



MODIFIED YIELD POINT SPECTRA SUBJECTED TO SEQUENCE-TYPE GROUND MOTIONS

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Abstract

This paper presents a new modified yield point spectra (YPS) method, which can be used to assure seismic design safety. Historical earthquake data have shown that structures suffering from aftershock ground motions may become damaged and even collapse. More than 267 mainshock-aftershock sequence-type ground motion records were employed to calculate the dynamical response of single-degree-of-freedom (SDOF) systems. This data and the strength reduction factor were used to assess ground-motion damage using ductility demand and accumulated damage demand. This research and resultant modified yield point spectra led to an assessment of strength demand under mainshock ground motions and mainshock-aftershock sequence-type ground motions, respectively. Findings demonstrated that the strength demand under mainshock-aftershock sequence-type ground motions will increase by 10% to 40%. Meanwhile, the modified yield point spectra (YPS) were used to assess damage of structures after earthquakes, and determine admissible design regions of strength and stiffness to satisfy performance-based design objectives. Finally, a test using an RC structure and the modified YPS method was employed to verify the rationality of the new method.

Keywords: yield point spectra, strength reduction factor, main-aftershock sequence-type ground motions, damage index



1. Introduction

Studies of historical earthquake damage have shown that many aftershocks occur in short intervals after a mainshock, so structures in a seismic region can also be subjected to sequence-type ground motions in short periods of time. Structures damaged after the mainshock will have different degrees of degradation based on their stiffness and ductility capacity; moreover, aftershocks have the potential to cause increasing damage and even collapse of a structure. Structures are often not repaired because the intervals between mainshock and aftershocks are too short; therefore, the effects of the mainshock-aftershock sequence-type ground motions should be considered in the design phase of any building.

Since the 1990s, investigations of many major earthquakes have found structures designed by the force-based seismic method can still be damaged under strong earthquakes or sequence-type earthquakes, resulting in extensive property loss and injuries to building occupants. Thus, current life-safety-based seismic design methods have failed to achieve their purpose. Hence, the correlation between deformation and seismic performance is a better design measurement to consider than that between force and seismic performance. The design method based on deformation can be a better guarantee of structural resistance to seismic activity. Therefore, this paper is introducing a design principle based on a displacement-based seismic design method, which includes the capacity spectrum method [1]. Experiments [2] have shown that yield displacements of RC structures were stable and consistent even when periods of vibration (and lateral stiffness) required to meet the performance objective differed substantially. Observations have also shown that yield displacement is a more stable and more useful parameter for seismic design. Accordingly, Aschheim [3] proposed an inelastic response spectrum based on the yield displacement, i.e., the yield point spectrum. As a modified form of capacity spectrum, yield point spectra (YPS) represent the relationship of the yield strength coefficient C_y , and the yield displacement u_y of a series of oscillators of varying natural frequency that are forced into motion by the same base vibration or shock. In this case, the abscissa is the yield displacement of the system, and the ordinate is the yield strength coefficient C_y , $C_y = F_y / (mg)$. Traditional response spectra use an estimate of the period of vibration, which is based on members' stiffness and mass, to determine the design lateral force. YPS can be used to determine the design lateral force based on the estimated yield displacement of the structure.

Besides displacement ductility, the cumulative damage resulting from inelastic cycles also plays an important role in determining the damage state of a structure. However, the YPS only consider the influence of displacement ductility. To take into account the cumulative damage, a modified YPS, which employs a direct damage model in the determination of the seismic demand for a target damage level, is introduced.

Currently, most seismic codes worldwide only consider single 'design earthquake' without taking into account the influence of the mainshock-aftershock sequence-type ground motions. Based on the empirical formula of existing strength reduction factor, this paper examines the characteristics of yield point spectra under mainshock-aftershock sequence-type ground motions. The authors also compare the strength demands of structures under both single earthquakes and sequence-type earthquakes. Then, a simplified expression of the modified yield point spectra is recommended. Finally, an RC frame is subjected to dynamic analysis to verify the reasonability of the method.

2. Determination of modified YPS

2.1 Sequence-type ground motions

A sequence-type ground motion record usually consists of one mainshock event and one or multiple aftershock events, which are called as one earthquake (mainshock only), a sequence of two earthquakes (mainshock plus one aftershock), a sequence of three earthquakes (mainshock plus two aftershocks), and so on. Scenario of mainshock plus one aftershock was commonly considered in previous studies [4–6]. Their results demonstrated that two-sequence earthquakes can provide valuable information about the influence of aftershock. Therefore, mainshock-aftershock sequence-type ground motion in this study is specified as one mainshock plus one aftershock.



With constraints of time and technology, no data of sequence-type ground motions were available before. Hence, artificial sequence-type ground motions or repeated strong ground motions were adopted to evaluate the effect of aftershocks by many researchers. However, it was determined that artificial sequence-type ground motions can lead to significant overestimation of maximum lateral drift demands. Record-to-record variability can also cause problems [7]. The level of overestimation depends on the approach for developing artificial sequences (repeated or randomized approach). The sequence-type ground motions used in this study are selected from the Pacific Earthquake Engineering Research Center (<http://peer.berkeley.edu/nga/>).

In this study, the following criteria were employed for identifying and selecting mainshock–aftershock seismic sequences: (a) use detailed information on the geological and geotechnical conditions of the site, (b) use sequence-type ground motions recorded from stations placed on free-field or low-rise buildings with negligible soil–structure interaction effects, and (c) choose sequence-type ground motions having a peak ground acceleration (PGA) of the mainshock horizontal component greater than 0.10 g and a PGA of the aftershocks greater than 0.05 g. The number of sequence-type ground motions, which are recorded on site classes A and D according to the site classification method of United States Geological Survey (USGS), was too small to meet the demands of this investigation. Thus, the sequence-type ground motions recorded on Site Classes B and C were used. A total of 267 sequence-type ground motions were obtained in this way and are listed in Table 1. Between two consecutive seismic events, a time gap was applied equal to 100 s. This gap is absolutely enough to cease the moving of any structure due to damping.

Table 1 –Number of recorded sequence-type ground motions used in this research

Earthquake name	Mainshock		Aftershock		Number	
	Time	M_w	Time	M_w	Site B	Site C
Hollister	1961/04/09 07:23	5.6	1961/04/09 07:25	5.5	0	1
Managua, Nicaragua	1972/12/23 06:29	6.2	1972/12/23 07:19	5.2	0	2
Imperial Valley	1979/10/15 23:16	6.5	1979/10/15 23:19	5.0	0	26
Livermore	1980/01/24 19:00	5.8	1980/01/27 02:33	5.4	0	1
Mammoth Lakes	1980/05/25 16:34	6.1	1980/05/25 16:49	5.7	2	4
Mammoth Lakes(1)	1983/01/07 01:38	5.3	1983/01/07 03:24	5.3	0	2
Coalinga	1983/05/02 23:42	6.4	1983/05/09 02:49	5.1	0	2
Chalfant Valley	1986/07/20 14:29	5.8	1986/07/21 14:42	6.2	0	3
Whittier Narrows	1987/10/01 14:42	6.0	1987/10/04 10:59	5.3	6	14
Superstition Hills	1987/11/24 05:14	6.2	1987/11/24 13:16	6.5	0	2
Northridge	1994/01/17 12:31	6.7	1994/01/17 12:32	6.1	14	13
Chichi	1999/09/20	7.6	1999/09/20 17:57	5.9	102	71
				Total	126	141

2.2 Elastic response spectra

For a comprehensive study of capacity demand under single earthquakes and sequence-type earthquakes, the elastic response spectra were obtained by time-history analysis of single-degree-of-freedom (SDOF) systems. The SDOF systems with a set of 60 periods between 0.1 and 6.0 s with an interval of 0.1 s were considered, and the viscous damping ratio was assumed to be 5%. Through time-history analyses and statistical average, elastic



response spectra of single earthquakes and sequence-type earthquakes on Site Classes B and C were obtained, as shown in Fig.1. The PGA of ground motions were 0.2 g in this case.

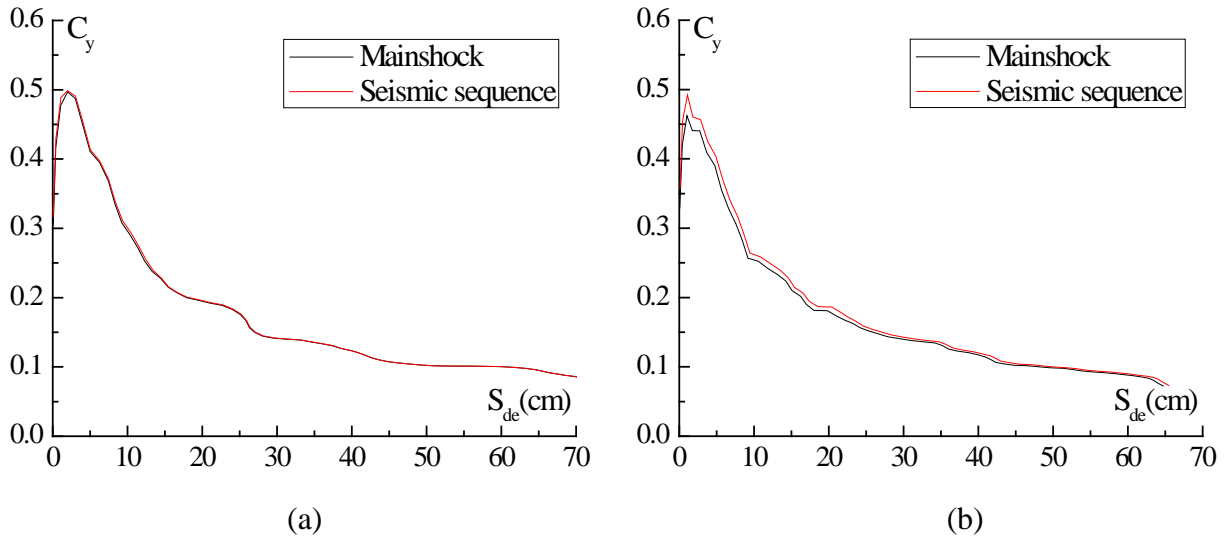


Fig. 1 – Mean elastic spectra: (a) Site Class B; (b) Site Class C

The difference of elastic spectra between mainshock ground motions and sequences-type ground motions are generally within 5% of all the overall mean spectra. Among yield displacement u_y , period T and yield strength coefficient C_y , two of the three parameters are known and the remaining one can be calculated; hence, the relationship is expressed as:

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{mu_y}{F_y}} = 2\pi\sqrt{\frac{u_y}{C_y g}} \quad (1)$$

2.3 Strength reduction factors

In order to determine a reasonable economic structural strength, strength reduction factors were introduced to evaluate inelastic spectra. Strength reduction factors R are defined as the ratio of the elastic strength demand F_e to the inelastic strength demand F_y . In the literature, the reduction factor is derived essentially from the ductility-based strength reduction factor R_μ . The required displacement ductility of the structure is μ for a prescribed level of ground motions. Through numerous investigations of R_μ , these equation parameters are widely used in seismic design.

However, the cumulative damage of nonlinear cycles also plays a significant role in determining the damage level of a structure. Some studies suggest that cumulative damage can be considered by modifying the ductility capacity, such as the equivalent ductility method or introducing a weighted ductility factor. These methods indirectly take into account the influence of cumulative damage. Some other studies consider the cumulative damage directly by employing a damage model in the determination of the seismic demand for a given damage level or performance level, the strength reduction factor obtained by this method is referred to as the damage-based strength reduction factor R_D , and R_D is defined as:

$$R = \frac{F_e}{F_{y,D}} = \frac{F_e(\mu = 1, D = 0)}{F_y(\mu = \mu_i, D = D_j)} \quad (2)$$

where $F_{y,D}$ is the inelastic strength demand to limit the inelastic response of the structure to a specified damage level D_j for a given ductility capacity μ_i . In this manuscript, the performance levels of a structure are defined using a damage index to take the cumulative damage of the structure into consideration. The Park-Ang model [8]



is employed, which consists of a linear combination of normalized historical maximum displacement and hysteretic energy dissipation.

Damage level of structures during an earthquake is divided into five levels of performance: 1) *Operational*, 2) *Immediate Occupancy*, 3) *Damage Control*, 4) *Life Safety* and 5) *Collapse Prevention*. By associating the damage levels with the damage index range of the Park–Ang damage model, the range of the damage index for each performance level may be given as shown in Table 2.

Table 2 – Damage index ranges for different performance levels

Performance level	Degree of damage	Damage index
Operational	Negligible	$0 < D < 0.2$
Immediate occupancy	Minor	$0.2 < D < 0.4$
Damage control	Moderate	$0.4 < D < 0.6$
Life Safety	Severe	$0.6 < D < 0.9$
Collapse prevention	Near collapse	$0.9 < D < 1.0$

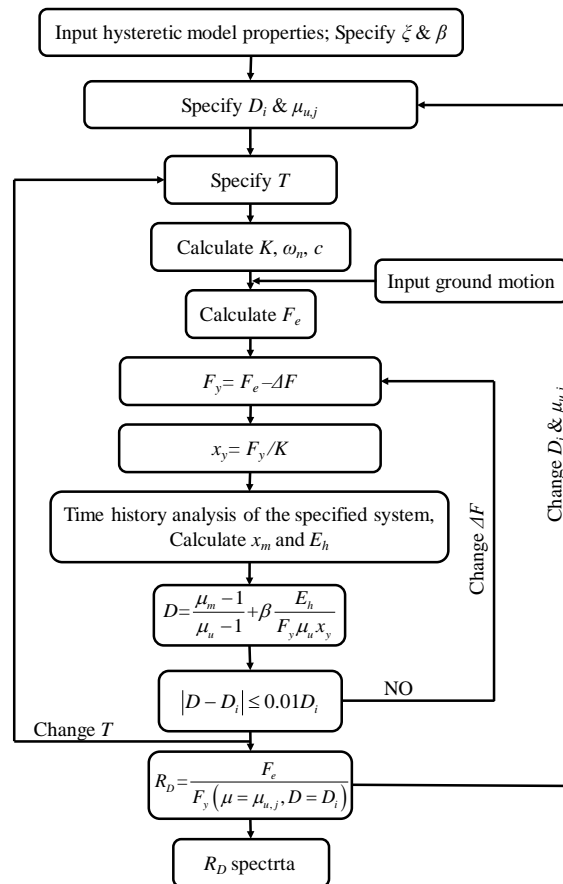


Fig. 2 – The flowchart for the computation of the strength reduction factor

For any ground motion input, a set of R_D spectra can be constructed for various levels of damage with a ductility capacity. Fig 2 shows the computational flowchart of the R_D factor. R_D is calculated by gradually reducing the applied strength from the corresponding elastic strength demand F_e until the specified D is achieved



within a tolerance (1% was used in this study). Based on the statistical results of R_D , the predictive model is expressed as:

$$R_D = 1 + D^{1.1} [0.2 \cdot (\mu - 1)]^{0.85} \frac{aT}{1 + bT + cT^2} \quad (3)$$

where the coefficients a , b and c are summarized in Table 3 for the cases of seismic sequences under consideration. Furthermore, the proposed empirical relation should satisfy the following boundary conditions:

$$R_D(T \rightarrow 0, D, \mu_u) = 1 \quad (4)$$

$$R_D(T, D = 0, \mu_u) = 1 \quad (5)$$

$$R_D(T, D, \mu_u = 1) = 1 \quad (6)$$

$$R_D(T \rightarrow \infty, D, \mu_u) = \tilde{R}_D \quad (7)$$

Table 3 – The values of $a-c$

Parameters		a	b	c
Site Class B	Single earthquake	19.96	6.41	-0.18
	Sequence-type earthquake	14.76	5.84	-0.28
Site Class C	Single earthquake	15.51	5.52	-0.30
	Sequence-type earthquake	9.41	3.60	-0.23

2.4 Modified YPS

While the elastic spectra are available, the yield strengths of structure corresponding to the specified displacement ductility and damage index can be determined approximately using R_D . Average yield strength can be calculated by Eq. (1) and the average yield strengths. Then, the modified YPS of nonlinear structures at the specified displacement ductility and damage index can be obtained. The modified YPS includes the constant ductility YPS and the constant damage YPS, as shown in Fig. 3 and Fig. 4.

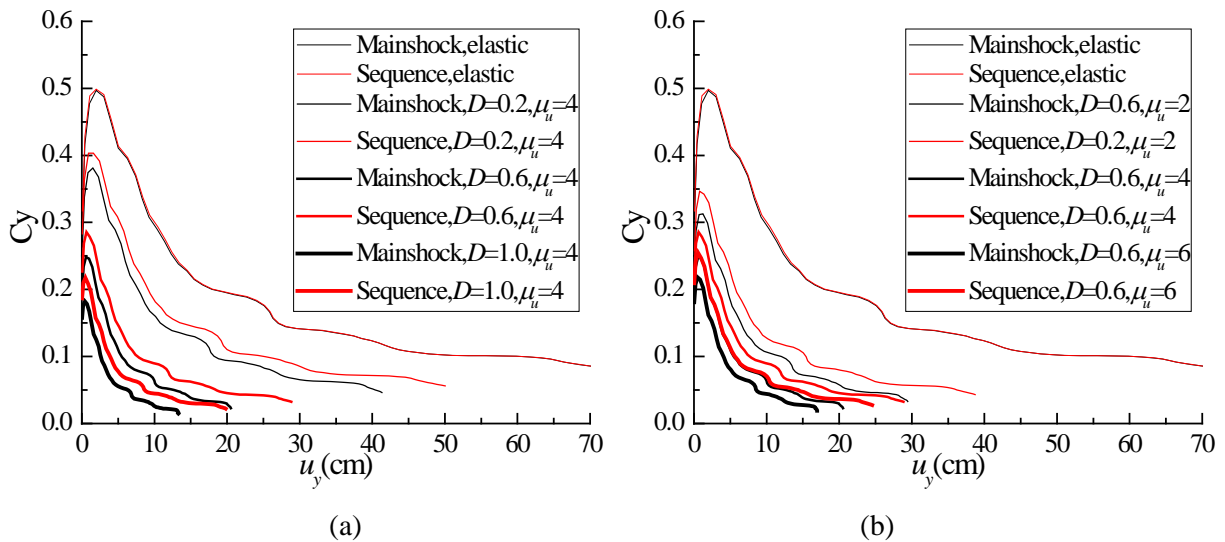


Fig. 3 – Modified YPS of Site Class B under mainshock–aftershock sequence-type ground motions and mainshock ground motions: (a) $\mu_u = 4$, (b) $D = 0.6$



Fig. 3a and Fig. 4a illustrate the constant ductility YPS of Site Class B and C, respectively. The two figures represent the YPS of elastic perfectly plastic systems when ductility is 4 and the damage indices are 0.4, 0.6 and 1.0. The yield strength demand of a structure decreases as the target damage index increases while ductility is constant. That is to say, with the same ductility, structural yield strength increases causing the damage index decrease, which results in less structural damage. The structure will stay at the elastic stage and suffer no damage when the yield strength of the structure is equal to or greater than the strength demand of the elastic spectrum. The spectra in Fig. 3b and Fig. 4b can be called the constant damage YPS, which represents the YPS of elastic perfectly plastic systems when the damage index is 0.6 and ductility readings are 2, 4 and 6 on Site Class B and C, respectively. Under same target damage index, the demand of yield strength decreases with the ductility of structure increases. This is consistent with the actual situation where the seismic performance of the structures with abundant ductility is better than that of the structures with poor ductility.

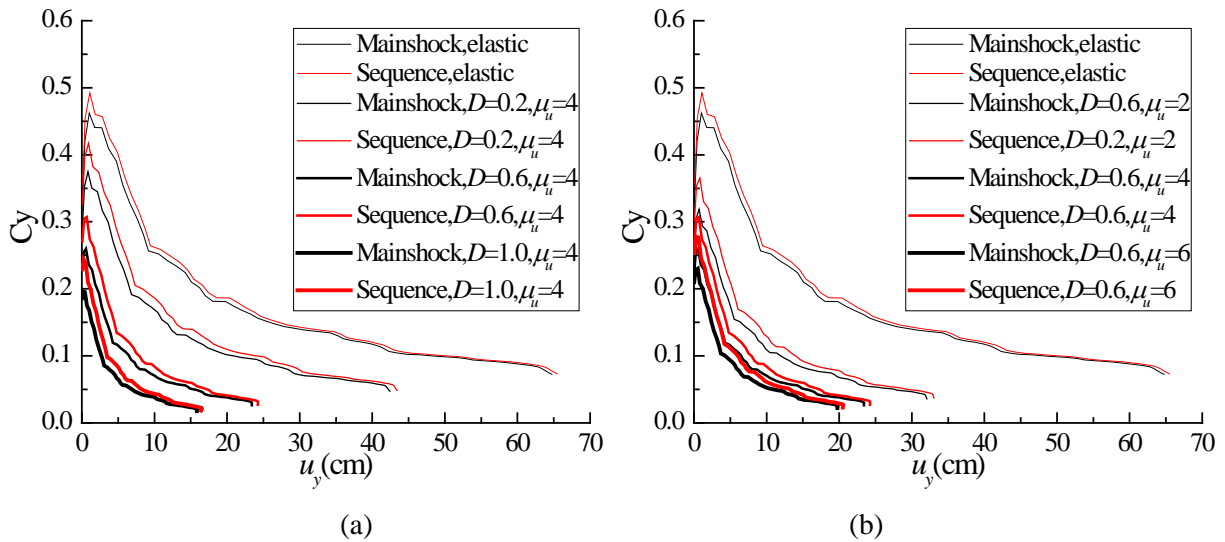


Fig. 4 – Modified YPS of Site Class C under mainshock–aftershock sequence-type ground motions and mainshock ground motions: (a) $\mu_u=4$, (b) $D=0.6$

Without considering the impact of damage, the difference between the yield strength demand of mainshock–aftershock and the yield strength demand of mainshock is less than 10% and can be ignored in seismic design and evaluation of structures; that is to say, the influence of aftershock is negligible. Nevertheless, the result did not correspond to the actual situation where the impact of aftershock was significant in the historical earthquake damage report. Taking into account both inductility and cumulative damage, the yield strength demand difference between mainshock–aftershock $F_{y,D,ma}$ and mainshock $F_{y,D,m}$ was great, and the ratio was between 1.1 and 1.4 under different conditions. With the same ductility, the ratio between $F_{y,D,ma}$ and $F_{y,D,m}$ increases with the increase of damage index of structures; moreover, with the same target damage index, the ratio between $F_{y,D,ma}$ and $F_{y,D,m}$ increases as ductility increases in structures.

Table 4 – The ratio of $F_{y,D,ma}/F_{y,D,m}$ under different ductility ratios and damage indices

	$\mu=4$			$D=0.6$		
	$D=0.2$	$D=0.6$	$D=1.0$	$\mu=2$	$\mu=4$	$\mu=6$
Site Class B	1.15	1.30	1.36	1.22	1.29	1.33
Site Class C	1.13	1.20	1.24	1.16	1.20	1.22



4. Application to Performance-Based Seismic Design

The purpose of this section is to illustrate performance-based seismic design using simple graphical constructions in conjunction with modified YPS. Performance objectives generally indicate performance limits in terms of peak displacement, maximum story drift, ductility and damage index, as well as other parameters. A four-storey reinforced concrete (RC) frame building, which was subjected to mainshock-aftershock sequence-type ground motions, was analyzed with the two performance levels of the structure remaining operational with damage control as described in Table 5. The performance level used in this example is associated with earthquake level, peak drift, damage index and system ductility. The values of these parameters are hypothetical values. It was not the objective of this paper to recommend ground motions, drifts, ductility, damage indices, or other values needed for performance-based design.

Table 5 – Performance objective of the 4-storey RC frame

Performance objective	Operational	Damage Control
Earthquake Level	0.2 g	0.4 g
Peak drift	1%	2%
Damage index	0.2	0.6
System ductility	2	6

Each storey of the RC frame was 3.8 m high. The building responded predominantly in a "first mode" and lacked the irregularities that could generate a significant torsional response. The first step in this research was to convert the MDOF system into that of an equivalent SDOF system. The mode-participation coefficient was 1.35. The yield strength coefficient and yield displacement of the equivalent SDOF system were 0.41g and 8.5 cm, respectively.

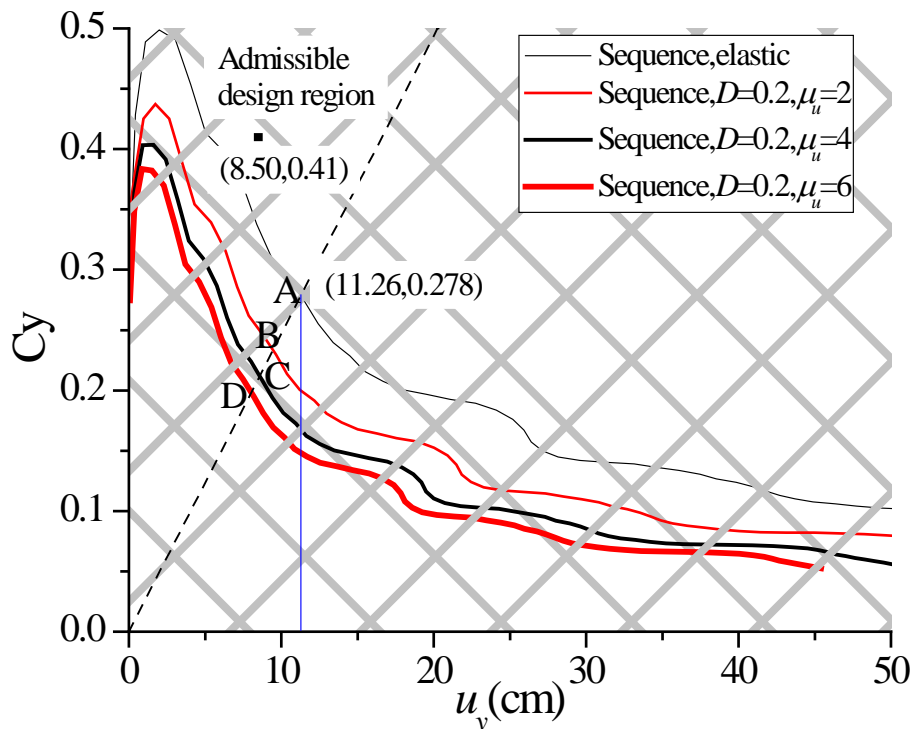


Fig. 5 – Admissible design region of the operational performance level subjected to sequence-type ground motions

The second step was to consider the operational performance level. A 1% drift corresponds to a peak roof displacement of 15.2 cm. So, the peak equivalent SDOF displacement was $15.2/1.35 = 11.26$ cm. A family of points is plotted on Fig. 5, where each point having the property of its yield displacement and displacement ductility equaled the peak displacement of 11.26 cm. Therefore, Point A indicates that the yield displacement of the elastic system was 11.26 cm. Point B corresponds to a ductility of 2 and the yield displacement was 5.63 cm. Points C and D corresponded to ductility readings of 4 and 6. The shaded areas in Figure 5 represent the inadmissible region, which is surrounded by the radial, the curve of ductility 2 and the abscissa.

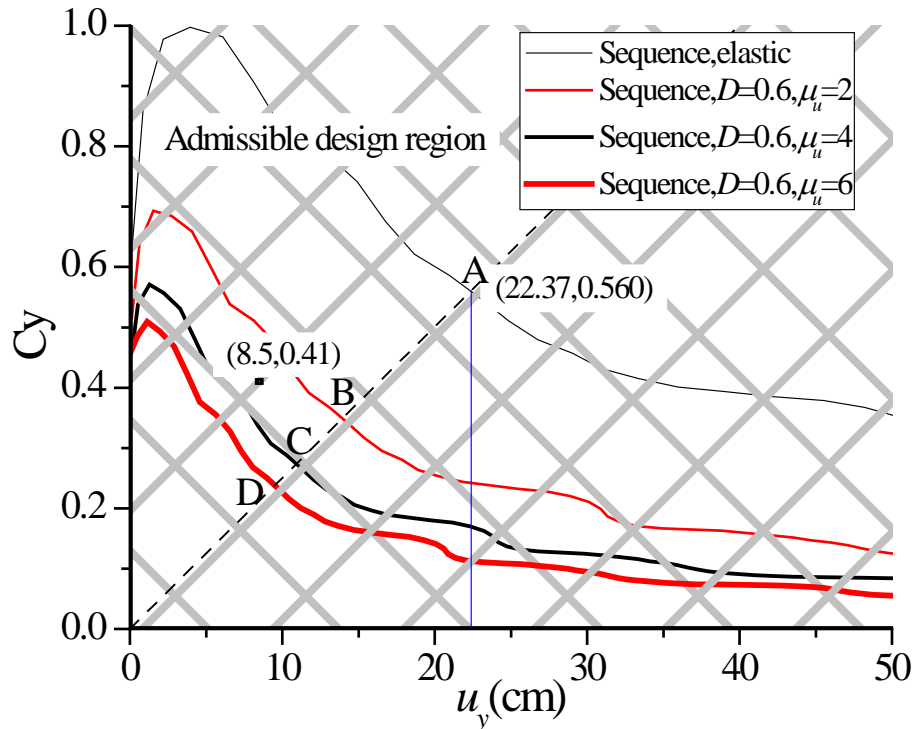


Fig. 6 – Admissible design region of the damage control performance level subjected to sequence-type ground motions

The method of constructing the admissible design region of the damage control performance level is the same as that of the operational performance level. To verify the capacity of the 4-storey RC frame, the yield point of the 4-storey RC frame is plotted in Fig. 5 and Fig. 6, and the yield point falls on the admissible design region. Thus, the 4-storey RC frame satisfies the performance objectives.

5. Conclusions

The purpose of this investigation was to introduce the modified yield point spectra for the mainshock–aftershock sequence-type ground motions. Based on time-history analysis of SDOF systems, the elastic spectra of the sequence-type earthquake and single earthquake were obtained. Then, regression formulas were constructed for the mean R_D spectra, which took into consideration the target damage limits for multiple performance levels as a function of the natural period of the system, the damage level and the system ductility. Finally, modified yield point spectra were proposed to research the strength demand under mainshock ground motions and mainshock-aftershock sequence-type ground motions. The following conclusions are drawn from this investigation:

(1) Compared to the yield strength $F_{y,D,m}$ of the structure subjected to mainshock ground motions, the yield strength $F_{y,D,ma}$ of the structure subjected to mainshock-aftershock ground motions increased by 10% to 40%. Thus, the impact of the aftershock can't be ignored.



(2) With same ductility, the ratio between $F_{y,D,ma}$ and $F_{y,D,m}$ increased with the increase of the damage indices of structures; with same target damage index, the ratio between $F_{y,D,ma}$ and $F_{y,D,m}$ increased with the increased ductility of structures.

(3) The modified YPS can be used to assess the structures subjected to different intensity earthquakes. Admissible design region of strength and stiffness were obtained to satisfy performance-based design objectives.

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