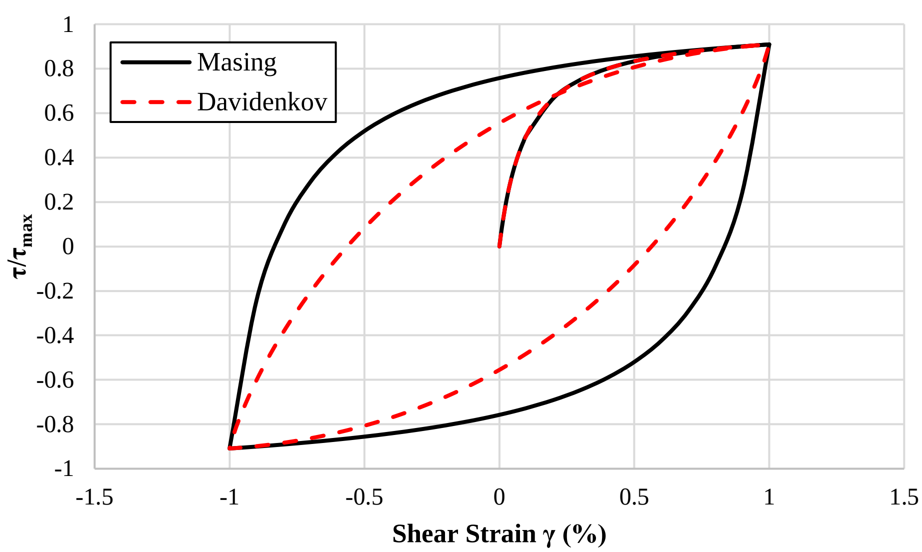

Fig. 1 – Comparison of hysteretic curves using Masing rule 2 and the Davidenkov (1938) model

parameters. Fig. 2 shows that the model predicts a maximum damping value of about 30%, which agrees with the findings of Hardin and Drnevich (1972) and Seed and Idriss (1972) [10]. This is a significant improvement over models that use the Masing rules and predict damping values of up to 60% at large shear strains. However, certain combinations of *a*, *b*, and γ_r may give unrealistic shear modulus reduction curves or damping curves. Therefore, the user should check the estimated curves for each layer to ensure a good match to measured data.

Another significant improvement of the hysteretic model implemented in QUIVER is its ability to match a specified shear strength at large shear strains. Most laboratory investigations of the dynamic properties of soils do not measure shear strains larger than 0.1 to 0.5% [11, 12]. When they are extrapolated to shear strains of 1-10% there is no guarantee that they will match the shear strength. Fig. 2 shows that the model in QUIVER reaches the specified shear strength at a shear strain of 10%.

2.2 Strain softening model

Fig. 3 shows the constitutive model used to simulate strain softening. The constitutive model consists of a visco-elastic linear loading and unloading model with strain softening post peak strength. QUIVER models strain softening using a trilinear function, where the user specifies the values of G_{max}, τ_{max}, τ_2, γ_2, τ_3, and γ_3 (see Fig. 3). This formulation allows great flexibility when modelling the effect of strain softening. One weakness with the

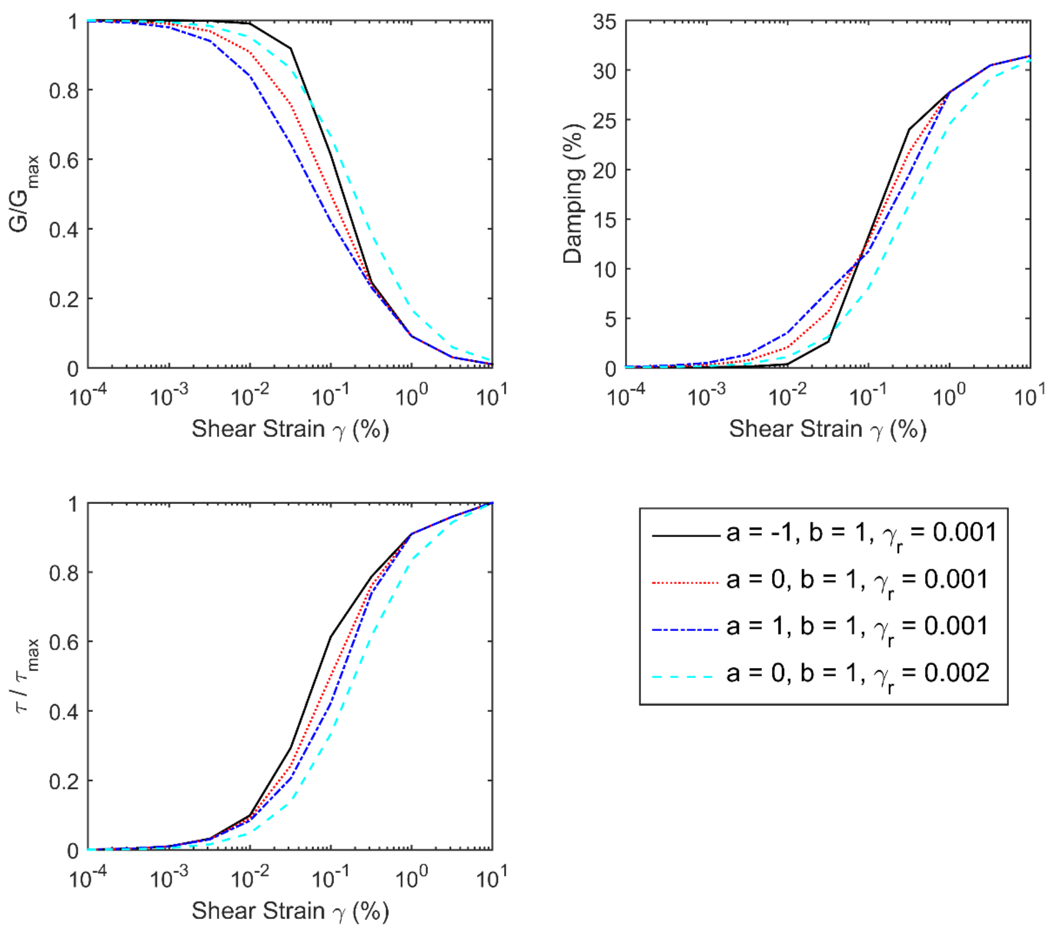

Fig. 2 – Shear modulus reduction, damping, and normalized shear stress versus shear strain curves for several different combinations of parameters

model is that there is no hysteretic damping and all damping must be specified with Rayleigh damping, which does not change with shear strain. However, strain softening will only affect the results at large shear strains when damping will also be large and the effect of small strain cycles will be minimal. Nevertheless, we are currently working on replacing the visco-elastic loading and unloading model with the hysteretic model described in section 2.1. This combined model will allow a more accurate description of soil stiffness and damping over a large strain range.

Another feature of the strain softening model is that it allows the user to tilt the entire soil column at a specified angle to evaluate slope stability during seismic shaking. For each layer, QUIVER calculates the effective normal stress (σ_n) in the direction normal to the slope, and a consolidation shear stress (τ_c) acting in the plane of the slope parallel to the dip. Similar to a 1D seismic site response analysis for flat ground, the earthquake motion is assumed to consist only of shear waves propagating perpendicular to the slope, and those propagating along the plane of the slope (surface waves) are ignored. The earthquake shaking then adds additional cyclic shear stresses acting on the plane of the slope. The sloping ground feature is not considered further in this paper, and all analyses are for level ground conditions.

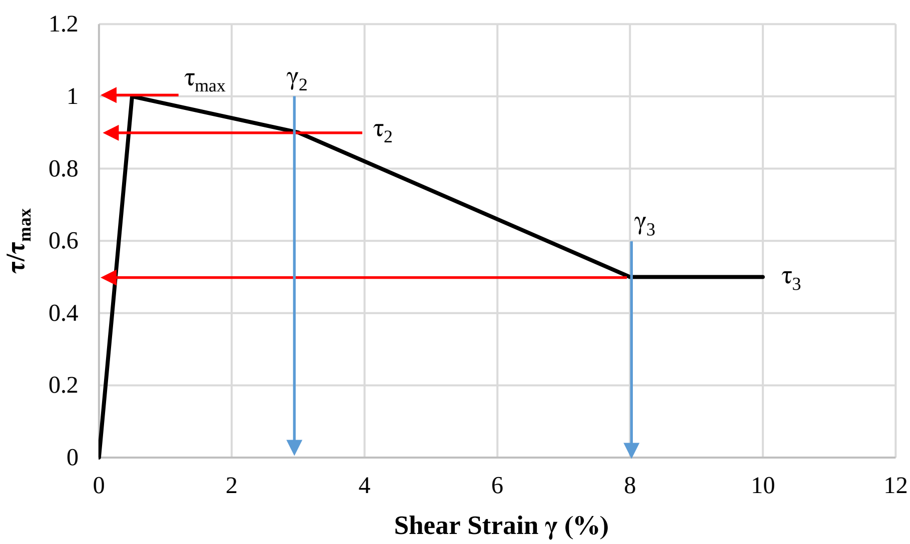

Fig. 3 – Strain softening constitutive model in QUIVER

3. Comparison of QUIVER with DEEPSOIL

This section compares results from QUIVER using the Hardin and Drnevich (1972) backbone curve and the Davidenkov (1938) model for unloading-reloading with the results of the General Quadratic/Hyperbolic (GQ/H) model [13] implemented in the 1D site response analysis program DEEPSOIL [14]. The GQ/H model describes the backbone curve and can accurately model both a given shear strength at failure and small strain nonlinearity as measured by laboratory experiments. In DEEPSOIL, the GQ/H model is coupled with the MRDF model [15] to describe the unload-reload behavior of the soil. The MRDF model applies a reduction factor similar to the model of Darendeli (2001) [11] to reduce the predicted damping at large shear strains and better match laboratory results.

3.1 Input parameters

We performed site response analyses for one soil profile and 19 two component horizontal ground motions for a total of 38 analyses. We took the ground motions from the PEER NGA West 2 online database [16]. All of the ground motion pairs have a directivity pulse. Table 1 lists the ground motion record sequence numbers in the PEER NGA West 2 database (RSN), moment magnitude (M_w), rupture distance (R_{rup}), time averaged shear wave velocity over the top thirty meters (Vs_{30}), peak ground acceleration (PGA), peak ground velocity (PGV), and duration between when 5% and 75% of the arias intensity is reached ($D_{5\text{-}75}$) for each of the acceleration time series.

Fig 4 shows the shear wave velocity and shear strength profile used for the site response analyses. We calculated the shear wave velocity profile from 0 to 35 meters depth using the model by Carlton and Tokimatsu (2014) [17] for a generic NEHRP E site. The shear wave velocity profile below 35 meters depth and the shear strength profile are based on common values for soft clays over stiff clayey sands. The unit weight, *a*, and *b* values for all layers were 19 kN/m^3, 0 and 1, respectively. Based on the recommendations of Stewart et al (2008), we selected the target frequencies for Rayleigh damping as the elastic site frequency and five times the elastic site frequency, and we used the input motions as recorded with no deconvolution and an elastic half-space underlying the soil profile. For the elastic half-space, we used Vs = 600 m/s, unit weight of 20 kN/m^3, and damping ratio of 1%. We adjusted the thickness of the soil layers so that the maximum frequency propagated through the site was 15 Hz.

Table 1 – Ground motions used in the site response analyses

RSN	M_w	R_{rup} (km)	V_{s30} (m/s)	Component 1			Component 2		
				PGA (g)	PGV (cm/s)	$D_{5\text{-}75}$ (s)	PGA (g)	PGV (cm/s)	$D_{5\text{-}75}$ (s)
285	6.90	8.18	650	0.19	34.7	4.9	0.13	23.6	5.2
451	6.19	0.53	561	1.28	78.5	1.7	0.70	52.9	3.0
459	6.19	9.87	663	0.29	36.5	4.5	0.22	11.2	3.1
983	6.69	5.43	526	0.57	76.2	4.2	0.99	67.4	3.6
1013	6.69	5.92	629	0.43	74.9	3.7	0.32	47.4	4.0
1475	7.62	56.12	570	0.12	37.9	12.0	0.09	26.7	11.8
1482	7.62	19.89	541	0.14	56.6	12.9	0.20	55.3	15.8
1487	7.62	35.00	520	0.30	42.0	10.1	0.40	34.4	9.2
1492	7.62	0.66	579	0.44	172.3	4.5	0.36	151.2	5.7
1502	7.62	16.59	646	0.12	55.3	19.5	0.11	42.7	19.4
1511	7.62	2.74	615	0.43	59.8	16.5	0.34	51.9	17.5
1519	7.62	6.98	539	0.12	45.0	14.9	0.11	40.5	16.1
1548	7.62	13.13	600	0.14	63.8	13.4	0.17	62.7	10.3
2734	6.20	6.20	553	0.34	43.5	2.6	0.32	32.9	4.4
3744	7.01	12.24	566	0.18	67.9	3.6	0.21	48.3	3.6
4097	6.00	2.99	648	0.35	53.2	2.3	0.21	26.0	2.8
4482	6.30	6.55	552	0.40	32.1	4.7	0.43	27.0	3.3
6928	7.00	25.67	650	0.36	30.3	7.6	0.23	17.7	7.4
8164	7.14	2.65	690	0.30	38.9	11.6	0.28	29.0	10.6

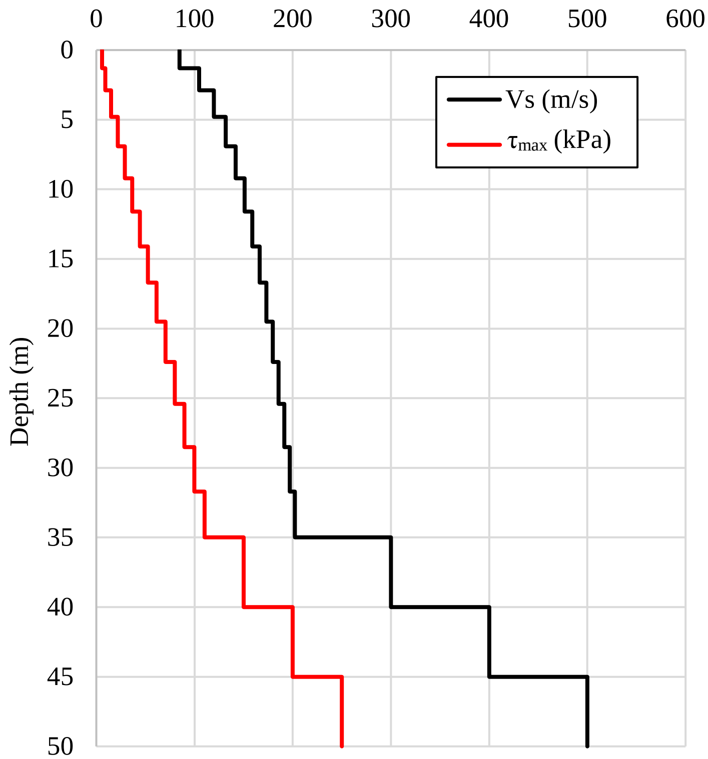

Fig. 4 – Soil profile used in site response analyses

3.2 Results

Fig 5 compares results from QUIVER and DEEPSOIL for several different ground motion parameters. Fig 5a shows the PGA at the soil surface calculated by QUIVER and DEEPSOIL versus the PGA of the input ground motion, as well as the curve from Seed et al (1997) [18] for E soil sites, the best fit line for the QUIVER data, and the 1:1 line. Seed et al (1997) classify E soil sites as sites with more than 3 meters of soft clay and various levels of shaking intensity, where soft clay is defined as cohesive soil with fines content ≥ 30%, plasticity index ≥ 20, and Vs < 180 m/s. Fig 5a shows that the best fit line for the QUIVER results follows the curve from Seed et al (1997) very well, and predicts higher PGA surface values than DEEPSOIL. The equation for the QUIVER best fit line is:

$$PGA_{surface} = 0.57 * PGA_{input}^{0.44} \tag{12}$$

The PGA surface values predicted by DEEPSOIL cross the 1:1 line at about 0.2 g, and show much more nonlinearity than the QUIVER results. This is a very interesting result because the models in QUIVER and DEEPSOIL give the same shear modulus reduction and damping curves and use the same Rayleigh damping values.

Fig 5b shows the total permanent displacement (u_{max}) at the end of shaking versus the PGV of the input ground motion. All of the input ground motion pairs have forward directivity and large velocity pulses that cause permanent displacement at the soil surface. We specifically selected ground motions with forward directivity to test the capability of QUIVER to estimate permanent displacements. Fig 5b shows that as the PGV of the input ground motion increases, the estimated value of u_{max} increases for both QUIVER and DEEPSOIL. However, in general, QUIVER estimates larger u_{max} values than DEEPSOIL.

Fig 5c shows the estimated arias intensity (I_a) at the soil surface versus the I_a of the input ground motion. As the I_a of the input ground motion increases the I_a predicted at the surface shows a nonlinear effect, or "bendover curve", similar to the relation between PGA at the soil surface and PGA of the input motion shown in Fig 5a. In general, QUIVER predicts larger values of I_a at the soil surface than DEEPSOIL, however there is much scatter.

Fig 5d shows the calculated $D_{5\text{-}75}$ at the soil surface versus the $D_{5\text{-}75}$ of the input ground motion. These values fall slightly above the 1:1 line, which means that the ground motion at the surface has a larger value of $D_{5\text{-}75}$ than the input motion. This is reasonable because soft soil sites tends to continue to vibrate after the initial shaking has stopped. Both QUIVER and DEEPSOIL predict similar values of $D_{5\text{-}75}$ at the soil surface.

Fig 6 compares the acceleration time series and response spectra predicted by QUIVER and DEEPSOIL at the soil surface for component 1 of ground motions RSN 1502 and RSN 1013. These ground motions are representative of the results for the other site response analyses. The results of RSN 1502 show that for lower intensity ground motions, QUIVER and DEEPSOIL predict very similar results. The main difference is that QUIVER predicts several peaks in the acceleration time series to be larger than those predicted by DEEPSOIL, which corresponds to the larger PGA surface values predicted by QUIVER even for PGA input values of 0.1 g shown in Fig 5a.

For larger intensity ground motions, as represented by RSN 1013, the two programs predict different results. Fig 6 shows that the acceleration time series predicted by DEEPSOIL for RSN 1013 saturates at around 0.2 g, whereas the one predicted by QUIVER predicts almost ± 0.4 g. After the first large acceleration pulse the phases become different, and QUIVER and DEEPSOIL predict the peaks occurring at different times. The response spectra reflects these differences, and shows that QUIVER predicts greater response spectral values for long periods than DEEPSOIL. For periods shorter than about 0.4 seconds, both programs give similar response spectral values.

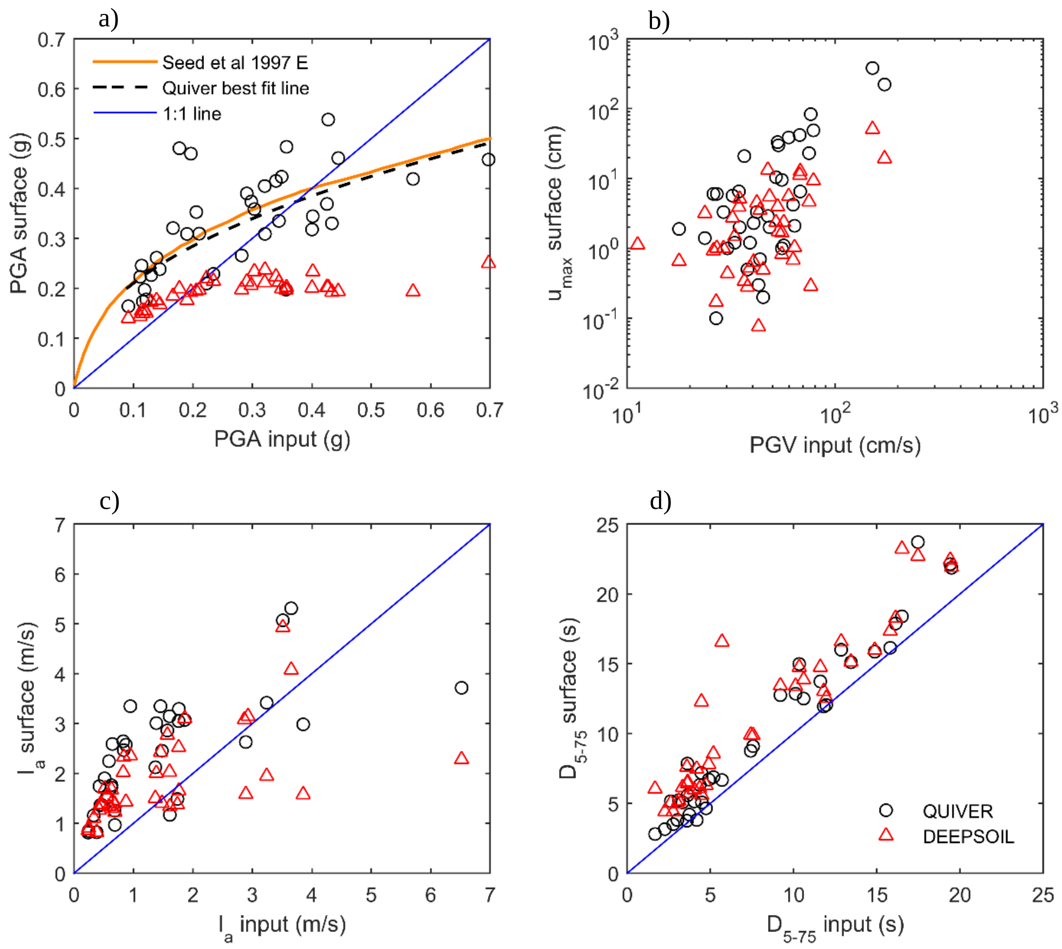

Fig. 5 – Summary comparison of QUIVER and DEEPSOIL results for all analyses

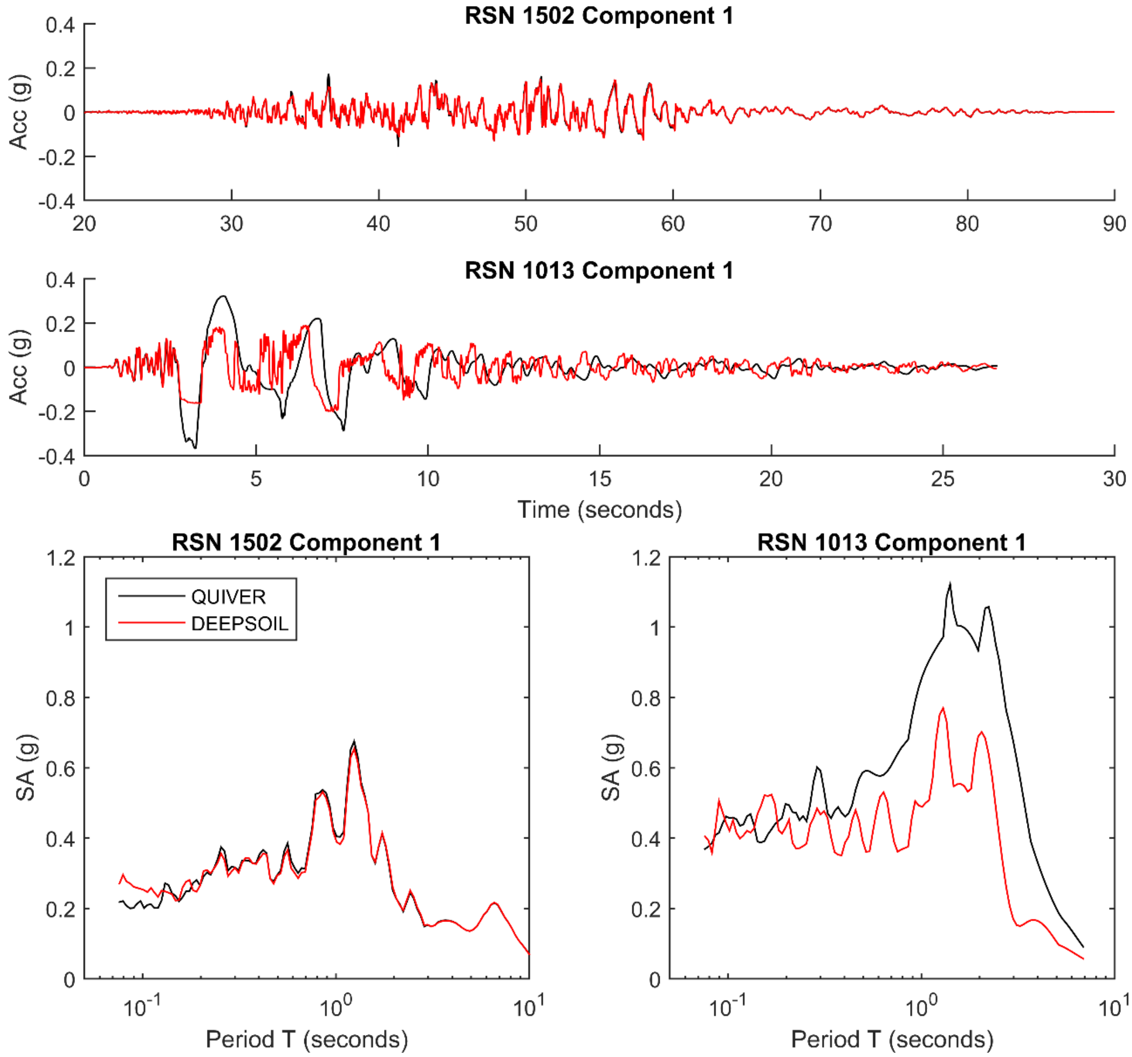

Fig. 6 – Comparison of predicted acceleration time series and response spectra for two analyses

4. Effect of strain softening on site response

4.1 Input parameters

To investigate the effect of strain softening, we conducted seismic site response analyses using the strain softening model of QUIVER and the ground motions listed in Table 1. We used the same soil layering and τ_{max} values as shown in Fig 4, but only ½ the G_{max} values. We reduced the G_{max} values and increased the Rayleigh damping to provide a better match between the strain softening model and the hysteretic model. We increased the Rayleigh damping because the strain softening model does not generate hysteretic damping, and therefore Rayleigh damping must account for damping at all shear strains and not only small strains. The unit weight for all layers was 19 kN/m^3, and we modeled the elastic half-space with Vs = 600 m/s, unit weight of 20 kN/m^3, and damping ratio of 1%, similar to the analyses using the hysteretic model.

We conducted analyses for three different sensitivity values (1, 2 and 4), where sensitivity (S_t) is defined as the ratio of peak shear strength to the residual shear strength (τ_{max} / τ_{res}). For all analyses, only the sensitivity of the top 35 meters was changed. For layers deeper than 35 meters we kept S_t = 1. Fig 7 shows the different strain softening models compared with the hysteretic model of option 1.

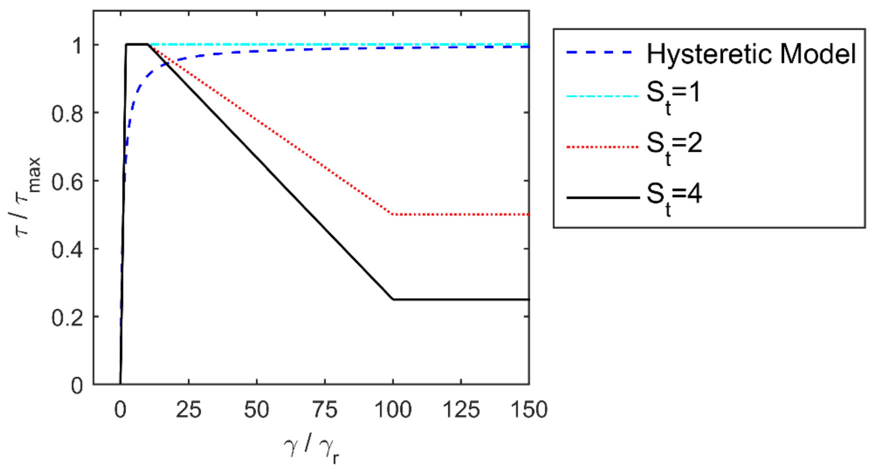

Fig. 7 – Strain softening models used in analyses

4.2 Results

Fig 8 compares the results of the site response analyses using different sensitivity values. Fig 8a shows the estimated PGA at the soil surface using all three sensitivity values versus the PGA of the input ground motion. As the input PGA increases the response becomes increasingly nonlinear, similar to the results of Fig 5a for the hysteretic model. At low values of input PGA the analyses with soil sensitivity of 2 and 4 predict slightly larger values of surface PGA, whereas at high values of input PGA the opposite is true. However, the mean difference between analyses with $S_t = 1$ and $S_t = 4$ is less than 2%, with a standard deviation of 4%. Therefore, the effect of soil sensitivity of 4 or less on PGA is most likely negligible.

Fig. 8b shows the effect of sensitivity on the estimated maximum permanent displacement versus the input PGV. The results show that as sensitivity increases the maximum predicted permanent displacement at the soil surface increases. The maximum predicted permanent displacement at the soil surface will tend to increase as sensitivity increases because the soil strength reduces more. This effect is greater for larger intensity motions that induce greater shear strains. The mean difference between predicted values of u_{max} for analyses with $S_t = 1$ and $S_t = 4$ is about 75%, which is significant. However, the standard deviation of the difference is about 100%, which implies a large amount of uncertainty and that in many cases the estimated u_{max} could be smaller for sensitive soils than for non-sensitive soils. The large uncertainty could be in part due to the fact that the ground motions used in the site response analyses have directivity pulses.

The effect of soil sensitivity on I_a changes with ground motion intensity, similar to the results for PGA. Fig. 8c presents the predicted surface I_a versus the input I_a for all three sensitivity values. At low values of input I_a the analyses predict slightly larger values of surface I_a for sensitive soils than for non-sensitive soils, whereas the opposite is true for high values of input I_a. This is because as sensitivity increases the soil loses strength at larger shear strains and becomes less stiff, which in turn reduces the amount of energy that can be propagated to the soil surface. The mean difference between predicted values of I_a for analyses with $S_t = 1$ and $S_t = 4$ is about 15%, with a standard deviation of 25%.

Fig 8d shows that sensitivity has a negligible effect on the predicted value of the surface $D_{5\text{-}75}$. It must be remembered, however, that these results are for one site only and a limited number of ground motions. Soft soil sites, and especially sensitive soils, are highly variable. This underlines the importance of having a tool such as QUIVER with which to conduct site specific analyses.

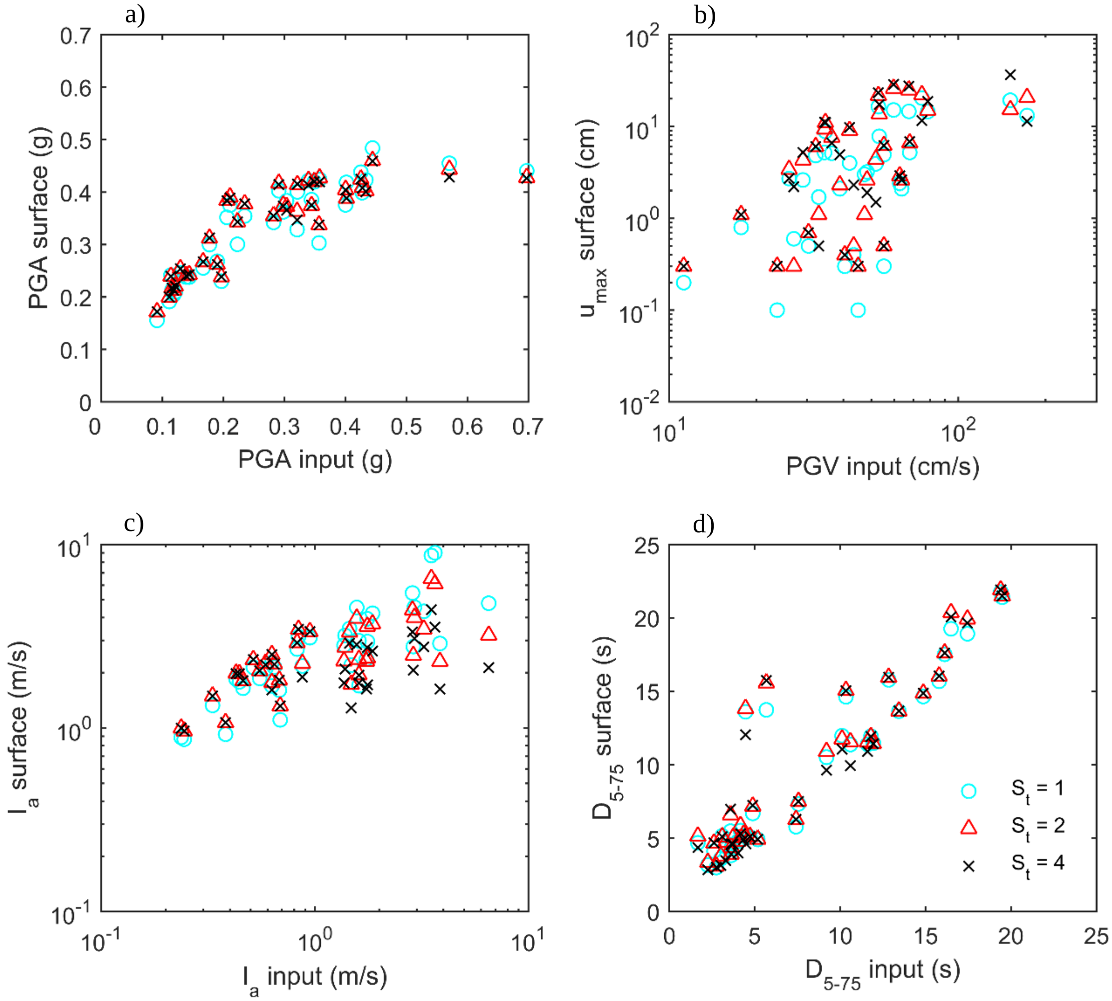

Fig. 8 – Effect of strain softening on different site response parameters

5. Summary and Conclusions

This paper presented the one-dimensional nonlinear seismic site response program QUIVER, which consists of two analysis options. The first option implements a normalized hyperbolic backbone curve based on the work of Hardin and Drnevich (1972) and uses Davidenkov's model (1938) to describe the unloading and reloading behavior. This model can accurately estimate both small strain nonlinearity and a given shear strength at failure. In addition, the Davidekov model (1938) gives narrower hysteresis loops that predict lower damping at large shear strains, which is more representative of laboratory test data than the extended Masing rules.

We compared the hysteretic model in QUIVER with the GQ/H model implemented in DEEPSOIL. For low intensity ground motions we found that both programs estimate the same response. However, for larger intensity ground motions, QUIVER estimates cycles with larger acceleration values than DEEPSOIL, even though both models give the same shear modulus reduction and damping curves and have the same Rayleigh damping.

The second option in QUIVER performs a nonlinear site response analysis using a visco-elastic linear loading-unloading model with a tri-linear model for strain softening post peak strength. The inclusion of a model for strain softening in QUIVER is unique and not available in most seismic site response programs. We investigated the effect of strain softening and found that as the soil sensitivity increases, u_{max} tends to increase and for large intensity motions I_a tends to decrease, however, due to the great amount of uncertainty, it is essential to conduct a site specific analysis that includes the effect of soil sensitivity.

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