16th World Conference on Earthquake Engineering, 16WCEE 2017

Santiago Chile, January 9th to 13th 2017

Paper N° 805

Registration Code: S-N1464709120

Evaluation of the dynamic response of a multi-layered poroelastic soil through the 'Thin Layer Method, TLM'

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Abstract

In this paper, dynamic response of a multi-layered poroelastic soil is studied through a semi analytical method named "Thin Layer Method, *TLM*". It is a semi-analytical method, through which the treatment of very complex problems becomes accessible without having to resort to numerical methods which have a major drawback in the presence of infinite geometries. The calculation method of the free field response of the layered soil is presented. The soil is subjected to the obliquely incident body wave. The interface stress between impervious elastic half-space and layered soil is estimated as the function of incident angle.

Some poroelastic multilayered soil profile amplifications as well as a parametric study are presented in order to analyses the effect of the incidence angle, the porosity and the saturation degree on the profile response.

Results show that porosity and the saturation degree variation affect mainly the horizontal amplification. This is true in the case of P_1wave (fast compressional wave) incidence. When SV wave incidence is considered, both horizontal and vertical responses are influenced but this influence is less important. This is explained by the fact that only compressional waves propagate in the interstitial pores. However, the effect of the incidence angle on the site response is important for both $P_1$and SV wave excitation, but it's more important in the case of SV-wave than in the case of P1-wave.

Keywords: Porous media, Thin Layer Method (TLM), Wave propagation, Saturation degree.

1. Introduction

Many materials encountered in civil, geophysical and biomechanical engineering can be considered as porous media consisting of an assemblage of solid particles and a pore space. The pore space may be filled with air (dry medium), a fluid (saturated medium) or both (unsaturated medium) [1].

The study of wave propagation in layered media has received considerable attention, especially in the context of exploration geophysics, seismology and engineering. The restriction to linear problems defined on horizontally layered media allows the use of integral transforms and the formulation of a layer and half space stiffness matrix that can be used in an exact stiffness formalism [2]. In spite of the aforementioned restrictions, this formalism allows for the solution of various important problems such as site amplification of plane harmonic waves, dispersion and attenuation of surface waves and harmonic and transient wave propagation due to a forced excitation [1].

The exact stiffness matrix, is a symmetric, which provides a significant benefit in reducing the number of operations. however, the calculations, necessarily involves a not always obvious numerical integration. An alternative method is the Thin-Layer Method (*TLM*) developed by Kausel et Peek [3]. The *TLM* is an effective numerical tool for the analysis of wave motions in laminated media. In a nutshell, the *TLM* combines the finite element method in the direction of layering together with analytical solutions for the remaining directions.

Based on Biot's theory, the soil profile is modelled as a two-phase porous medium system consisting of an elastic skeleton and an incompressible fluid phase. Firstly, the calculation method of free field response of the layered soil is presented. The soil is subjected to the obliquely incident body wave, where the interface stress between elastic half space and fluid-filled poroelastic layers soil is estimated as the function of incident angle. secondly, the amplification function is calculated for various incident angles. Finally, the amplification function is calculated for various parameters such as porosity and saturation degree.

2. Seismic wave propagation through porous media

2.1. Governing equations

Formulation of stresses in impervious elastic half-space:

Consider a layered system as shown in Fig. 1 layered soil is supported on impervious elastic half space, and subjected to obliquely incident body wave, we define the displacement vector $\boldsymbol{U}$ and stress vector $\boldsymbol{S}$ in elastic half-space as

$$\boldsymbol{U} = \{u_x \quad u_y \quad iu_z\} \tag{1}$$

$$\boldsymbol{S} = \{\tau_{xx} \quad \tau_{yz} \quad i\sigma_z\} \tag{2}$$

In case that the plane waves are propagation in elastic half-space, $\boldsymbol{U}$ and $\boldsymbol{S}$ are given as follows

$$\begin{Bmatrix} \boldsymbol{U} \\ \boldsymbol{S} \end{Bmatrix} = \begin{Bmatrix} \overline{\boldsymbol{U}} \\ \overline{\boldsymbol{S}} \end{Bmatrix} exp\{i(\omega t - kx - ly)\} \tag{3}$$

Where ω is the circular frequency, k and l are the wave numbers of x-direction and y-direction, respectively. We can set $l = 0$ without loss of generality. $\overline{\boldsymbol{U}}$ and $\overline{\boldsymbol{S}}$ are the functions of only z, and the components are

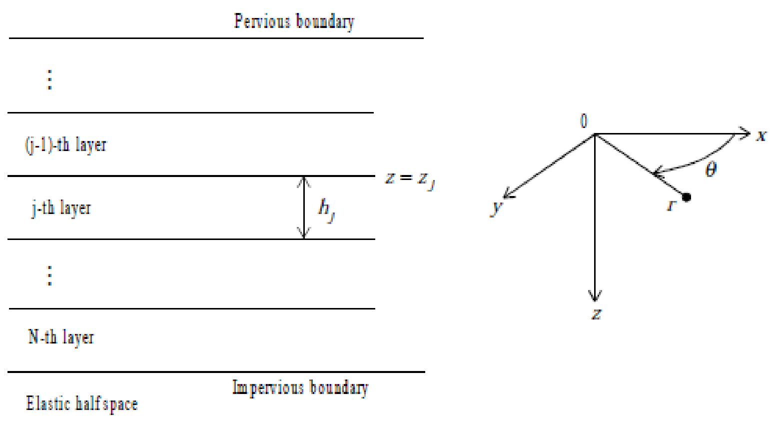

Fig. 1–Coordinate system for fluid-filled poroelastic layers

$$\overline{\boldsymbol{U}} = \{\bar{u}_x \quad \bar{u}_y \quad i\bar{u}_z\} \tag{4}$$

$$\overline{\boldsymbol{S}} = \{\bar{\tau}_{xx} \quad \bar{\tau}_{yz} \quad i\bar{\sigma}_z\} \tag{5}$$

According to Kausel and Roesset [2], the relationship of the displacement $\boldsymbol{U}$ and the stress $\boldsymbol{S}$ at the interface between layered soil and lower half space is

$$\overline{\boldsymbol{S}}|_{z=0} = \boldsymbol{K}\,\overline{\boldsymbol{U}}|_{z=0} \tag{6}$$

where $\boldsymbol{K}$ is the stiffness matrix.

In case on descending wave (radiation wave), x and z component of stiffness matrix $\boldsymbol{K}$, which is related to P-wave and SV wave case, is

$$\boldsymbol{K}_1 = 2G\,k\left[\frac{1-s^2}{2(1-ps)}\begin{Bmatrix} p & 1 \\ 1 & s \end{Bmatrix} - \begin{Bmatrix} 0 & 1 \\ 1 & 0 \end{Bmatrix}\right] \tag{7}$$

the y-component of stiffness matrix $\boldsymbol{K}$, which corresponds to SH-wave case is

$$\boldsymbol{K}_1 = G\,ks \tag{8}$$

where

$$p = \sqrt{1-\left(\frac{\omega}{k\,V_P}\right)^2}, \qquad s = \sqrt{1-\left(\frac{\omega}{k\,V_s}\right)^2} \tag{9}$$

where G is the shear modulus, V_s and V_p are S and P-wave velocities of the half-space soil, respectively.

In case of ascending wave (incident wave), the x and z-component of stiffness matrix $\boldsymbol{K}$, which indicates P and SV-wave case, is

$$\boldsymbol{K}_0 = 2G\,k\left[\frac{1-s^2}{2(1-ps)}\begin{Bmatrix} -p & 1 \\ 1 & -s \end{Bmatrix} - \begin{Bmatrix} 0 & 1 \\ 1 & 0 \end{Bmatrix}\right] \tag{10}$$

the y-component of stiffness matrix $\boldsymbol{K}$, which corresponds to SH-wave case is

$$\boldsymbol{K}_0 = -G\,ks \tag{11}$$

decomposing the displacement vector $\overline{\boldsymbol{U}}$ into the component by incident wave $\overline{\boldsymbol{U}}_0$ and the component by radiation wave $\overline{\boldsymbol{U}}_1$, Eq. (6) can be described as follows

$$\overline{\boldsymbol{S}}|_{z=0} = \boldsymbol{K}_0\,\overline{\boldsymbol{U}_0}|_{z=0} + \boldsymbol{K}_1\,\overline{\boldsymbol{U}_1}|_{z=0} = -(\boldsymbol{K}_1 - \boldsymbol{K}_0)\,\overline{\boldsymbol{U}_0}|_{z=0} + \boldsymbol{K}_1\,(\overline{\boldsymbol{U}_0}|_{z=0} + \overline{\boldsymbol{U}_1}|_{z=0}) \tag{12}$$

so we can write

$$\overline{\boldsymbol{S}}|_{z=0} = -\boldsymbol{F} + \boldsymbol{K}_1\,\overline{\boldsymbol{U}}|_{z=0} \tag{13}$$

$$\boldsymbol{F} = (\boldsymbol{K}_1 - \boldsymbol{K}_0)\,\overline{\boldsymbol{U}}_0|_{z=0} \tag{14}$$

it can be seen from Eq. (14) that $\boldsymbol{F}$ depends only on the displacement wave, $\boldsymbol{F}$ is calculated as following.

2.2. Amplification function

In case of incident P and SV-wave, the incident displacement $\boldsymbol{U}_0$ is described by using potential function $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, which are associated with P and SV-wave motions, as

$$\boldsymbol{U}_0 = \begin{Bmatrix} u_{0x} \\ iu_{0z} \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & -\frac{\partial}{\partial z} \\ i\frac{\partial}{\partial z} & i\frac{\partial}{\partial x} \end{bmatrix}\begin{Bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\psi} \end{Bmatrix} \tag{15}$$

where u_{0x}, u_{0z} are x and z component of incident displacement field $\mathbf{U}_0$. Potential function $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, are

$$\begin{Bmatrix} \boldsymbol{\phi} \\ \boldsymbol{\psi} \end{Bmatrix} = \begin{Bmatrix} a\;exp(krz) \\ \gamma\;exp(ksz) \end{Bmatrix} exp\;i(\omega t - kx) \tag{16}$$

where a,γ are x the amplitude of potential function $\boldsymbol{\phi}$ and $\boldsymbol{\psi}$, respectively. Substitute of Eq. (16) into Eq. (15) and by using Eq. (3), yield

$$\overline{\boldsymbol{U}}_0 = \begin{Bmatrix} \bar{u}_{0x} \\ i\bar{u}_{0z} \end{Bmatrix} = \begin{bmatrix} -ik & -ks \\ ikr & k \end{bmatrix} \begin{Bmatrix} a\ exp(krz) \\ \gamma\ exp(ksz) \end{Bmatrix} \tag{17}$$

where $\bar{u}_{0x}$, $\bar{u}_{0z}$ are x and z component of incident displacement field $\overline{\mathbf{U}}_0$. $\mathbf{F}$ is obtained by the substitution of Eq. (17) in Eq. (14) as follows

$$\boldsymbol{F} = 2k^2 G\ \frac{1-s^2}{1-s} \begin{bmatrix} -ir & -rs \\ irs & s \end{bmatrix} \begin{Bmatrix} a \\ \gamma \end{Bmatrix} \tag{18}$$

Calculation method of displacement in fluid-filled poroelastic layers: The displacement vector $\overline{\boldsymbol{U}}$ of thin layer interface and load vector $\overline{\mathbf{P}}$ of thin layer interface is related as follows

$$(\boldsymbol{A}k^2 + \boldsymbol{B}k + \boldsymbol{C})\overline{\boldsymbol{U}} = \overline{\boldsymbol{P}} \tag{19}$$

where $$\{\overline{\boldsymbol{U}}\} = \{\overline{\boldsymbol{U}}_1, \dots\dots, \overline{\boldsymbol{U}}_{n+1}\}^T \qquad \{\overline{\boldsymbol{P}}\} = \{\overline{\boldsymbol{P}}_1, \dots\dots, \overline{\boldsymbol{P}}_{n+1}\}^T \tag{20}$$

where n is the number of thin layers.

Matrix **A, B**, and **C** are given by [3]. when layered soil is subjected to obliquely incident body wave, can represent the interface load between half-space and layered soil through Eq. (13), and so we can set as.

$$\overline{\boldsymbol{P}}_{n+1} = -\boldsymbol{S} = \boldsymbol{F} - \boldsymbol{K}_1 \overline{\boldsymbol{U}}_{n+1} \qquad \overline{\boldsymbol{P}}_j = 0 \quad j = 1, \dots\dots. n+1 \tag{21}$$

Eq. (19) is then described as

$$(\boldsymbol{A}k^2 + \boldsymbol{B}k + \boldsymbol{C}_1 + \boldsymbol{C}_2 + \boldsymbol{K}_W^1) \begin{Bmatrix} \overline{\boldsymbol{U}}_1 \\ \overline{\boldsymbol{U}}_{n+1} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \boldsymbol{F} \end{Bmatrix} \tag{22}$$

$\boldsymbol{K}_W^1$ and $\boldsymbol{F}$ can be assumed as a generalized dashpot effect and external load effect, respectively, $\{\overline{\boldsymbol{U}}\}$ can be obtained by solving matrix Eq. (22) directly.

Table 1 – Elements of matrix equations [4]

$\boldsymbol{A} = \frac{h}{6}\begin{bmatrix} -\frac{2}{F}n^2 & . & . & -\frac{1}{F}n^2 & . & . \\ . & 2G & . & . & 2G & . \\ . & . & 2(\lambda+2G) & . & . & 2(\lambda+2G) \\ -\frac{1}{F}n^2 & . & . & -\frac{2}{F}n^2 & . & . \\ . & 2G & . & . & 2G & . \\ . & . & 2(\lambda+2G) & . & . & 2(\lambda+2G) \end{bmatrix}$	$\boldsymbol{B} = \frac{1}{6}\begin{bmatrix} . & . & 2nT & . & . & 2nT \\ . & . & -3(\lambda-G) & . & . & -3(\lambda+G) \\ 2nT & 3(\lambda-G) & . & 2nT & -3(\lambda+G) & . \\ . & . & 2nT & . & . & 2nT \\ . & . & -3(\lambda+G) & . & . & -3(\lambda-G) \\ 2nT & 3(\lambda+G) & . & 2nT & -3(\lambda-G) & . \end{bmatrix}$
$\boldsymbol{C}_1 = \begin{bmatrix} -\frac{hn^2}{3R} & \frac{(2R+Q)n}{2R} & . & -\frac{hn^2}{6R} & -\frac{Qn}{2R} & . \\ \frac{(2R+Q)n}{2R} & -\frac{\lambda+2G}{h} & . & \frac{Qn}{2R} & -\frac{(\lambda+2G)}{h} & . \\ . & . & \frac{G}{h} & . & . & -\frac{G}{h} \\ -\frac{hn^2}{6R} & RRR\frac{Qn}{2R}RR & . & -\frac{hn^2}{3R} & -\frac{(2R+Q)n}{2R} & . \\ R-\frac{Qn}{2R}RR & -\frac{(\lambda+2G)}{h} & . & -\frac{(2R+Q)n}{2R} & \frac{(\lambda+2G)}{h} & . \\ . & . & -\frac{G}{h} & . & . & \frac{G}{h} \end{bmatrix}$	$\boldsymbol{C}_2 = \begin{bmatrix} -\frac{n^2}{hF} & -\frac{i\omega bn}{2F} & . & \frac{n^2}{hF} & -\frac{i\omega bn}{2F} & . \\ -\frac{i\omega bn}{2F} & -\frac{h}{3}M & . & \frac{i\omega bn}{2F} & \frac{h}{6}M & . \\ . & . & \frac{h}{3}M & . & . & \frac{h}{6}M \\ -\frac{hn^2}{6R} & \frac{i\omega bn}{2F} & . & -\frac{n^2}{hF} & \frac{i\omega bn}{2F} & . \\ -\frac{i\omega bn}{2F} & \frac{h}{6}M & . & \frac{i\omega bn}{2F} & \frac{h}{3}M & . \\ . & . & \frac{h}{6}M & . & . & \frac{h}{3}M \end{bmatrix}$

$$T = \left(\frac{Q}{R} + \frac{i\omega b}{F}\right) h$$

$$F = i\omega b - \omega^2 \rho_{22}$$

$$M = i\omega b - \rho_{11}\omega^2 + \frac{b^2\omega^2}{F}$$

Where n: porosity

λ, G : Lame's modulus of soil skeleton

β: bulk modulus of pore water

ω : circular frequency

$$R = n\beta \qquad Q = (1-n)\beta$$

$b = \rho_f g n^2 / k$ $\quad$ g: gravitational acceleration

$$\rho_{11} = (1-n)\rho \qquad \rho_{22} = n\rho_f$$

Where ρ: mass density of soil particle

ρ_f: mass density of pore water

$\hat{k}$: permeability

h: tickness of thin layer

3. Numerical examples

A fluid-filled poroelastic layered soil profile model corresponding to sand layer overlying an impervious elastic half-space are used.

The geomechanic characteristics of the two type soil nature and the half space are summarized in table (2)

Table 2 – Material properties of soil

Surface Layer 30 m	Poroelastic	Mass density of particle	(t/m^3)	2.65
		Shear modulus of soil skeleton	(t/m^2)	5000
		Poisson's ratio of soil skeleton	-	0.4
		Porosity	-	0.45
		Permeability	(m/sec)	$1.0x10^{-6}$
		Bulk modulus of pore water	(t/m^2)	$2.2\ x10^{-6}$
Half space	Elastic	Mass density	(t/m^3)	2.65
		Shear modulus	(t/m^2)	12000
		Poisson's ratio	-	0.3

3.1. Incidence of P_1wave

Fig. 2 show the horizontal and vertical amplification functions corresponding in the case of P_1-wave excitation for various angles of incidence.

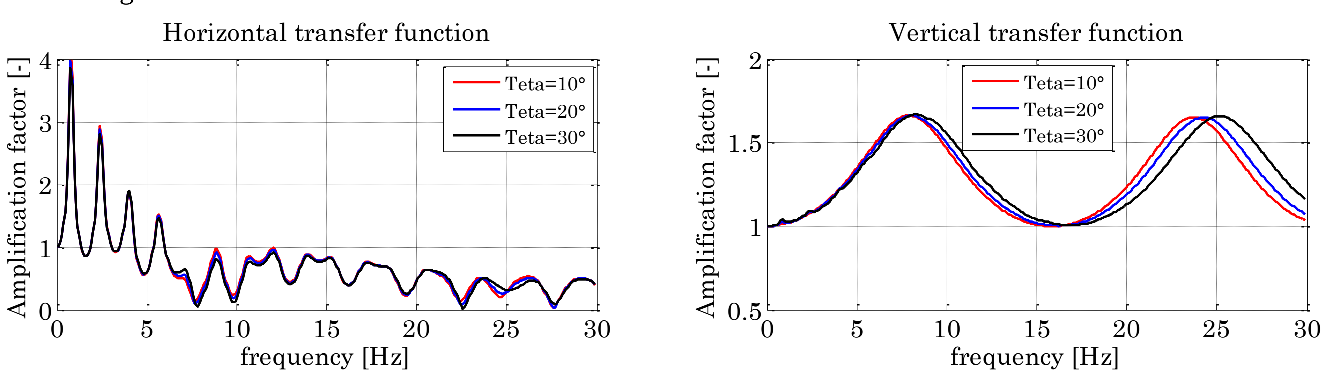

Fig. 2 – Amplification functions of the sand layer for various angles of incidence of the P_1wave

3.2. Incidence of SV wave

Fig. 3 show the horizontal and vertical amplification functions corresponding in the case of SV-wave excitation for various angles of incidence.

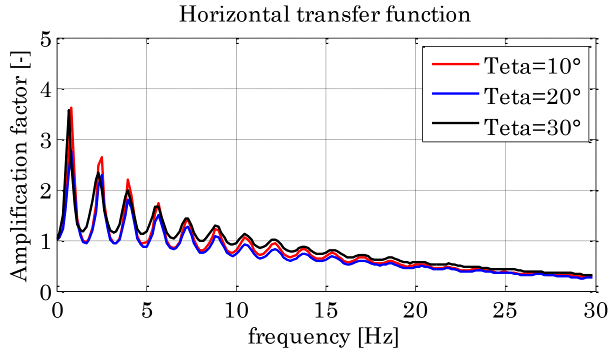

Fig. 3 – Amplification functions of the sand layer for various angles of incidence of the SV wave

4. Parametric Study

In this section a parametric study is presented in order to analyses the effect of the variation on the porosity and the saturation degree on the response on de proelastic soil profile.

4.1. Effect of porosity

The porosity is varied from 0 to 0.5 for a completely saturated soil layer. It's clear from Fig. 4 that the increase in the porosity of the soil layer causes more significant amplification of the dynamic response. This effect is much more pronounced in the case of P_1-wave than in the case of SV-wave.

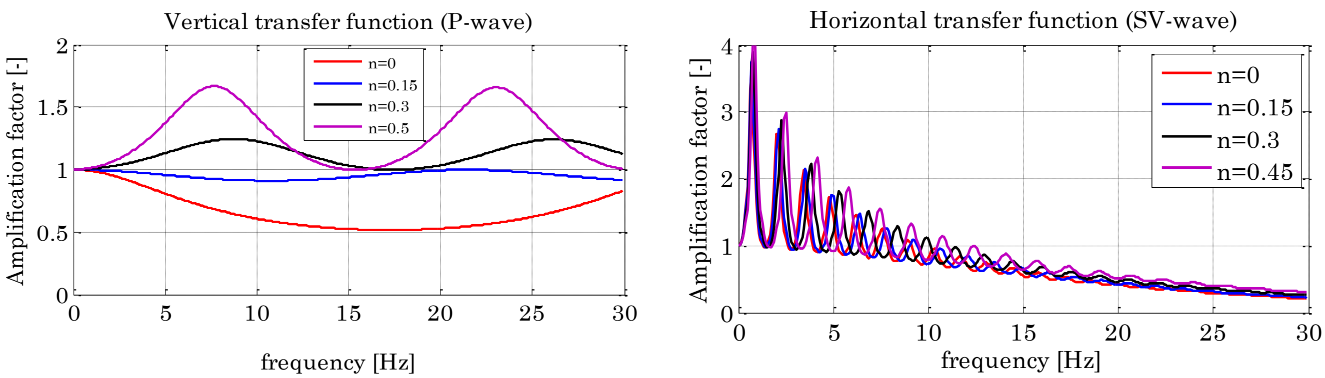

Fig. 4 – Effect of the porosity on the amplification functions – Incidence of vertical waves.

4.2. Saturation degree effect

Partially saturated soil represents a three-phase medium (mixture of solid, fluid and gas). So far there is no generally accepted theory for wave propagation in such soil medium. In this paper, a simplified approach is followed, in which is the theory for a two-phase medium is used, but with reduced bulk modulus of the fluid, as in [5, 6, 7]. Let S_r be the degree of saturation. The relative proportions of the constituent volumes are defined as

$$n = \frac{V_V}{V_T} \tag{23}$$

$$S_r = \frac{V_w}{V_V} \tag{24}$$

where n is the porosity of the soil, and V_V, V_w and V_T are respectively the volumes of pores, pore water and the total volume.

In this study, a high degree of saturation is considered > 90%, assuming the embedded air in the pore water is in the form of bubbles uniformly distributed through the fluid. In this case, the bulk modulus of fluid K_f can be written as

$$K_f = \frac{1}{\frac{1}{K_w}+\frac{1-S_r}{P_a}} \tag{25}$$

Where K_w is the bulk modulus of pore water and P_a is the absolute fluid pressure.

With the following characteristics: saturation degree varies from 95% to 100%, the amplification function in the case of vertical incidence of a P_1wave, is affected by the fluctuations of the degree of saturation (Fig. 5).

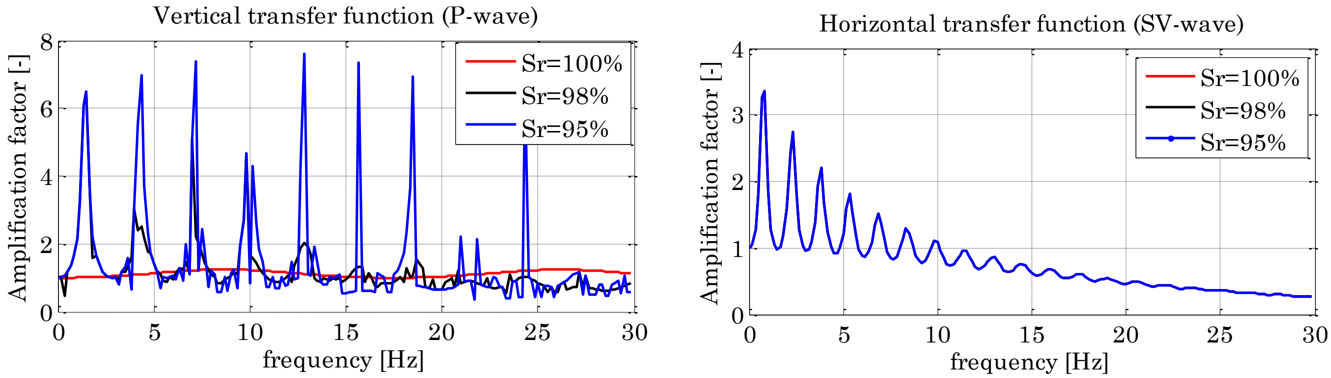

Fig. 5 – Effect of the saturation degree on the amplification functions - Incidence of vertical waves.

On the other hand, in the case of vertical incidence of a SV-wave no effect is noted. This is completely logical, since only the compressional waves propagate in the interstitial fluid.

5. Conclusion

The Thin Layer Method, TLM is used in this paper to analyze the effect of the incidence angle, the porosity and the saturation degree on the site amplification of a poroelastic multilayered soil profile. This method to calculate the free field response of the layered soil is comprehensive in formulations, and matrix operation are easy. The soil profile is modelled as a two-phase porous medium system consisting of a viscoelastic skeleton and an incompressible fluid phase.

The principal conclusions of the parametric study are:

- The porosity and the saturation degree variation affect mainly the horizontal amplification. This is true in the case of P_1wave incidence. When SV wave incidence is considered, both horizontal and vertical responses are influenced but this influence is less important.
- The effect of the incidence angle on the site response is important for both P_1 and SV wave excitation, but it's more important in the case of SV-wave.

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